Power Electronics
Three Phase Controlled Rectifiers

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Table of contents

1. Controlled Three Phase Half Wave Rectifiers
2. Controlled Three Phase Half Wave Rectifiers with Freewheeling Diode
3. Controlled Three Phase Full Wave Rectifiers
4. Three Phase Full Wave Half Controlled Rectifiers
The thyristor will conduct (ON state), when the anode-to-cathode voltage is positive and a firing current pulse is applied to the gate terminal. Delaying the firing pulse by an angle $\alpha$ controls the load voltage.

The possible range for gating delay is between $\alpha = 0^\circ$ and $\alpha = 180^\circ$, but because of commutation problems in actual situations, the maximum firing angle is limited to around $160^\circ$. 
Controlled Three Phase Half Wave Rectifiers

- When the load is resistive, current \( i_d \) has the same waveform of the load voltage. As the load becomes more and more inductive, the current flattens and finally becomes constant. The thyristor goes to the non-conducting condition (OFF state) when the following thyristor is switched ON, or the current, tries to reach a negative value.
Controlled Three Phase Half Wave Rectifiers

Continuous & Discontinuous Conduction in Three-Phase Controlled Rectifier

For resistive load

- $0^\circ \leq \alpha \leq 30^\circ$, output voltage is continuous.
- $30^\circ \leq \alpha \leq 120^\circ$, output voltage is discontinuous and has some intervals in which output voltage is zero.
- $\alpha > 150^\circ$, output voltage is zero.

For Inductive load

- There is no discontinuous conduction mode for three-phase controlled rectifier if $L \gg R$.
- But if $L \approx R$ or firing angle is very large, discontinuities can be seen in output as output voltage can become zero in certain intervals (those intervals in which inductor has quickly dissipated its energy and firing angle hasn’t reached).

Resistive load
- Firing angle $\alpha = 0^\circ$
- Firing angle $\alpha = 30^\circ$
- Firing angle $30^\circ \leq \alpha \leq 150^\circ$ ($\alpha = 60^\circ$)

RL load
The RL load voltage is modified by changing firing angle \( \alpha \). When \( \alpha < 90^\circ \), \( V_{dc} \) is positive and when \( \alpha > 90^\circ \), the average dc voltage becomes negative. In such a case, the rectifier begins to work as an inverter and the load needs to be able to generate power reversal by reversing its dc voltage.
Controlled Three Phase Half Wave Rectifiers

For RL Load

Let

\[ V_{an} = V_m \sin \omega t \]
\[ V_{bn} = V_m \sin (\omega t - 2\pi/3) \]
\[ V_{cn} = V_m \sin (\omega t - 4\pi/3) \]

\( T_1 \) is triggered at \( \omega t = \left( \frac{\pi}{6} + \alpha \right) = (30^0 + \alpha) \)

\( T_2 \) is triggered at \( \omega t = \left( \frac{5\pi}{6} + \alpha \right) = (150^0 + \alpha) \)

\( T_3 \) is triggered at \( \omega t = \left( \frac{7\pi}{6} + \alpha \right) = (270^0 + \alpha) \)

Each thyristor conducts for 120° or \( \frac{2\pi}{3} \) radians
Controlled Three Phase Half Wave Rectifiers

For RL Load

Load current is always continuous. The dc component of the output voltage is the average value, and load current is the resistor voltage divided by resistance.

\[
V_{dc} = \frac{3}{2\pi} \int_{\pi/6 + \alpha}^{5\pi/6 + \alpha} V_m \sin \omega t \, d\omega t = \frac{3\sqrt{3} V_m}{2\pi} \cos \alpha
\]

\[
I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3} V_m}{2\pi R} \cos \alpha
\]

The *rms* component of the output voltage and current waveforms are determined from

\[
V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\pi/6 + \alpha}^{5\pi/6 + \alpha} (V_m \sin \omega t)^2 \, d\omega t} = \sqrt{3} V_m \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha}
\]

\[
I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{\sqrt{3} V_m}{\sqrt{R^2 + (\omega L)^2}} \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha}
\]
In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T₁ is triggered at \( \omega t = (30° + \alpha) \) and T₁ conducts up to \( \omega t = 180° \). When the phase supply voltage decreases to zero, the load current falls to zero and the thyristor T₁ turns off. Thus T₁ conducts from \( \omega t = (30° + \alpha) \) to \( (180°) \).

