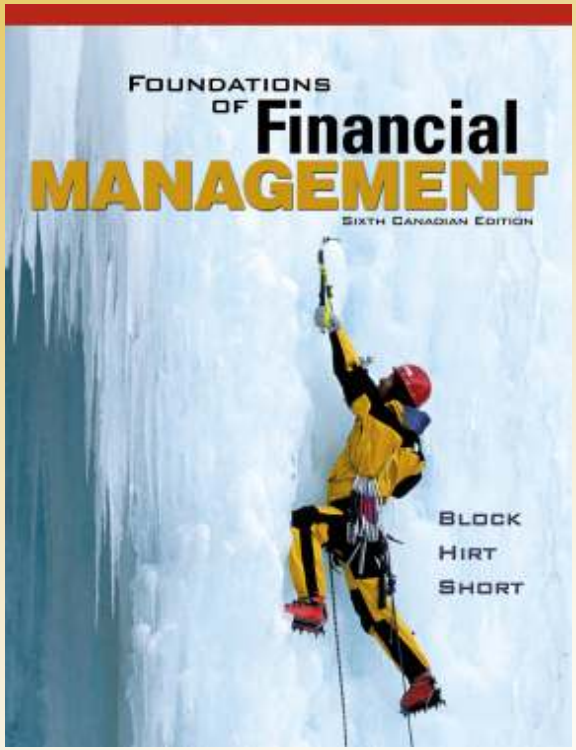
Chapter 9

The Time Value of Money



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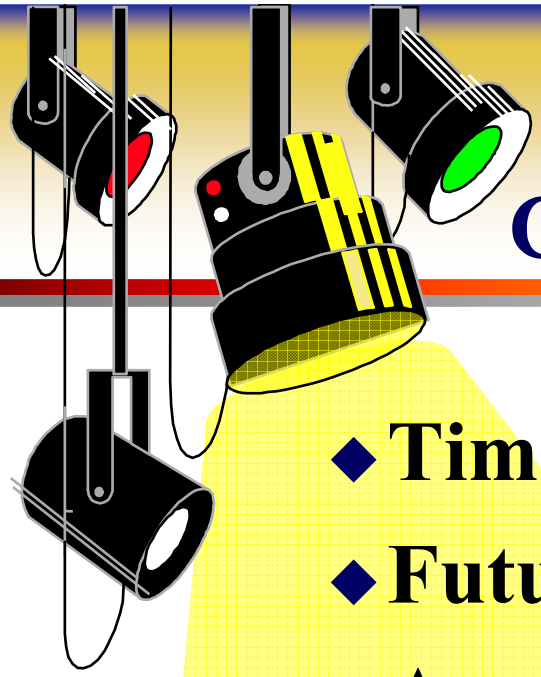


Chapter 9 - Outline

- Time Value of Money
- Future Value and Present Value
- Annuities
- Time-Value-of-Money Formulas
- Adjusting for Non-Annual Compounding
- Compound Interest Tables
- Summary and Conclusions

Time Value of Money

- The basic idea behind the concept of time value of money is:
  - $1 received today is worth more than $1 in the future

OR

  - $1 received in the future is worth less than $1 today

Why?

  - because interest can be earned on the money

- The connecting piece or link between present (today) and future is the interest or discount rate

