

# Philadelphia University Department of Basic Sciences and Mathematics 

| Academic Year: | $2016-2017$ | Course Name: | Numerical Analysis |
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| Semester: | Second Semester | Course Number: | 250371 |
| Exam: | Second Exam | Instructor Name: | Feras Awad |
| Exam Date: | $03 / 05 / 2017$ | Student Name: | - |
| Exam Day: | Wednesday | University ID: | - |
| Mark: | $[20]$ | Serial: | - |

1. For a function $f(x)$, the Newton divided differences are given by

$$
\begin{array}{llll}
\hline x_{0}=0.0 & f\left[x_{0}\right] & f\left[x_{0}, x_{1}\right] & f\left[x_{0}, x_{1}, x_{2}\right]=\frac{50}{7} \\
x_{1}=0.4 & f\left[x_{1}\right] & f\left[x_{1}, x_{2}\right]=10 & \\
x_{2}=0.7 & f\left[x_{2}\right]=6 & \\
\hline
\end{array}
$$

(a) (1 point) Find the polynomial $p(x)$ that interpolates $f(x)$ at the nodes $x_{0}, x_{1}$, and $x_{2}$.
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(b) (3 points) Determine the missing values in the table.
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2. (4 points) Develop a formula for the first derivative $f^{\prime}(x)$ in terms of $f(x-h), f(x)$, and $f(x+2 h)$. What is the order of error of this formula ?
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3. Consider the following table of values of a function $f(x)$.

| $x$ | 1.0 | 1.2 | 1.4 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | 1.0000 | 1.2625 | 1.6595 |

(a) (1 point) Use the forward-difference formula to approximate the value of $f^{\prime}(1.0)$.
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(b) (1 point) Use the central-difference formula to approximate the value of $f^{\prime}(1.2)$.
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(c) (2 points) Approximate the value of $\int_{1.0}^{1.4} f(x) d x$ using the mid-point rule.
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4. (5 points) The quadrature formula $\int_{-1}^{1} f(x) d x=c_{0} f(-1)+c_{1} f(0)+c_{2} f(1)$ is exact for all polynomials of degree less than or equal to 2 . Determine $c_{0}, c_{1}$, and $c_{2}$.
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5. (3 points) The Trapezoidal rule applied to $\int_{0}^{2} f(x) d x$ gives the value 4 , and Simpson's rule gives the value 2 . Find the value of $f(1)$ ?
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