Chapter 7

Acid-Base Equilibria
Acid-Base Theories

• **Arrhenius theory** (Nobel Prize)

An acid is any substance that ionizes (partially or completely) in water to give *hydrogen ions, H\(^+\)* (which associate with the solvent to give hydronium ions, H\(_3\)O\(^+\))

\[
\text{HA} + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{A}^-
\]

• A base: any substance that ionizes in water to give *hydroxyl ions* OH\(^-\). Weak (partially ionized) bases generally ionize as follows:

\[
\text{B} + \text{H}_2\text{O} \rightleftharpoons \text{BH}^+ + \text{OH}^-
\]

• strong bases such as metal hydroxides (e.g., NaOH) dissociate as

\[
\text{M(OH)}_n \rightleftharpoons \text{M}^{n+} + n\text{OH}^-
\]
THEORY OF SOLVENT SYSTEMS

- solvent ionizes to give a cation and an anion; for example,
  \[ \text{2H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{OH}^- \]
  \[ \text{2NH}_3 \rightleftharpoons \text{NH}_4^+ + \text{NH}_2^- \]

- According to this theory:
  **Acid** is defined as a solute that yields the cation of the solvent
  **Base** is a solute that yields the anion of the solvent.

- Thus, \( \text{NH}_4\text{Cl} \) is a strong acid in liquid ammonia similar to \( \text{HCl} \) in water:
  \[ \text{HCl} + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^- \]
• NaNH$_2$ is a strong base in ammonia (similar to NaOH in water);
• both of these compounds ionize to give the solvent cation and anion, respectively.
• Ethanol ionizes as follows:
• $2\text{C}_2\text{H}_5\text{OH} \rightleftharpoons \text{C}_2\text{H}_5\text{OH}_2^+ + \text{C}_2\text{H}_5\text{O}^-$
• Hence, sodium ethoxide, NaOC$_2$H$_5$, is a strong base in this solvent.
BRONSTED-LOWRY THEORY

• The theory of solvent systems is suitable for ionizable solvents, but it is not applicable to acid-base reactions in nonionizable solvents such as benzene or dioXane.

• In 1923, Bronsted and Lowry separately described what is now known as the Bronsted-Lowry theory.

• This theory states that an acid is any substance that can donate a proton, and a base is any substance that can accept a proton.

• Thus, we can write a "half-reaction"

\[
\text{Acid} = \text{H}^+ + \text{Base}
\]

The acid and the base are called “Conjugate pairs”
## Bronsted acid-base reactions

<table>
<thead>
<tr>
<th>Solvent</th>
<th>Acid₁</th>
<th>Base₂</th>
<th>→</th>
<th>Acid₂</th>
<th>Base₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH₃ (liq.)</td>
<td>HOAc</td>
<td>NH₃</td>
<td>→</td>
<td>NH₄⁺</td>
<td>OAc⁻</td>
</tr>
<tr>
<td>H₂O</td>
<td>HCl</td>
<td>H₂O</td>
<td>H₃O⁺</td>
<td>Cl⁻</td>
<td></td>
</tr>
<tr>
<td>H₂O</td>
<td>NH₄⁺</td>
<td>H₂O</td>
<td>H₃O⁺</td>
<td>NH₃</td>
<td></td>
</tr>
<tr>
<td>H₂O</td>
<td>H₂O</td>
<td>OAc⁻</td>
<td>HOAc</td>
<td>OH⁻</td>
<td></td>
</tr>
<tr>
<td>H₂O</td>
<td>HCO₃⁻</td>
<td>OH⁻</td>
<td>H₂O</td>
<td>CO₃²⁻</td>
<td></td>
</tr>
<tr>
<td>C₂H₅OH</td>
<td>NH₄⁺</td>
<td>C₂H₅O⁻</td>
<td>C₂H₅OH</td>
<td>NH₃</td>
<td></td>
</tr>
<tr>
<td>C₆H₆</td>
<td>H picrate</td>
<td>C₆H₅NH₂</td>
<td>C₆H₅NH₃⁺</td>
<td>picrate</td>
<td></td>
</tr>
</tbody>
</table>
LEWIS THEORY

• Acid is a substance that can accept an electron pair.
• Base is a substance that can donate an electron pair. The latter frequently contains an oxygen or a nitrogen as the electron donor.
• Thus, non-hydrogen-containing substances are included as acids.
• Examples of acid-base reactions in the Lewis theory are as follows:
• \( H^+ \text{(solvated)} + :NH_3 \rightarrow H:NH_3^+ \)
Lewis Acid

\[
\text{AlCl}_3 + :O \rightarrow \text{Cl}_3\text{Al}:\text{OR}_2
\]

\[
\text{H} \underset{O:}{\overset{+ \text{H}^+}{\rightarrow}} \text{H}_2\text{O}:\text{H}^+
\]

\[
\text{H}^+ + :\text{OH}^- \rightarrow \text{H}:\text{OH}
\]
Acid-Base Equilibria in Water

• When an acid or base is dissolved in water, it will **dissociate**, or **ionize**.

• The amount (degree) of ionization is dependent on the strength of the acid.

• A "strong" electrolyte is completely dissociated, while a "weak" electrolyte is partially dissociated.
Strong and weak electrolytes

<table>
<thead>
<tr>
<th>Strong</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>HC$_2$H$_3$O$_2$ (acetic acid)</td>
</tr>
<tr>
<td>HClO$_4$</td>
<td>NH$_3$</td>
</tr>
<tr>
<td>H$_2$SO$_4$$^a$</td>
<td>C$_6$H$_5$OH (phenol)</td>
</tr>
<tr>
<td>HNO$_3$</td>
<td>HCHO$_2$ (formic acid)</td>
</tr>
<tr>
<td>NaOH</td>
<td>C$_6$H$_5$NH$_2$ (aniline)</td>
</tr>
<tr>
<td>NaC$_2$H$_3$O$_2$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$The first proton is completely ionized in dilute solution, but the second proton is partially ionized ($K_i = 10^{-2}$).
Dissociation of weak acids