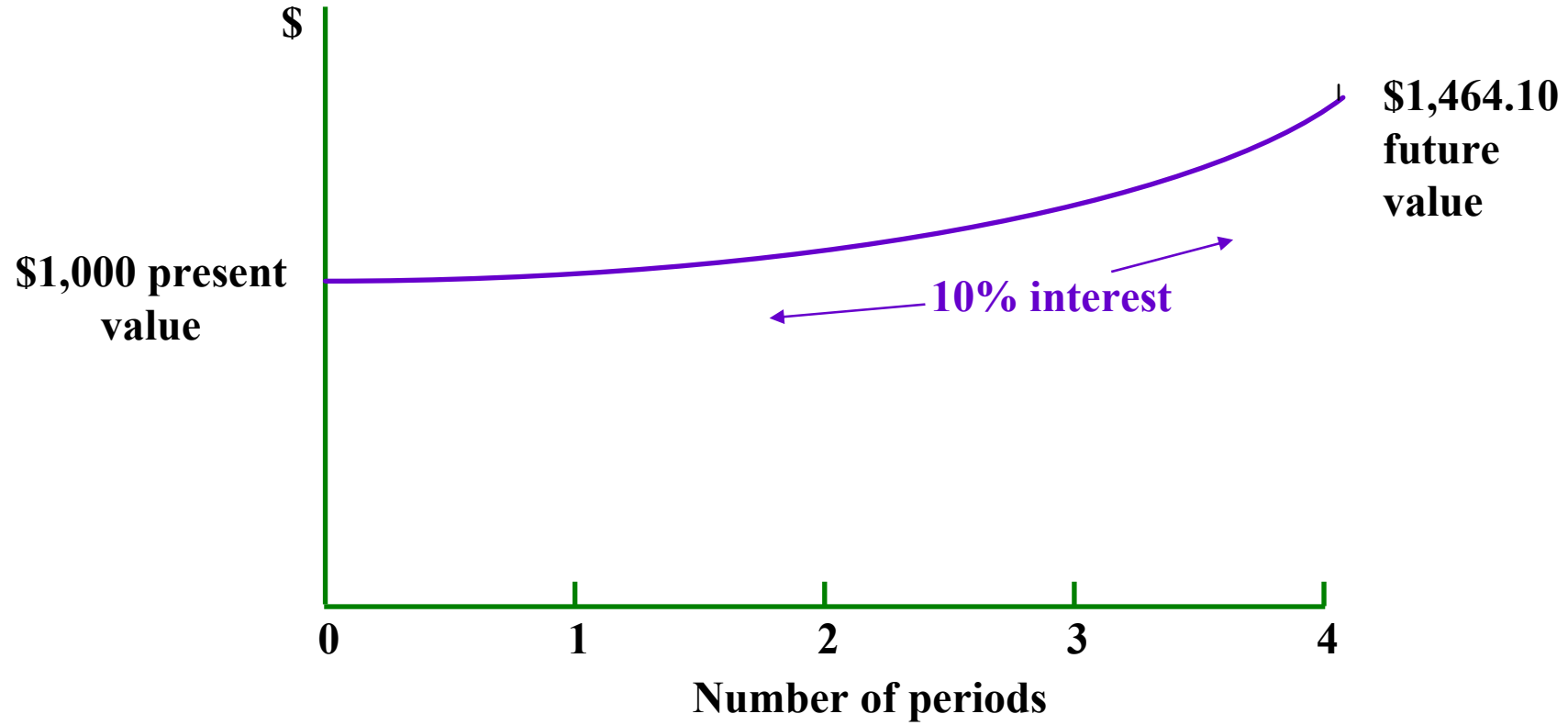
Future Value and Present Value

- **Future Value (FV)** is what money today will be worth at some point in the future

- **Present Value (PV)** is what money at some point in the future is worth today

Figure 9-1

Relationship of present value and future value



Future value of $1 ($FV_{IF}$)

Periods	1%	2%	3%	4%	6%	8%	10%
1	1.010	1.020	1.030	1.040	1.060	1.080	1.100
2	1.020	1.040	1.061	1.082	1.124	1.166	1.210
3	1.030	1.061	1.093	1.125	1.191	1.260	1.331
4	1.041	1.082	1.126	1.170	1.262	1.360	1.464
5	1.051	1.104	1.159	1.217	1.338	1.469	1.611
10	1.105	1.219	1.344	1.480	1.791	2.159	2.594
20	1.220	1.486	1.806	2.191	3.207	4.661	6.727

An expanded table is presented in Appendix A

Present value of $1 ($PV_{IF}$)