1- when \( \alpha \leq 30° \)

\[
V_{dc} = \frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m \sin \omega t \, d\omega t = \frac{3\sqrt{3}V_m}{2\pi} \cos \alpha
\]

\[
I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R} \cos \alpha
\]

\[
V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} (V_m \sin \omega t)^2 \, d\omega t} = \sqrt{3}V_m \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha}
\]

\[
I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{3}V_m}{R} \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha}
\]
Controlled Three Phase Half Wave Rectifiers

For Resistive Load

2- when $\alpha \geq 30^\circ$

\[ V_{dc} = \frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} V_m \sin \omega t \, d\omega t = \frac{3V_m}{2\pi} (1 + \cos \left(\frac{\pi}{6} + \alpha\right)) \]

\[ I_{dc} = \frac{V_{dc}}{R} = \frac{3V_m}{2\pi R} \left(1 + \cos \left(\frac{\pi}{6} + \alpha\right)\right) \]

\[ V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} (V_m \sin \omega t)^2 \, d\omega t} = \sqrt{\frac{3V_m^2}{4\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} (1 - \cos 2\omega t) \, d\omega t} \]

\[ V_{rms} = V_m \sqrt{\frac{3}{4\pi} \left(\frac{5\pi}{6} - \alpha + \frac{1}{2} \sin \left(\frac{\pi}{3} + 2\alpha\right)\right)} = \]

\[ I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R} \sqrt{\frac{3}{4\pi} \left(\frac{5\pi}{6} - \alpha + \frac{1}{2} \sin \left(\frac{\pi}{3} + 2\alpha\right)\right)} \]
Controlled Three Phase Half Wave Rectifiers with Freewheeling Diode
Controlled Three Phase Full Wave Rectifiers

- Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals.

- The three thyristors \((T_1, T_3, \text{and} T_5)\) will not work together at the same time or two of them also will not work together at the same time.
- The three thyristors \((T_2, T_4, \text{and} T_6)\) will not work together at the same time or two of them also will not work together at the same time.
- \((T_1 \text{ and } T_4), (T_3 \text{ and } T_6) \text{ or } (T_5 \text{ and } T_2)\) will not work together at the same time.
- Each thyristor is triggered at an interval of \(\frac{2\pi}{3}\).
- Each thyristors pair \((T_6 & T_1), (T_1 & T_2), (T_2 & T_3), (T_3 & T_4), (T_4 & T_5), \text{or} \ (T_5 & T_6)\) is triggered at an interval of \(\frac{\pi}{3}\).
- The frequency of output ripple voltage is \(6f_S\).
Controlled Three Phase Full Wave Rectifiers

- If $T_1$ is triggered at $(30 + \alpha)$, $T_3$ will be triggered at $(30 + \alpha + 120)$ and $T_5$ will be triggered at $(30 + \alpha + 240)$. $T_4$ will be triggered at $(30 + \alpha + 180)$, $T_6$ will be triggered at $(30 + \alpha + 120 + 180)$ and $T_2$ will be triggered at $(30 + \alpha + 240 + 180)$.

<table>
<thead>
<tr>
<th>Firing Angle</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$T_5$</th>
<th>$T_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>30°</td>
<td>90</td>
<td>150°</td>
<td>210</td>
<td>270°</td>
<td>330°</td>
</tr>
<tr>
<td>30°</td>
<td>60°</td>
<td>120°</td>
<td>180°</td>
<td>240°</td>
<td>300°</td>
<td>360°</td>
</tr>
<tr>
<td>60°</td>
<td>90°</td>
<td>150°</td>
<td>210°</td>
<td>270°</td>
<td>330°</td>
<td>390°</td>
</tr>
<tr>
<td>90°</td>
<td>120°</td>
<td>180°</td>
<td>240°</td>
<td>300°</td>
<td>360°</td>
<td>420°</td>
</tr>
</tbody>
</table>
Thyristors are numbered in the order in which they are triggered.
The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34, ……
Controlled Three Phase Full Wave Rectifiers