Thermodynamic acidity constant, \( K^o_a \)

\[
\text{HOAc} + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{OAc}^-
\]

\[
K^o_a = \frac{a_{\text{H}_3\text{O}^+} \cdot a_{\text{OAc}^-}}{a_{\text{HOAc}} \cdot a_{\text{H}_2\text{O}}}
\]

In dilute solutions, the activity of water remains essentially constant, and is taken as unity at standard state.

\[
K^o_a = \frac{a_{\text{H}_3\text{O}^+} \cdot a_{\text{OAc}^-}}{a_{\text{HOAc}}}
\]
Thermodynamic autoprotolysis/Self-ionization constant

Pure water ionizes slightly, or undergoes autoprotolysis:

\[ 2\text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{OH}^- \]

\[
K_w^o = \frac{a_{\text{H}_3\text{O}^+} \cdot a_{\text{OH}^-}}{a_{\text{H}_2\text{O}}^2}
\]

\[
K_w^o = a_{\text{H}_3\text{O}^+} \cdot a_{\text{OH}^-}
\]
Molar equilibrium constant/ Acid dissociation constant, $K_a$

$$K_a = \frac{[H^+][OAc^-]}{[HOAc]}$$

HCl $\rightarrow$ H$^+$ + Cl$^-$

HOAc $\rightleftharpoons$ H$^+$ + OAc$^-$

The product of the hydrogen ion concentration and the hydroxyl ion concentration in aqueous solution is *always* equal to $1.0 \times 10^{-14}$

Except when the hydrogen ion concentration from the acid is very small, 10-6 M or less, any contribution to [H+] from the ionization of water can be neglected.

$$[H^+] [OH^-] = 1.0 \times 10^{-14}$$
A \(1.0 \times 10^{-3}\) \(M\) solution of hydrochloric acid is prepared. What is the hydroxyl ion concentration?
The pH scale

• It is more convenient to compress the acidity scale by placing it on a logarithm basis.

• The pH of a solution was defined by Sorenson as

\[
pH = -\log [H^+] \\
pOH = -\log [OH^-]
\]
\[ K_w = [H^+][OH^-] \]

\[-\log K_w = -\log[H^+][OH^-] = -\log[H^+] - \log[OH^-] \]

\[ pK_w = pH + pOH \]

\[ 14.00 = pH + pOH \]
Example

1. Calculate the pOH and the pH of a $5.0 \times 10^{-2}$ M solution of NaOH

2. Calculate the pH of a solution prepared by mixing 2.0 mL of a strong acid solution of pH 3.00 and 3.0 mL of a strong base of pH 10.00.
Example

• The pH of a solution is 9.67. Calculate the hydrogen ion concentration in the solution.

\[
[H^+] = 10^{-pH}
\]
**pH and acidity and alkalinity**

- When \([H^+] = [OH^-]\), the solution neutral.
- If \([H^+] > [OH^-]\), the solution acidic.
- If \([H^+] < [OH^-]\), the solution is alkaline.
- In pure water at 25°C \([H^+] = [OH^-]\),
  \[pH = pOH = 7\]
- A solution of \(pH < 7\) is acidic
- A solution of \(pH > 7\) is alkaline (or basic)
Comments on basicity and acidity

• Is there a pH of negative value?

• How about the pH value of a solution that contains $1 \times 10^{-8}$ M HCl?

• If the concentration of an acid or base is much less than $10^{-7}$ M, then its contribution to the acidity or basicity will be negligible compared to the contribution from water.
Example

• Calculate the pH and pOH of a 1.0 X 10^{-7} M solution of HCl.

\[ \text{HCl} \rightarrow \text{H}^+ + \text{Cl}^- \]
\[ \text{H}_2\text{O} \rightleftharpoons \text{H}^+ + \text{OH}^- \]
\[ [\text{H}^+][\text{OH}^-] = 1.0 \times 10^{-14} \]
\[ [\text{H}^+]_{\text{H}_2\text{O} \text{ diss.}} = [\text{OH}^-]_{\text{H}_2\text{O} \text{ diss.}} = x \]

Since the hydrogen ions contributed from the ionization of water are not negligible compared to the HCl added,

\[ [\text{H}^+] = C_{\text{HCl}} + [\text{H}^+]_{\text{H}_2\text{O} \text{ diss.}} \]
\[
([H^+]_{\text{HCl}} + x)(x) = 1.0 \times 10^{-14}
\]
\[
(1.00 \times 10^{-7} + x)(x) = 1.0 \times 10^{-14}
\]
\[
x^2 + 1.00 \times 10^{-7} x - 1.0 \times 10^{-14} = 0
\]

\[
x = \frac{-1.00 \times 10^{-7} \pm \sqrt{1.0 \times 10^{-14} + 4(1.0 \times 10^{-14})}}{2} = 6.2 \times 10^{-8} \text{ M}
\]

Therefore, the total H\(^+\) concentration = \((1.00 \times 10^{-7} + 6.2 \times 10^{-8}) = 1.62 \times 10^{-7} \text{ M}:

\[
pH = -\log 1.62 \times 10^{-7} = 7 - 0.21 = 6.79
\]

\[
pOH = 14.00 - 6.79 = 7.21
\]

or, since [OH\(^-\)] = x,

\[
pOH = -\log(6.2 \times 10^{-8}) = 8 - 0.79 = 7.21
\]
Comments

• The calculation in this example is more academic than practical because carbon dioxide from the air dissolved in water exceeds these concentrations.

• Since carbon dioxide in water forms an acid, extreme care would have to be taken to remove and keep this from the water, to have a solution of $10^{-7} \, M$ acid.