Periods	1%	2%	3%	4%	6%	8%	10%
1	0.990	0.980	0.971	0.962	0.943	0.926	0.909
2	0.980	0.961	0.943	0.925	0.890	0.857	0.826
3	0.971	0.942	0.915	0.889	0.840	0.794	0.751
4	0.961	0.924	0.888	0.855	0.792	0.735	0.683
5	0.951	0.906	0.863	0.822	0.747	0.681	0.621
10	0.905	0.820	0.744	0.676	0.558	0.463	0.386
20	0.820	0.673	0.554	0.456	0.312	0.215	0.149

An expanded table is presented in Appendix B

2 Questions to Ask in Time Value of Money Problems

1. **Future Value or Present Value?**
   - **Future Value: Present (Now) → Future**
   - **Present Value: Future → Present (Now)**

2. **Single amount or Annuity?**
   - **Single amount: one-time (or lump) sum**
   - **Annuity: equal amount per year for a number of years**

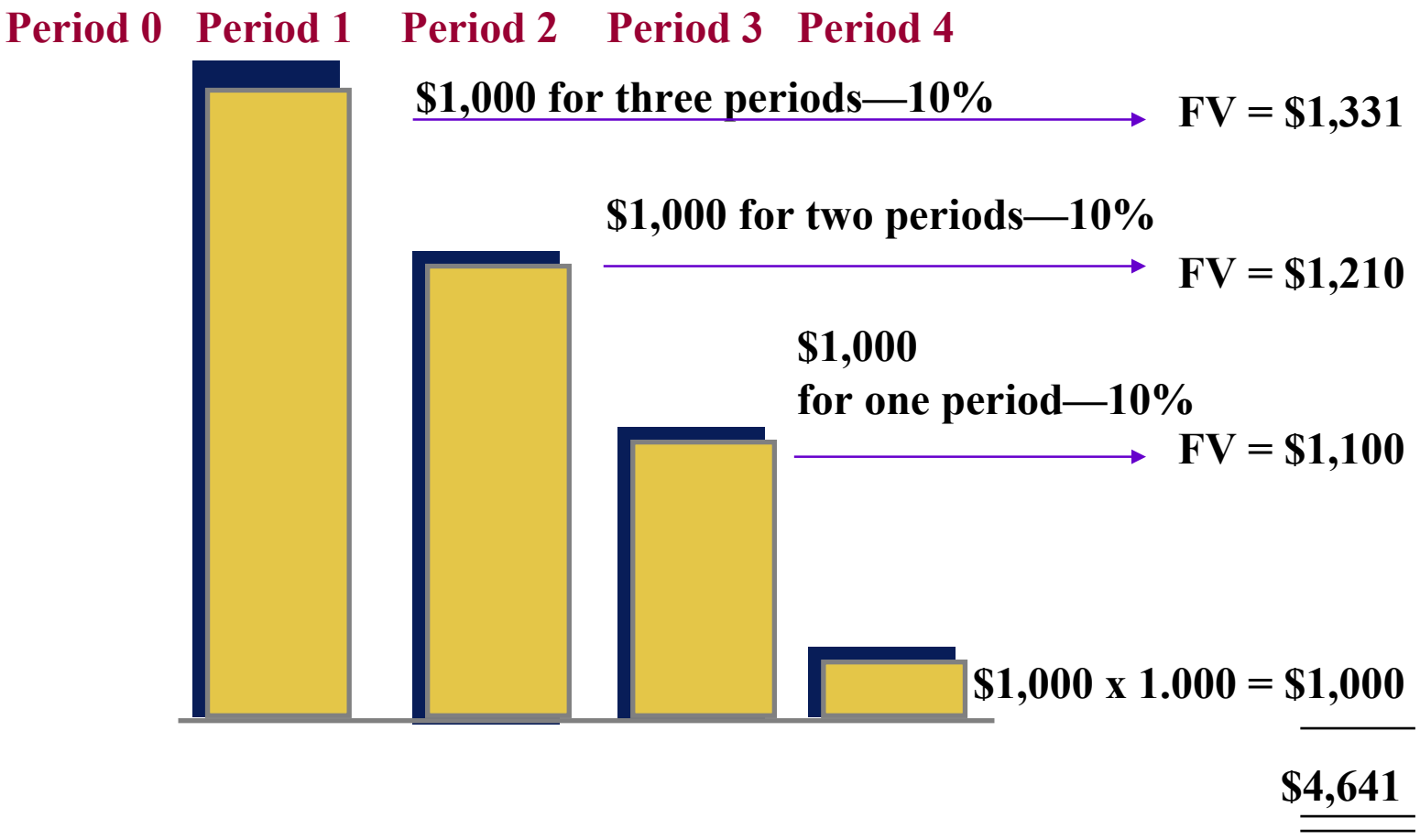
Annuities

Annuity: a stream or series of equal payments to be received in the future

- **The payments are assumed to be received at the end of each period (unless stated otherwise)**
- **A good example of an annuity is a lease, where a fixed monthly charge is paid over a number of years**

Figure 9-2

Compounding process for annuity



Future value of an annuity of $1 ($FV_{IFA}$)

Periods	1%	2%	3%	4%	6%	8%	10%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.010	2.020	2.030	2.040	2.060	2.080	2.100
3	3.030	3.060	3.091	3.122	3.184	3.246	3.310
4	4.060	4.122	4.184	4.246	4.375	4.506	4.641
5	5.101	5.204	5.309	5.416	5.637	5.867	6.105
10	10.462	10.950	11.464	12.006	13.181	14.487	15.937
20	22.019	24.297	26.870	29.778	36.786	45.762	57.275
30	34.785	40.588	47.575	56.085	79.058	113.280	164.490

An expanded table is presented in Appendix C

Present value of an annuity of $1 ($PV_{IFA}$)

