Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2016–2017	**Course Name:**	Numerical Analysis
Semester:	Second Semester	**Course Number:**	250371
Exam:	Final Exam	**Instructor Name:**	Feras Awad
Exam Date:	15/06/2017	**Student Name:**	_____
Exam Day:	Thursday	**University ID:**	_____
Mark:	[40]	**Serial:**	_____

1. (3 points) Find $\|\vec{x}\|_1$ and $\|\vec{x}\|_2$ if $\vec{x} = \left(3, -4, 0, \frac{3}{2}\right)^t \in \mathbb{R}$.

2. (3 points) Let $A = \begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix}$. Evaluate $\|A\|_\infty$.

3. (4 points) Prove that the sequence of vectors $\{\vec{x}^{(k)}\}_{k=1}^{\infty}$ where $\vec{x}^{(k)} = \left(\frac{1}{k}, e^{1-k}, -\frac{2}{k^2}\right)^t$ converges, and find its limit.

4. (7 points) Find the first **two** iterations of the **Jacobi** method for the following linear system, using $\vec{x}^{(0)} = (0,0,0)^t$.

$$\begin{aligned} 10x_1 - x_2 &= 9 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6 \end{aligned}$$

[2]

5. (6 points) Use **divided difference** method to show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

[3]

6. (6 points) Suppose that $f(0) = 1$, $f(1/2) = 5/2$, $f(1) = 2$, and $f(1/4) = f(3/4) = \alpha$. Find α if the **composite Trapezoidal rule** with $n = 4$ gives the value 7/4 for $\int_0^1 f(x)\,dx$.

7. (4 points) It can be shown that the equation

$$\frac{3}{2}x - 6 - \frac{1}{2}\sin(2x) = 0$$

has a unique real root.Find an interval on which this unique real root is guaranteed to exist.

8. Consider the initial–value problem

$$y' = te^{3t} - 2y \quad , \quad 0 \le t \le 1 \quad , \quad y(0) = 0.$$

(a) (3 points) Show that the IVP has a unique solution on the convex set

$$\mathbf{D} = \{(t, y) : t \in [0, 1], y \in \mathbb{R}\}.$$

(b) (4 points) Use Euler's method to approximate the solution for the IVP with $h = 0.5$