• We usually neglect the contribution of water to the acidity in the presence of an acid since its ionization is suppressed in the presence of the acid.
pH and temperature

- Does pH change with temperature? If yes, why?
- $K_w = 5.5 \times 10^{-13}$ at $100^\circ C$

\[
[H^+] = [OH^-] = \sqrt{5.5 \times 10^{-13}} = 7.4 \times 10^{-7} \text{ M}
\]

$pH = pOH = 6.13$

$pK_w = 12.26 = pH + pOH$

- The pH of blood at body temperature ($37^\circ C$) is 7.35 - 7.45.
- This value represents a slightly more alkaline solution relative to neutral water than the same value would be at room temperature.
- At $37^\circ C$, $K_w = 2.5 \times 10^{14}$ and $pK_w = 13.60$.
- The pH (and pOH) of a neutral solution is $13.60/2 = 6.80$.
- The $[H^+] = [OH^-] = 1.6 \times 10^{-7} \text{ M}$

- Since a neutral blood solution at $37^\circ C$ would have pH 6.8
pH and temperature

• Since a neutral blood solution at 37°C would have pH 6.8.
• A blood pH of 7.4 is more alkaline at 37°C by 0.2 pH units than it would be at 25°C.
• This is important when one considers that a change of 0.3 pH units in the body is extreme.
• The HCl concentration in the stomach is about 0.1 to 0.02 M.
• Since, the pH at 0.02 M would be 1.7.
• It will be the same regardless of the temperature since the hydrogen ion concentration is the same and the same pH would be measured at either temperature.
• But, while the pOH would be 14.0 - 1.7 = 12.3 at 25°C, it is 13.6 - 1.7 = 11.9 at 37°C.
pH of the weak acids and bases

• pH and pOH are determined readily from the concentration of the acid or base.
• Weak acids (or bases) are only partially ionized.
• Most organic acids and bases, as found in clinical applications, are weak.
• The ionization constant can be used to calculate the amount ionized and, from this, the pH.
pH of the weak acids

\[ \text{HOAc} \rightleftharpoons \text{H}^+ + \text{OAc}^- \]

\[
\frac{[\text{H}^+][\text{OAc}^-]}{[\text{HOAc}]} = 1.75 \times 10^{-5}
\]

Calculate the pH and pOH of a \(1.00 \times 10^{-3}\) M solution of acetic acid.

\[ \text{HOAc} \rightleftharpoons \text{H}^+ + \text{OAc}^- \]

<table>
<thead>
<tr>
<th></th>
<th>(1.00 \times 10^{-3})</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change ((x = \text{mmol/mL}))</td>
<td>(-x)</td>
<td>(+x)</td>
<td>(+x)</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>(1.00 \times 10^{-3} - x)</td>
<td>(x)</td>
<td>(x)</td>
</tr>
</tbody>
</table>

\[
\frac{(x)(x)}{1.00 \times 10^{-3} - x} = 1.75 \times 10^{-5}
\]
• If \( x \) less than about 10 or 15% of the acid is ionized, the expression may be simplified by neglecting \( x \) compared with \( C \) (10\(^{-3} \) M in this case).

• The simplification applies if \( K_a \) is smaller than about \( 0.01 \) \( C \), that is, smaller than \( 10^{-4} \) at \( C = 0.01 \) M, \( 10^{-3} \) at \( C = 0.1 \) M, and so forth.

• Under these conditions, the error in calculation is 5% or less (results come out too high)

• If \( C_{HA} > 100K_a \), \( x \) can be neglected compared to \( C_{HA} \).

\[
\frac{x^2}{1.00 \times 10^{-3}} = 1.75 \times 10^{-15}
\]

\[
x = 1.32 \times 10^{-4} \text{ M} \equiv [H^+]
\]

In General

\[
x = [H^+] = \sqrt{k_a C_A}
\]
pH of the weak bases

The basicity constant $K_b$ for ammonia is $1.75 \times 10^{-5}$ at 25°C. (It is only coincidental that this is equal to $K_a$ for acetic acid.) Calculate the pH and pOH for a $1.00 \times 10^{-3}$ M solution of ammonia.

$$\begin{align*}
\text{NH}_3 + \text{H}_2\text{O} & \rightleftharpoons \text{NH}_4^+ + \text{OH}^- \\
(1.00 \times 10^{-3} - x) & \quad x \\
\frac{[\text{NH}_4^+][\text{OH}^-]}{[\text{NH}_3]} & = 1.75 \times 10^{-5}
\end{align*}$$
pH of salts of weak acids and bases

• Salts of weak acids or weak bases are completely ionized
  • NaOAc → Na⁺ + OAc⁻
  • NH₄Cl → NH₄⁺ + Cl⁻
  • BHCl → BH⁺ + Cl⁻
• Anions of salts of weak acids are Bronsted bases
  • Oac⁻ + H₂O ⇌ HOAc⁻ + OH⁻
• Cations of salts of weak bases are Bronsted acids
  • BH⁺ + H₂O ⇌ B + H₃O⁺
• Anions and cations of strong acids or strong bases are neutral
This ionization is known as **hydrolysis** and its equilibrium constant may be called **Hydrolysis constant** or the **basicity constant** of the salt.

Thus for a conjugate acid-base, always:

\[ K_w = K_a \times K_b \]
pH of salts of weak acids, HA

\[ A^- + H_2O \rightleftharpoons HA + OH^- \]

\[ \frac{[HA][OH^-]}{[A^-]} = \frac{K_w}{K_a} = K_b \]

\[ A^- + H_2O \rightleftharpoons HA + OH^- \]

\[ (C_{A^-} - x) \quad x \quad x \]

\[ K_b = \frac{k_w}{k_a} = \frac{x^2}{C_{A^-} - x} \approx \frac{x^2}{C_{A^-}} \]

\[ \frac{[OH^-][OH^-]}{C_{A^-}} = \frac{K_w}{K_a} = K_b \]

\[ [OH^-] = \sqrt{\frac{K_w}{K_a} \cdot C_{A^-}} = \sqrt{K_b \cdot C_{A^-}} \]

\[ C_{A^-} = C_{salt} \]

This equation holds only if:

\[ C_A > 100 \cdot k_b \text{ and } x \text{ can be neglected compared to } C_{A^-} \]
pH of salts of weak bases, BHA