Periods	1%	2%	3%	4%	6%	8%	10%
1	0.990	0.980	0.971	0.962	0.943	0.926	0.909
1	0.990	0.980	0.971	0.962	0.943	0.926	0.909
2	1.970	1.942	1.913	1.886	1.833	1.783	1.736
3	2.941	2.884	2.829	2.775	2.673	2.577	2.487
4	3.902	3.808	3.717	3.630	3.465	3.312	3.170
5	4.853	4.713	4.580	4.452	4.212	3.993	3.791
8	7.652	7.325	7.020	6.773	6.210	5.747	5.335
10	9.471	8.983	8.530	8.111	7.360	6.710	6.145
20	18.046	16.351	14.877	13.590	11.470	9.818	8.514
30	25.808	22.396	19.600	17.292	13.765	11.258	9.427

An expanded table is presented in Appendix D

Table 9-1
Relationship of present value to annuity

Year	Beginning Balance	Annual Interest (6 percent)	Annual Withdrawal	Ending Balance
1	$10,000.00	$600.00	$2,886.00	$7,714.00
2	7,714.00	462.84	2,886.00	5,290.84
3	5,290.84	317.45	2,886.00	2,722.29
4	2,722.29	163.71	2,886.00	0

Table 9-2
Payoff table for loan (amortization table)

Period	Beginning Balance	Annual Payment	Annual Interest (8%)	Repayment on Principal	Ending Balance
1	$40,000	$4,074	$3,200	$ 874	$39,126
2	39,126	4,074	3,130	944	38,182
3	38,182	4,074	3,055	1,019	37,163

Determining the Yield on an Investment (a)

	Formula	Appendix
Future value—single amount . . (9-1)	$FV = PV(1+i)^n$	A
Present value—single amount . (9-3)	$PV = FV\left[\frac{1}{(1+i)^n}\right]$	B
Future value—annuity . . . . . (9-4a)	$FV_A = A\left[\frac{(1+i)^n - 1}{i}\right]$	C
Future value—annuity in advance . . . . . (9-4b)	$FV_A = A_{BGN}\left[\frac{(1+i)^{n+1} - (1+i)}{i}\right]$	–
Present value—annuity . . . . . (9-5a)	$PV_A = A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$	D

Determining the Yield on an Investment (b)

