Philadelphia University
Faculty of Science
Department of Mathematics

Academemic Year: *2023–2024*
Semester: *First Semester*
Exam: *Mid-Term Exam*
Instructor: *Dr.Hani Qwareeq*
Dr.Abdullah Alsoboh

Course Name: *Calculus 3*
Course Number: *250202*
Duration: *9:45–10:45 am*
Date: *29/11/2023*
Section $1, 2, 3, 4$

Student Name:..... **Student ID:**.....

This exam consists of 4 PAGES, 2 QUESTIONS and (30+3 Bonus) total marks.
Please show all work needed to arrive at your solutions. GOOD LUCK

Question #1: [22 **marks**] **T**his question consists of 11 multiple choice questions (2 Mark for each), where each question has 5 options, only one of which is correct. Put the answer symbol in the table below.

Question #	1	2	3	4	5	6	7	8	9	10	11
Answer Symbol											

1. The domain of $\vec{r}(t) = \sqrt{5+t}\,\vec{i} + \sqrt{5-t}\,\vec{j} + t^2\,\vec{k}$ is

   a) $t \leq 5$ b) $|t| \leq 5$ c) $|t| \geq 5$ d) $t \geq 5$ e) None of the above

2. One of the following equations represent the equation of plane through the point $P(2, -10, 3)$ and perpendicular to the line

$$x = 5 + 4t, y = 2 + 2t, z = 1 + 6t$$

   a) $4x + 2y + 6z = 6$ b) $4x + 2y + 6z = -5$

   c) $4x + 2y + 6z = 12$ d) $4x + 2y + 6z = -6$ e) None of the above

3. The radius of the sphere described by the equation $(x+3)^2 + y^2 + 2y + (z-10)^2 = 3$.

   a) 2 b) 1 c) $\sqrt{3}$ d) 3 e) None of the above

4. The distance from the point $\mathcal{S}(5, 1, 4)$ to the plane $2x + 2y + z = 3$ is

   a) $\frac{11}{9}$ b) $\frac{13}{9}$ c) $\frac{11}{3}$ d) $\frac{13}{3}$ e) None of the above

5. Give a geometric description of the set of points whose coordinates satisfy the given condition $x^2 + y^2 + z^2 > 1$

   a) All points outside the cylinder with radius 1
   b) All points outside the sphere of radius 1
   c) All points inside the sphere of radius 1
   d) All points in space
   e) None of the above

6. The unit vector $\overrightarrow{\mathbf{u}}$ that have the same direction of $\overrightarrow{\mathbf{v}} = \langle 4, 1, 2\sqrt{2} \rangle$ is

   a) $\left\langle \frac{4}{5}, 1, \frac{2\sqrt{2}}{5} \right\rangle$ b) $\left\langle \frac{4}{5}, \frac{1}{5}, \frac{2\sqrt{2}}{5} \right\rangle$ c) $\left\langle \frac{4}{7}, 1, \frac{2\sqrt{2}}{7} \right\rangle$ d) $\left\langle \frac{4}{7}, \frac{1}{7}, \frac{2\sqrt{2}}{7} \right\rangle$ e) None of the above

7. If $\langle 3\mathbf{a} - 1, 6, 2\mathbf{c} + 1 \rangle = \langle -4, -2\mathbf{b}, \mathbf{c} \rangle$. Then, the values of **a**, **b** and **c** are

   a) $\mathbf{a} = -1, \mathbf{b} = 3, \mathbf{c} = -\mathbf{1}$ b) $\mathbf{a} = 1, \mathbf{b} = -3, \mathbf{c} = \mathbf{1}$
   c) $\mathbf{a} = -1, \mathbf{b} = -3, \mathbf{c} = -\mathbf{1}$ d) $\mathbf{a} = -1, \mathbf{b} = -3, \mathbf{c} = \mathbf{1}$
   e) None of the above

8. The symmetric equations of the line given by

   $$L : x = 1 + 3t, y = 3 - 4t, z = 1 - 2t \quad \text{is}$$

   a) $\frac{x-1}{3} = \frac{y-3}{-4} = \frac{z-1}{-2}$ b) $\frac{x-1}{-3} = \frac{y-3}{4} = \frac{z+1}{2}$
   c) $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-1}{2}$ d) $\frac{x+1}{3} = \frac{y-3}{-4} = \frac{z-1}{-2}$
   e) None of the above

9. Find the triple scalar product $(\vec{u} \times \vec{v}) \cdot \vec{w}$ of the given vectors :

   $$\vec{u} = \vec{i} + \vec{j} + \vec{k}; \quad \vec{v} = 8\vec{i} + 5\vec{j} + 2\vec{k}; \quad \vec{w} = 9\vec{i} + 6\vec{j} + 4\vec{k}.$$

   a) 75 b) -3 c) -9 d) 25 e) Cannot be calculated

10. Convert the spherical coordinate $S(10, \pi/2, \pi/3)$ **to rectangular coordinate**.

    a) $(5, 5\sqrt{3}, 10)$ b) $(10, 5, 5\sqrt{3})$
    c) $(0, 5\sqrt{3}, 5)$ d) $(5, 5\sqrt{3}, 0)$
    e) None of the above

11. Convert the rectangular coordinate $R(4\sqrt{3}, 4, -4)$ **to cylindrical coordinate**

    a) $\left(4, \frac{\pi}{3}, 4\right)$ b) $\left(8, \frac{\pi}{6}, 4\right)$
    c) $\left(8, \frac{\pi}{3}, -4\right)$ d) $\left(8, \frac{\pi}{6}, -4\right)$
    e) None of the above

From Cylindrical to Rectangular	**From Spherical to Cylindrical**	**From Spherical to Rectangular**
$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$	$\begin{cases} r = \rho\sin\phi \\ \theta = \theta \\ z = \rho\cos\phi \end{cases}$	$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$
From Rectangular to Cylindrical	**From Cylindrical to Spherical**	**From Rectangular to Spherical**
$\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan\theta = \frac{y}{x} \\ z = z \end{cases}$	$\begin{cases} \rho = \sqrt{r^2 + z^2} \\ \theta = \theta \\ \tan\phi = \frac{r}{z} \end{cases}$	$\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \tan\theta = \frac{y}{x} \\ \cos\phi = \frac{z}{\rho} \end{cases}$

Question #2: [11 **marks**] **A**nswer each of the following parts **/11**

1. [**6** marks] Find the traces in the plane $x = 0, x = 1$ for the quadratic equation $2x^2 + y^2 + z^2 = 4$, what is the name of this surface?

2. [**5** marks] Are these two lines parallel, perpendicular , skew?

$$L_1 : x = 1 + 7t, \quad y = 3 + t, \quad z = 5 - 3t$$
$$L_2 : x = 4 - t, \quad y = 6, \quad z = 7 + 2t$$