BHA $\rightarrow$ BH$^+$ + A$^-$

\[
\text{BH}^+ + \text{H}_2\text{O} \rightleftharpoons \text{B} + \text{H}_3\text{O}^+ \\
K_H = K_a = \frac{[\text{B}][\text{H}_3\text{O}^+]}{[\text{BH}^+]}
\]

\[
K_a = \frac{[\text{B}][\text{H}_3\text{O}^+][\text{OH}^-]}{[\text{BH}^+][\text{OH}^-]}
\]

\[
\text{B} + \text{H}_2\text{O} \rightleftharpoons \text{BH}^+ + \text{OH}^-
\]

\[
K_b = \frac{[\text{BH}^+][\text{OH}^-]}{[\text{B}]}
\]

\[
\frac{[\text{B}][\text{H}_3\text{O}^+]}{[\text{BH}^+]^2} = \frac{K_w}{K_b} = K_a
\]

\[
K_a = \frac{K_w}{K_b} = \frac{1.0 \times 10^{-14}}{1.75 \times 10^{-5}} = 5.7 \times 10^{-10}
\]
\[ C_{BH^+} = C_{BHA} - C_B = C_{\text{salt}} - x \]

\[ BH^+ + H_2O \rightleftharpoons B + H_3O^+ \]

\[ C_{\text{salt}} - x \quad x \quad x \]

\[ K_a = \frac{x^2}{C_{\text{salt}} - x} \approx \frac{x^2}{C_{\text{salt}}} = \frac{[H_3O^+][H_3O^+]}{C_{\text{salt}}} = \frac{[H^+][H^+]}{C_{\text{salt}}} \]

\[ K_a = \frac{[H^+][H^+]}{C_{\text{salt}}} = \frac{[H^2]}{K_w} = \frac{K_w}{K_b} \]

\[ [H^+] = \sqrt{\frac{K_w}{K_b}} \cdot C_{\text{salt}} \]

\[ C_{\text{salt}} = C_{BH^+} \]
pH for NH₄Cl

\[ \text{NH}_4\text{Cl} \rightarrow \text{NH}_4^+ + \text{Cl}^- \quad \text{(ionization)} \]

\[ \text{NH}_4^+ + \text{H}_2\text{O} \rightleftharpoons \text{NH}_4\text{OH} + \text{H}^+ \quad \text{(hydrolysis)} \]

\[ (\text{NH}_4^+ + \text{H}_2\text{O} \rightleftharpoons \text{NH}_3 + \text{H}_3\text{O}^+) \]

\[
\frac{[\text{NH}_4\text{OH}][\text{H}^+]}{[\text{NH}_4^+]} = K_a = \frac{K_w}{K_b} = \frac{1.0 \times 10^{-14}}{1.75 \times 10^{-5}} = 5.7 \times 10^{-10}
\]
Buffers (Buffer solutions)

• A buffer is a solution that resists change in pH when a small amount of an acid or base is added or when the solution is diluted.

• A buffer solution consists of a mixture of a weak acid and its conjugate base or a weak base and its conjugate acid at predetermined concentrations or ratios.

• That is, we have a mixture of a weak acid and its salt or a weak base and its salt.
pH of the acetate Buffer

\[ \text{HOAc} \rightleftharpoons H^+ + \text{OAc}^- \]

\[ [H^+] = K_a \frac{[\text{HOAc}]}{[\text{OAc}^-]} \]

\[ -\log[H^+] = -\log K_a - \log \frac{[\text{HOAc}]}{[\text{OAc}^-]} \]

\[ \text{pH} = pK_a - \log \frac{[\text{HOAc}]}{[\text{OAc}^-]} \]

\[ \text{pH} = pK_a + \log \frac{[\text{OAc}^-]}{[\text{HOAc}]} \]
pH of a buffer of weak acid solution containing its salt

Henderson-Hasselbalch equation

\[ \text{HA} \rightleftharpoons \text{H}^+ + \text{A}^- \]

\[
pH = pK_a + \log \frac{[\text{A}^-]}{[\text{HA}]} 
\]

\[
pH = pK_a + \log \frac{[\text{conjugate base}]}{[\text{acid}]} 
\]

\[
pH = pK_a + \log \frac{[\text{proton acceptor}]}{[\text{proton donor}]} 
\]

\[
pH = pK_a + \log \frac{[\text{Salt}]}{[\text{Acid}]} 
\]
Example

Calculate the pH of a buffer prepared by adding 10 mL of 0.10 M acetic acid to 20 mL of 0.10 M sodium acetate.

$$pH = pK_a + \log \frac{[\text{proton acceptor}]}{[\text{proton donor}]}$$

$$pH = pK_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$$
Example

Calculate the pH of a solution prepared by adding 25 mL of 0.10 M sodium hydroxide to 30 mL of 0.20 M acetic acid.

\[
\text{HOAc} + \text{NaOH} \rightleftharpoons \text{NaOAc} + \text{H}_2\text{O}
\]

\[
pH = pK_a + \log \frac{[\text{Salt}]_{\text{formed}}}{[\text{Acid}]_{\text{left}}}
\]

\[
pH = pK_a + \log \frac{[\text{Salt}]_{\text{formed}}}{[\text{Acid}]_{\text{left}}}
\]
Buffering mechanism

\[
pH = \text{constant} + \log \frac{[A^-]}{[HA]}
\]

\[\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-
\]

- If the solution is diluted, the ratio remains constant, and so the pH of the solution does not change.
- If a small amount of a strong acid is added, it will combine with an equal amount of the \( A^- \) to convert it to HA.
- The change in the ratio \([A^-]/[HA]\) is small and hence the change in pH is small.
- If a small amount of a strong base is added, it will combine with part of the HA to form an equivalent amount of \( A^- \).
- Again, the change in the ratio is small.
Buffering capacity

\[ \text{pH} = \text{constant} + \log \frac{[A^-]}{[HA]} \]

- The amount of acid or base that can be added without causing a large change in pH is governed by the **Buffering capacity**.
- **Buffering capacity** is determined by the concentrations of \([HA]\) and \([A^-]\).
  - The higher their concentrations, the more acid or base the solution can tolerate.
- The buffer capacity (buffer intensity, buffer index) of a solution is defined as

\[ \beta = \frac{dC_{\text{BOH}}}{dpH} = -\frac{dC_{\text{HA}}}{dpH} \]
Buffering capacity

• The buffer capacity is a positive number.
• The larger it is, the more resistant the solution is to pH change.
• The buffering capacity is governed by the ratio of HA to A⁻.
• The buffer is maximum when the ratio is unity.