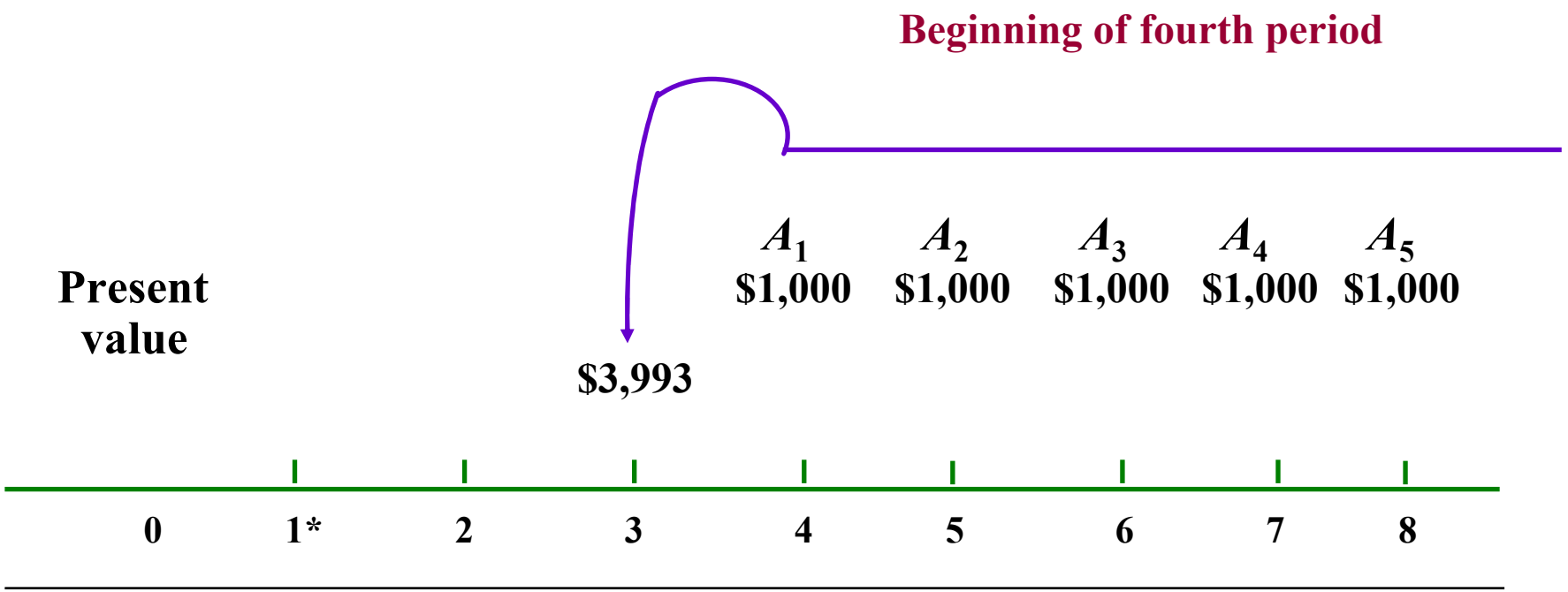
	Formula	Appendix
Present value—annuity in advance . . . . . (9-5b)	$PV_A = A_{BGN}\left[\frac{(1+i)-\frac{1}{(1+i)^{n-1}}}{i}\right]$	–
Annuity equalling a future value . . . . . (9-6a)	$A = FV_A\left[\frac{i}{(1+i)^n-1}\right]$	C
Annuity in advance equalling a future value . . . . . (9-6b)	$A_{BGN} = FV_A\left[\frac{i}{(1+i)^{n+1}-(1+i)}\right]$	–
Annuity equalling a present value . . . . . (9-7a)	$A = PV_A\left[\frac{i}{1-\frac{1}{(1+i)^n}}\right]$	D
Annuity in advance equalling a present value . . . . . (9-7b)	$A_{BGN} = PV_A\left[\frac{i}{(1+i)-\frac{1}{(1+i)^{n-1}}}\right]$	–