- T₁ is triggered at $\omega t = (30 + \alpha)$, T₆ is already conducting when T₁ is turned ON.
- During the interval $(30 + \alpha)$ to $(90 + \alpha)$, T₁ and T₆ conduct together & the output load voltage is equal to $v_o = v_{ab} = (v_{an} - v_{bn})$.
- T₂ is triggered at $\omega t = (90 + \alpha)$, T₆ turns off naturally as it is reverse biased as soon as T₂ is triggered. During the interval $(90 + \alpha)$ to $(150 + \alpha)$, T₁ and T₂ conduct together & the output load voltage $v_o = v_{ac} = (v_{an} - v_{cn})$.
- T₃ is triggered at $\omega t = (150 + \alpha)$, T₁ turns off naturally as it is reverse biased as soon as T₃ is triggered. During the interval $(150 + \alpha)$ to $(210 + \alpha)$, T₂ and T₃ conduct together & the output load voltage $v_o = v_{bc} = (v_{bn} - v_{cn})$.
- T₄ is triggered at $\omega t = (210 + \alpha)$, T₂ turns off naturally as it is reverse biased as soon as T₄ is triggered. During the interval $(210 + \alpha)$ to $(270 + \alpha)$, T₃ and T₄ conduct together & the output load voltage $v_o = v_{ba} = (v_{bn} - v_{an})$.
- T₅ is triggered at $\omega t = (270 + \alpha)$, T₃ turns off naturally as it is reverse biased as soon as T₅ is triggered. During the interval $(270 + \alpha)$ to $(330 + \alpha)$, T₄ and T₅ conduct together & the output load voltage $v_o = v_{ca} = (v_{cn} - v_{an})$.
- T₆ is triggered at $\omega t = (330 + \alpha)$, T₄ turns off naturally as it is reverse biased as soon as T₆ is triggered. During the interval $(330 + \alpha)$ to $(390 + \alpha)$, T₅ and T₆ conduct together & the output load voltage $v_o = v_{cb} = (v_{cn} - v_{bn})$. 
When $\alpha = 0$
Controlled Three Phase Full Wave Rectifiers

When $\alpha = 30$

When $\alpha = 60$
Controlled Three Phase Full Wave Rectifiers

When $\alpha = 90$

Output Voltage when $\alpha = 90$ for Resistive load

Output Voltage when $\alpha = 90$ for RL load
Controlled Three Phase Full Wave Rectifiers

Thyristor one ($T_1$) voltage for different firing angles.
Controlled Three Phase Full Wave Rectifiers

Let

\[ V_{an} = V_m \sin \omega t \quad V_{bn} = V_m \sin(\omega t - 2\pi/3) \quad V_{bn} = V_m \sin(\omega t - 4\pi/3) \]

\[ V_{ab} = \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) \quad V_{bc} = \sqrt{3}V_m \sin(\omega t - \frac{\pi}{2}) \]

\[ V_{ca} = \sqrt{3}V_m \sin(\omega t - \frac{7\pi}{6}) \]

The dc component of the output voltage and current can be found as

\[ V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha \]

\[ I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R} \cos \alpha \]

The rms component of the output voltage and current waveforms are determined from

\[ V_{rms} = \sqrt{\frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} (\sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}))^2 d\omega t} = \sqrt{3}V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha} \]

\[ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \sqrt{3}V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha} \]
Controlled Three Phase Full Wave Rectifiers

*Special case: resistive load α>60°*

The dc component of the output voltage and current can be found as

\[ V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{\pi} \cos\left(\frac{\pi}{3} + \alpha\right) \]

\[ I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R} \cos\left(\frac{\pi}{3} + \alpha\right) \]

The rms component of the output voltage and current waveforms are determined from

\[ V_{rms} = \sqrt{\frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} \left(\sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})\right)^2 d\omega t} \]

\[ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} \]
Example: A three-phase controlled rectifier has an input voltage which is 480V\textsubscript{rms} at 60 Hz. The load is modeled as a series resistance and inductance with R=10 \, \Omega and L=50\, \text{mH}. Determine the delay angle required to produce an average current of 50 A in the load.

\[ V_{dc} = I_{dc}R = 50 \times 10 = 500V \]
\[ \sqrt{3}V_{rms} = 480V \]
\[ \alpha = cos^{-1} \left( \frac{V_{dc}\pi}{3\sqrt{3}V_m} \right) = cos^{-1} \left( \frac{500\pi}{3\sqrt{2480}} \right) = 39.5^o \]
Three Phase Full Wave Half Controlled Rectifiers

- 3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

- The power factor of 3-phase semi-converter decreases as the trigger angle $\alpha$ increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.
Three Phase Full Wave Half Controlled Rectifiers

- Thyristor \( T_1 \) is forward biased when the phase supply voltage \( v_{an} \) is positive and greater than the other phase voltages \( v_{bn} \) and \( v_{cn} \). The diode \( D_1 \) is forward biased when the phase supply voltage \( v_{cn} \) is more negative than the other phase supply voltages.