That is, when the pH = pKₐ
Example

- A buffer solution is 0.20 \( M \) in acetic acid and in sodium acetate. Calculate the change in pH upon adding 1.0 mL of 0.10 \( M \) hydrochloric acid to 10 mL of this solution.

\[
pH = pK_a + \log \frac{[A^-]}{[HA]}
\]

\[pK_a = 4.76\]
pH of a buffer of a weak base solution containing its salt

- A mixture of a weak base and its salt acts as a buffer in the same manner as a weak acid and its salt.

Consider an equilibrium between a base and its conjugate acid BH⁺ (same as before). Then write a $K_a$ expression for the conjugate (Bronsted) acid.

\[
BH^+ = B + H^+
\]

\[
K_a = \frac{[B][H^+]}{[BH^+]} = \frac{K_w}{K_b}
\]

\[
[H^+] = K_a \cdot \frac{[BH^+]}{[B]} = \frac{K_w}{K_b} \cdot \frac{[BH^+]}{[B]}
\]

\[
-\log[H^+] = -\log K_a - \log \frac{[BH^+]}{[B]} = -\log \frac{K_w}{K_b} - \log \frac{[BH^+]}{[B]}
\]
pH of a weak base solution containing its salt

\[ \text{pH} = pK_a + \log \frac{[B]}{[BH^+]} = (pK_w - pK_b) + \log \frac{[B]}{[BH^+]} \]

Since \( \text{pH} = pK_w - \text{pOH} \)

\[ \text{pOH} = pK_b + \log \frac{[BH^+]}{[B]} = pK_b + \log \frac{[\text{proton donor}]}{[\text{proton acceptor}]} \]
Buffering mechanism for a buffer of weak base solution and its salt

- When a strong acid is added, it combines with some of the base $B$ to form the salt $BH^+$.
- Conversely, when a strong base is added, it combines with $BH^+$ to form $B$.
- Since the change in the ratio will be small, the change in pH will be small.
- Again, the buffering capacity is maximum when $pH = pK_a$; that is $pH = 14 - pK_b$ or $pOH = pK_b$.

\[ BH^+ = B + H^+ \]

\[ pH = pK_a + \log \frac{[B]}{[BH^+]} \]
Example

Calculate the volume of concentrated ammonia and the weight of ammonium chloride you would have to take to prepare 100 mL of a buffer at pH 10.00 if the final concentration of salt is to be 0.200 M. Given that the molarity of concentrated ammonia is 14.8 M.

\[
\text{pH} = pK_a + \log \frac{[\text{proton acceptor}]}{[\text{proton donor}]}
\]

\[
= (14.00 - pK_b) + \log \frac{[\text{NH}_3]}{[\text{NH}_4^+]} + \log \frac{[\text{NH}_3]}{0.200 \text{ mol/L}}
\]

\[
10.0 = (14.00 - 4.76) + \log \frac{[\text{NH}_3]}{0.200 \text{ mol/L}}
\]

\[
[NH_3] = 1.16 \text{ M}
\]

The molarity of concentrated ammonia is 14.8 M. Therefore,

\[
100 \text{ mL} \times 1.16 \text{ mmol/mL} = 14.8 \text{ mmol/mL} \times \text{mL NH}_3
\]

\[
\text{mL NH}_3 = 7.8 \text{ mL}
\]
Example

How many grams ammonium chloride and how many milliliters 3.0\textit{M} sodium hydroxide should be added to 200 mL water and diluted to 500 mL to prepare a buffer of pH 9.50 with a salt concentration of 0.10 \textit{M}?  

\[
\text{NH}_4\text{Cl} \quad + \quad \text{NaOH} \quad \rightarrow \quad \text{NH}_3 \quad + \quad \text{NaCl} \quad + \quad \text{H}_2\text{O}
\]

Find the concentration of \(\text{NH}_3\) formed from the equation, \([\text{NH}_3]\) = 0.18 M

Total # mmoles of \(\text{NH}_4\text{Cl}\) added =

\[
\text{# mmoles of } \text{NH}_4\text{Cl left}_{(\text{final})} + \text{# mmoles of } \text{NH}_3 \text{ formed}_{(\text{final})}
\]

Total # mg \(\text{NH}_4\text{Cl}\) added = Total # mmoles \(\text{NH}_4\text{Cl}\) x MM (\(\text{NH}_4\text{Cl}\))

Volume of NaOH added = # mmoles of NaOH added /M (NaOH)

= # mmoles of \(\text{NH}_3\) formed/M (NaOH)
Polyprotic acids and their salts

- Polyfunctionaly acids or bases are substances that have more than one ionizable proton or hydroxide ion.
- These substances ionize stepwise, and an equilibrium constant can be written for each step.

\[
\begin{align*}
H_3PO_4 & \rightleftharpoons H^+ + H_2PO_4^- & K_{a1} = 1.1 \times 10^{-2} = \frac{[H^+][H_2PO_4^-]}{[H_3PO_4]} \\
H_2PO_4^- & \rightleftharpoons H^+ + HPO_4^{2-} & K_{a2} = 7.5 \times 10^{-8} = \frac{[H^+][HPO_4^{2-}]}{[H_2PO_4^-]} \\
HPO_4^{2-} & \rightleftharpoons H^+ + PO_4^{3-} & K_{a3} = 4.8 \times 10^{-13} = \frac{[H^+][PO_4^{3-}]}{[HPO_4^{2-}]} 
\end{align*}
\]

The stepwise $K_a$ values of polyprotic acids get progressively smaller as the increased negative charge makes dissociation of the next proton difficult.
Polyprotic acids and their salts

• The overall ionization is the sum of these individual steps and the overall ionization constant is the product of the individual ionization constants:

\[ H_3PO_4 \rightleftharpoons 3H^+ + PO_4^{3-} \]

\[ K_a = K_{a1}K_{a2}K_{a3} = 4.0 \times 10^{-22} = \frac{[H^+][PO_4^{3-}]}{[H_3PO_4]} \]

• In order to make precise pH calculations, the contributions of protons from each ionization step must be taken into account.
• Exact calculation is difficult and requires a tedious iterative procedure
Buffer calculations for polyprotic acids

• The anion on the right side in each ionization step can be considered the salt (conjugate base) of the acid from which it is derived.