Adjusting for Non-Annual Compounding

- **Interest is often compounded quarterly, monthly, or semiannually in the real world**

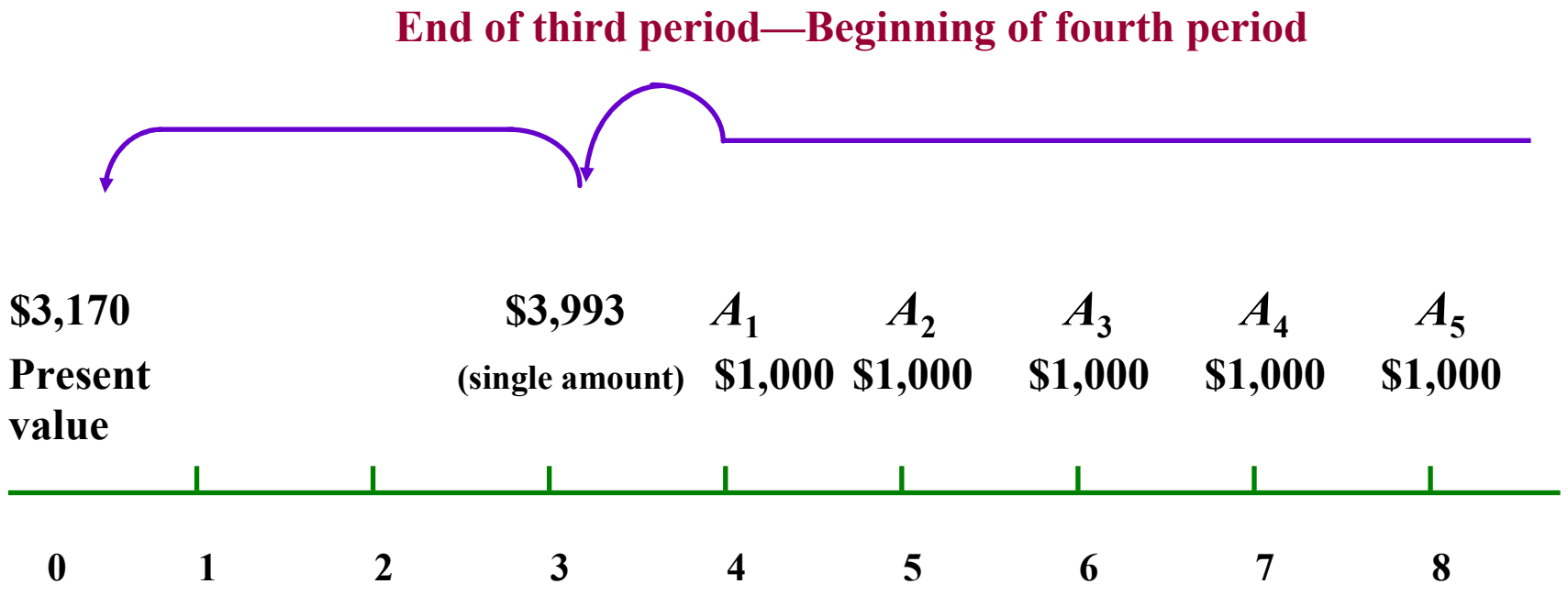
- **Since the time value of money tables assume annual compounding, an adjustment must be made:**
  - **the number of years is multiplied by the number of compounding periods**
  - **the annual interest rate is divided by the number of compounding periods**

The present value of a deferred annuity ($1,000 per year to be paid 4 - 8 years in the future) (first step)



*Each number represents the end of the period; that is, 4 represents the end of the fourth period

The present value of a deferred annuity ($1,000 per year to be paid 4 - 8 years in the future) (second step)



Summary and Conclusions

- The financial manager uses the time value of money approach to value cash flows that occur at different points in time

- A dollar invested today at compound interest will grow a larger value in future. That future value, discounted at compound interest, is equated to a present value today

- Cash values may be single amounts, or a series of equal amounts (annuity)