- Thyristor \( T_2 \) is forward biased when the phase supply voltage \( v_{bn} \) is positive and greater than the other phase voltages. Diode \( D_2 \) is forward biased when the phase supply voltage \( v_{an} \) is more negative than the other phase supply voltages.

- Thyristor \( T_3 \) is forward biased when the phase supply voltage \( v_{cn} \) is positive and greater than the other phase voltages. Diode \( D_3 \) is forward biased when the phase supply voltage \( v_{bn} \) is more negative than the other phase supply voltages.

- The frequency of the output supply waveform is \( 3f_S \), where \( f_S \) is the input ac supply frequency. The trigger angle \( \alpha \) can be varied from 0 to 180°.
Three Phase Full Wave Half Controlled Rectifiers

For $\alpha > 60^\circ$

| During the time period $\pi/6 \leq \omega t \leq 7\pi/6$ (i.e. $30^\circ \leq \omega t \leq 210^\circ$) thyristor $T_1$ is forward biased. If $T_1$ is triggered at $\omega t = \pi/6 + \alpha$, $T_1$ and $D_1$ conduct together and the line to line supply voltage $v_{ac}$ appears across the load. At $\omega t = 7\pi/6$, $v_{ac}$ starts to become negative and the free wheeling diode $D_m$ turns on and conducts. The load current continues to flow through the free wheeling diode $D_m$ and thyristor $T_1$ and diode $D_1$ are turned off. |
| If the free wheeling diode $D_m$ is not connected across the load, then $T_1$ would continue to conduct until the thyristor $T_2$ is triggered at $\omega t = 5\pi/6 + \alpha$ and the free wheeling action is accomplished through $T_1$ and $D_2$, when $D_2$ turns on as soon as $v_{an}$ becomes more negative at $\omega t = 7\pi/6$. |
Three Phase Full Wave Half Controlled Rectifiers

Waveforms for $\alpha=90^\circ$
For $\alpha<60^\circ$

If the trigger angle $\alpha \leq \pi/3$ each thyristor conducts for $2\pi/3$ and the free wheeling diode $D_m$ does not conduct.

Waveforms for $\alpha=30^\circ$
Three Phase Full Wave Half Controlled Rectifiers

For $\alpha<60^\circ$

If the trigger angle $\alpha \leq \pi/3$ each thyristor conducts for $2\pi/3$ and the free wheeling diode $D_m$ does not conduct.

Waveforms for $\alpha=30^\circ$
Three Phase Full Wave Half Controlled Rectifiers

Let

\[ V_{an} = V_m \sin \omega t \quad V_{bn} = V_m \sin(\omega t - 2\pi/3) \quad V_{bn} = V_m \sin(\omega t - 4\pi/3) \]

For \(\alpha > 60^\circ\) and Discontinuous Output Voltage

\[ V_o = V_{ac} = \sqrt{3}V_m \sin(\omega t - \frac{\pi}{6}) \]

The dc component of the output voltage and current can be found as

\[ V_{dc} = \frac{3}{2\pi} \int_{\pi/6}^{\pi/6 + \alpha} \sqrt{3}V_m \sin(\omega t - \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos \alpha) \]

\[ I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R} (1 + \cos \alpha) \]

The rms component of the output voltage and current waveforms are determined from

\[ V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\pi/6}^{\pi/6 + \alpha} \left(\sqrt{3}V_m \sin(\omega t - \frac{\pi}{6})\right)^2 d\omega t} = \frac{3V_m}{2} \sqrt{\frac{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}{2\pi}} \]

\[ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{3V_m}{2\sqrt{R^2 + (\omega L)^2}} \sqrt{\frac{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}{2\pi}} \]
Three Phase Full Wave Half Controlled Rectifiers

For $\alpha \leq 60^\circ$ and Continuous Output Voltage

$$V_o = V_{ab} = \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})$$

The dc component of the output voltage and current can be found as

$$V_{dc} = \frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos \alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R} (1 + \cos \alpha)$$

The $rms$ component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{3}{2\pi} \left[ \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2}} (V_{ab})^2 d\omega t + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (V_{ac})^2 d\omega t \right]} = \frac{3V_m}{2} \sqrt{\frac{2}{3} + \frac{\sqrt{3}(\cos \alpha)^2}{\pi}}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{3V_m}{2\sqrt{R^2 + (\omega L)^2}} \sqrt{\frac{2}{3} + \frac{\sqrt{3}(\cos \alpha)^2}{\pi}}$$