• That is, in, $H_2PO_4^-$ is the salt of the acid $H_3PO_4$

  $H_3PO_4 \rightleftharpoons H^+ + H_2PO_4^-$

• $HPO_4^{2-}$ is the salt of the acid $H_2PO_4^-$

  $H_2PO_4^- \rightleftharpoons H^+ + HPO_4^{2-}$

• $PO_4^{3-}$ is the salt of the acid $HPO_4^{2-}$

  $HPO_4^{2-} \rightleftharpoons H^+ + PO_4^{3-}$

So each of these pairs constitutes a buffer system.
Buffers of orthophosphoric acid

• Orthophosphate buffers can be prepared over a wide pH range.
• The optimum buffering capacity of each pair occurs at a pH corresponding to its $pK_a$.
• The $\text{HPO}_4^{2-} / \text{H}_2\text{PO}_4^-$ couple is an effective buffer system in the blood.
Example

• The pH of blood is 7.40. What is the ratio of $[\text{HPO}_4^{2-}] / [\text{H}_2\text{PO}_4^{-}]$ in the blood (assume/ 25°C)?

\[
\text{pH} = \text{p}K_a + \log \frac{[\text{proton acceptor}]}{[\text{proton donor}]}
\]

\[
\text{p}K_{a2} = 7.12
\]

\[
\text{pH} = 7.12 + \log \frac{[\text{HPO}_4^{2-}]}{[\text{H}_2\text{PO}_4^{-}]}
\]
Example

Calculate the pH of a 0.100 M $\text{H}_3\text{PO}_4$ solution.

- The pH of a solution of $\text{H}_3\text{PO}_4$ can be calculated same as any weak monoprotic acid.
- The $\text{H}^+$ from the first ionization step effectively suppresses the other two ionization steps, so that the $\text{H}^+$ contribution from them is negligible compared to the first ionization.
- The quadratic equation must be solved because $K_{\text{al}}$ is relatively large.

$$
\text{H}_3\text{PO}_4 \approx \text{H}^+ + \text{H}_2\text{PO}_4^- \\
0.100 - x \quad x \quad x
$$

$$
\frac{(x)(x)}{0.100 - x} = 1.1 \times 10^{-2}
$$

Approximation is not acceptable
Fractions of Dissociation at a given pH: $\alpha$ Values

• Consider the following equilibria for $\text{H}_3\text{PO}_4$

\[
\text{H}_3\text{PO}_4 \rightleftharpoons \text{H}^+ + \text{H}_2\text{PO}_4^- \\
\text{H}_2\text{PO}_4^- \rightleftharpoons \text{H}^+ + \text{HPO}_4^{2-} \\
\text{HPO}_4^{2-} \rightleftharpoons \text{H}^+ + \text{PO}_4^{3-}
\]

• At any given pH, all the four phosphoric acid species will coexist in equilibrium with one another.

• By changing the pH, the equilibria shift, and the relative concentrations change.

• It is possible to derive general equations for calculating the fraction of the acid that exists in a given form, from the given hydrogen ion concentration.
Fractions of Dissociation at a given pH: $\alpha$ Values

- For a given total analytical concentration of phosphoric acid, $C_{H_3PO_4}$ we can write

$$C_{H_3PO_4} = [PO_4^{3-}] + [HPO_4^{2-}] + [H_2PO_4^-] + [H_3PO_4]$$

$$\alpha_0 = \frac{[H_3PO_4]}{C_{H_3PO_4}} \quad \alpha_1 = \frac{[H_2PO_4^-]}{C_{H_3PO_4}} \quad \alpha_2 = \frac{[HPO_4^{2-}]}{C_{H_3PO_4}}$$

$$\alpha_3 = \frac{[PO_4^{3-}]}{C_{H_3PO_4}} \quad \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

- The $\alpha$'s are the fractions of each species present at equilibrium.
- Note that the subscripts denote the number of dissociated protons or the charge on the species.
Fractions of Dissociation at a given pH: $\alpha$ Values

\[ \begin{align*}
H_3PO_4 & \rightleftharpoons H^+ + H_2PO_4^- \\
H_2PO_4^- & \rightleftharpoons H^+ + HPO_4^{2-} \\
HPO_4^{2-} & \rightleftharpoons H^+ + PO_4^{3-} \\
\end{align*} \]

\[ \begin{align*}
K_a1 &= \frac{[H^+][H_2PO_4^-]}{[H_3PO_4]} \\
K_a2 &= \frac{[H^+][HPO_4^{2-}]}{[H_2PO_4^-]} \\
K_a3 &= \frac{[H^+][PO_4^{3-}]}{[HPO_4^{2-}]} \\
\end{align*} \]

\[ \begin{align*}
[PO_4^{3-}] &= \frac{K_{a3}[HPO_4^{2-}]}{[H^+]} \\
[HPO_4^{2-}] &= \frac{K_{a2}[H_2PO_4^-]}{[H^+]^2} \\
[H_2PO_4^-] &= \frac{K_{a1}[H_3PO_4]}{[H^+]^3} \\
\end{align*} \]

\[ \begin{align*}
C_{H_3PO_4} &= [PO_4^{3-}] + [HPO_4^{2-}] + [H_2PO_4^-] + [H_3PO_4] \\
C_{H_3PO_4} &= \frac{K_{a1}K_{a2}K_{a3}[H_3PO_4]}{[H^+]^3} + \frac{K_{a1}K_{a2}[H_3PO_4]}{[H^+]^2} + \frac{K_{a1}[H_3PO_4]}{[H^+]} + [H_3PO_4] \\
\end{align*} \]
Fractions of Dissociation at a given pH: α Values

\[ C_{H_3PO_4} = \frac{K_{a1}K_{a2}K_{a3}[H_3PO_4]}{[H^+]^3} + \frac{K_{a1}K_{a2}[H_3PO_4]}{[H^+]^2} + \frac{K_{a1}[H_3PO_4]}{[H^+]} + [H_3PO_4] \]

\[ C_{H_3PO_4} = [PO_4^{3-}] + [HPO_4^{2-}] + [H_2PO_4^-] + [H_3PO_4] \]

\[ \alpha_0 = \frac{[H_3PO_4]}{C_{H_3PO_4}} \]

\[ \frac{1}{\alpha_0} = \frac{K_{a1}K_{a2}K_{a3}}{[H^+]^3} + \frac{K_{a1}K_{a2}}{[H^+]^2} + \frac{K_{a1}}{[H^+]} + 1 \]

\[ \alpha_0 = \frac{[H^+]^3}{[H^+]^3 + K_{a1}[H^+]^2 + K_{a1}K_{a2}[H^+] + K_{a1}K_{a2}K_{a3}} \]
Fractions of Dissociation at a given pH: $\alpha$ Values

\[
\alpha_1 = \frac{K_{a1}[H^+]^2}{[H^+]^3 + K_{a1}[H^+]^2 + K_{a1}K_{a2}[H^+] + K_{a1}K_{a2}K_{a3}}
\]

\[
\alpha_2 = \frac{K_{a1}K_{a2}[H^+]}{[H^+]^3 + K_{a1}[H^+]^2 + K_{a1}K_{a2}[H^+] + K_{a1}K_{a2}K_{a3}}
\]

\[
\alpha_3 = \frac{K_{a1}K_{a2}K_{a3}}{[H^+]^3 + K_{a1}[H^+]^2 + K_{a1}K_{a2}[H^+] + K_{a1}K_{a2}K_{a3}}
\]
Example

Calculate the equilibrium concentration of the different species in a 0.10 M phosphoric acid solution at pH 3.00 ([H$^+$] = 1.0 $\times$ 10$^{-3}$ M).

\[
\begin{align*}
\alpha_0 &= \frac{(1.0 \times 10^{-3})^3}{(1.0 \times 10^{-3})^3 + (1.1 \times 10^{-2})(1.0 \times 10^{-3})^2 + (1.1 \times 10^{-2})(7.5 \times 10^{-8})(1.0 \times 10^{-3}) + (1.1 \times 10^{-2})(7.5 \times 10^{-8})(4.8 \times 10^{-13})} \\
&= \frac{1.0 \times 10^{-9}}{1.2 \times 10^{-8}} = 8.3 \times 10^{-2}
\end{align*}
\]

[H$_3$PO$_4$] = $C_{H_3PO_4} \alpha_0 = 0.10 \times 8.3 \times 10^{-2} = 8.3 \times 10^{-3}$ M

\[
\alpha_1 = 0.92
\]

[H$_2$PO$_4^-$] = $C_{H_3PO_4} \alpha_1 = 0.10 \times 0.92 = 9.2 \times 10^{-2}$ M

\[
\alpha_2 = 6.9 \times 10^{-5}
\]

[HPO$_4^{2-}$] = $C_{H_3PO_4} \alpha_2 = 0.10 \times 6.9 \times 10^{-5} = 6.9 \times 10^{-6}$ M

\[
\alpha_3 = 3.3 \times 10^{-14}
\]

[PO$_4^{3-}$] = $C_{H_3PO_4} \alpha_3 = 0.10 \times 3.3 \times 10^{-14} = 3.3 \times 10^{-15}$ M

We see that at pH 3, the majority (91%) of the phosphoric acid exists as H$_2$PO$_4^-$ and 8.3% exists as H$_3$PO$_4$. Only 3.3 $\times$ 10$^{-12}$% exists as PO$_4^{3-}$!
Salts of polyprotic acids

- Salts of acids such as $\text{H}_3\text{PO}_4$ may be acidic or basic.

- The protonated salts pose both acidic and basic properties ($\text{H}_2\text{PO}_4^-$, $\text{HPO}_4^-$)

- The unprotonated salt is simply a Brensted base that hydrolyzes ($\text{PO}_4^{3-}$).

Amphoteric Salts.

- $\text{H}_2\text{PO}_4^-$ possesses both acidic and basic properties.

- That is, it is amphoteric. It ionizes as a weak acid and it also is a Brensted base that hydrolyzes:

$$H_2\text{PO}_4^- \rightleftharpoons H^+ + \text{HPO}_4^{2-}$$

\[ K_{a2} = \frac{[H^+][\text{HPO}_4^{2-}]}{[H_2\text{PO}_4^-]} = 7.5 \times 10^{-8} \]

\[ K_b = \frac{K_w}{K_{a1}} = \frac{[\text{H}_3\text{PO}_4][\text{OH}^-]}{[\text{H}_2\text{PO}_4^-]} = \frac{1.00 \times 10^{-14}}{1.1 \times 10^{-2}} = 9.1 \times 10^{-13} \]

$$H_2\text{PO}_4^- + \text{H}^+ \rightleftharpoons \text{H}_3\text{PO}_4$$
Salts of polyprotic acids

• The solution of $\text{H}_2\text{P}0_4^-$ could be either alkaline or acidic, depending on which ionization is more extensive.

• Since $K_{a2}$ for the first ionization is nearly $10^5$ greater than $K_b$ for the second ionization, the solution in this case will obviously be acidic.
[H\(^+\)] in a solution of amphoteric salts, H\(_2\)PO\(_4\)\(^-\)

\[
\text{H}_2\text{PO}_4^- \rightleftharpoons \text{H}^+ + \text{HPO}_4^{2-}
\]

\[
\text{H}_2\text{PO}_4^- + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{PO}_4 + \text{OH}^-
\]

\[
\text{H}_2\text{PO}_4^- + \text{H}^+ \rightleftharpoons \text{H}_3\text{PO}_4
\]

\[
C_{\text{H}^+} = [\text{H}^+]_{\text{total}} = [\text{H}^+]_{\text{H}_2\text{O}} + [\text{H}^+]_{\text{H}_2\text{PO}_4^-} - [\text{OH}^-]_{\text{H}_2\text{PO}_4^-}
\]

OR

\[
[\text{H}^+] = [\text{OH}^-] + [\text{HPO}_4^{2-}] - [\text{H}_3\text{PO}_4]
\]

- [OH\(^-\)] will be counted only if pH is around 7.
- In the case of acidic solution [H\(^+\)] is negligible therefore no need to include [OH\(^-\)]
In the above equation substitute values from equilibrium constants in the right hand side of the equation.

\[
[H^+] = \frac{K_w}{[H^+]} + \frac{K_{a2}[H_2PO_4^-]}{[H^+]} - \frac{[H_2PO_4^-][H^+]}{K_{a1}}
\]

Multiplying each side of the equation by \([H^+]\), collecting the terms containing \([H^+]\) on the left side, and solving for \([H^+]^2\)

\[
[H^+]^2 = \frac{K_w + K_{a2}[H_2PO_4^-]}{1 + \frac{[H_2PO_4^-]}{K_{a1}}}
\]
If the dissociation of $H_2O$ is neglected, the $K_{a1}K_w$ will be very small and can be removed from the numerator of the equation above.

In most cases $[HA^-] \gg K_{a1}$; thus $K_{a1}$ can be neglected from the denominator.
Consequently, \([H^+]\) for a solution of \(\text{NaH}_2\text{PO}_4\) (\(\text{H}_2\text{PO}_4^-\))

This equation is valid on the basis that the stepwise dissociation constants differ by factors of 100 or more

\[
[H^+] \approx \sqrt{K_{a1}K_{a2}} = \sqrt{1.1 \times 10^{-2} \times 7.5 \times 10^{-8}} = 2.9 \times 10^{-5} \text{ M}
\]

- \((\text{pH} = 4.54)\) thus, pH is approximately independent of the salt concentration
- This would be the approximate pH of a \(\text{NaH}_2\text{PO}_4\) solution.

- Similarly, \(\text{HPO}_4^{2-}\) (\(\text{Na}_2\text{HPO}_4\)) is both an acid and a base.
- The \(K\) values involved here are \(K_{a2}\) and \(K_{a3}\) of \(\text{H}_3\text{PO}_4\)
- Again, stepwise dissociation constants differ by factors of 100 or more

\[
[H^+] \approx \sqrt{K_{a2}K_{a3}} = \sqrt{7.5 \times 10^{-8} \times 4.8 \times 10^{-13}} = 1.9 \times 10^{-10}
\]

- pH of a \(\text{Na}_2\text{HPO}_4\) solution is 9.72
Unprotonated Salt

• Unprotonated phosphate \( \text{Na}_3\text{PO}_4, (\text{PO}_4^{3-}) \) is a fairly strong Brensted base in solution and ionizes as follows:

\[
\text{PO}_4^{3-} + \text{H}_2\text{O} \rightleftharpoons \text{HPO}_4^{2-} + \text{OH}^-
\]

\[
K_b = \frac{K_w}{K_{a3}}
\]

• The constant \( K_{a3} \) is very small, and so the equilibrium lies significantly to the right.

• Because \( K_{a3} \ll K_{a2} \), hydrolysis of \( \text{HPO}_4^- \) is suppressed by the \( \text{OH}^- \) from the first step.

• Thus, pH of \( \text{PO}_4^{3-} \) can be calculated just as for a salt of a monoprotic weak acid

• Because \( K_b \) is relatively large, the quadratic equation must be solved, that is, \( \text{PO}_4^{3-} \) is quite a strong base.
Example

- Calculate the pH of 0.100 M Na₃PO₄.

\[
\begin{align*}
\text{PO}_4^{3-} + \text{H}_2\text{O} & \rightleftharpoons \text{HPO}_4^{2-} + \text{OH}^- \\
0.100 - x & \quad x \quad x \\
[\text{HPO}_4^{2-}][\text{OH}^-] & = K_b = \frac{K_w}{K_{a3}} = \frac{1.0 \times 10^{-14}}{4.8 \times 10^{-13}} = 0.020 \\
\frac{(x)(x)}{0.100 - x} & = \frac{1.0 \times 10^{-14}}{4.8 \times 10^{-13}} = 0.020 \\
The \text{concentration is only five times } K_b, \text{ so the quadratic equation is used:} \\
x^2 + 0.020x - 2.0 \times 10^{-3} & = 0 \\
x & = \frac{-0.200 \pm \sqrt{(0.020)^2 - 4(-2.0 \times 10^{-3})}}{2} \\
x & = [\text{OH}^-] = 0.036 \text{ M} \\
pH & = 12.56
\end{align*}
\]
Physiological buffers

• The pH of the blood in a healthy individual remains remarkably constant at 7.35 to 7.4
• This is because the blood contains a number of buffers that protect against pH change due to the presence of acidic or basic metabolites.
• From a physiological viewpoint, a change of ±0.3 pH unit is extreme.
• Acid metabolites are ordinarily produced in greater quantities than basic metabolites, and carbon dioxide is the principal one.
• The buffering capacity of blood for handling CO$_2$ is estimated to be distributed among various buffer systems as follows:
  – Hemoglobin and oxyhemoglobin, 62%
  – $H_2PO_4^-/HPO_4^{2-}$ 22%
  – Plasma protein, 11%
  – Bicarbonate, 5%.
Buffers for Biological and Clinical Measurements

• Many biological reactions of interest occur in the pH range of 6 to 8.
• A number, particularly specific enzyme reactions that might be used for analyses may occur in the pH range of 4 to 10 or even greater.
• The proper selection of buffers for the study of biological reactions or for use in clinical analyses can be critical in determining whether or not they influence the reaction.
• A buffer must have the correct $pK_a$, near physiological pH so the ratio of $[A^-]/[HA]$ in the Henderson-Hasselbalch equation is not too far from unity, and it must be physiologically compatible.

\[
\text{pH} = 6.10 + \log \frac{[\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3]} 
\]
Phosphate Buffers

• Biological systems usually contain some phosphate already, and phosphate buffers will not interfere in many cases.
• By choosing appropriate mixtures of $\text{H}_3\text{PO}_4/\text{H}_2\text{PO}_4^-$, $\text{H}_2\text{PO}_4^-$ /$\text{HPO}_4^{2-}$, or $\text{HPO}_4^{2-} / \text{PO}_4^{3-}$
• phosphate will precipitate or complex many polyvalent cations, and it frequently will participate in or inhibit a reaction.
• It should not be used, for example, when calcium is present if its precipitation would affect the reaction of interest.
Tris Buffers

• It is that prepared from tris(hydroxymethyl)aminomethane and its conjugate acid (the amino group is protonated).

• It is a primary standard and has good stability, has a high solubility in physiological fluids, is nonhygroscopic, does not absorb CO$_2$ appreciably, does not precipitate calcium salts, does not appear to inhibit many enzyme systems, and is compatible with biological fluids.