Advanced Measurement Systems & Sensors (0640732)

Lecture (3)

Sensor Characteristics

(Part Two)

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3. Computation of Stimulus:

The main objective of sensing is to determine a value of the input stimulus (s) from the value of the sensor output signal (S). This can be done by two methods.

1. From the inverted transfer function {s = F(S)} or its approximation, or
2. From a direct transfer function {S = f(s)} by use of the iterative computation.

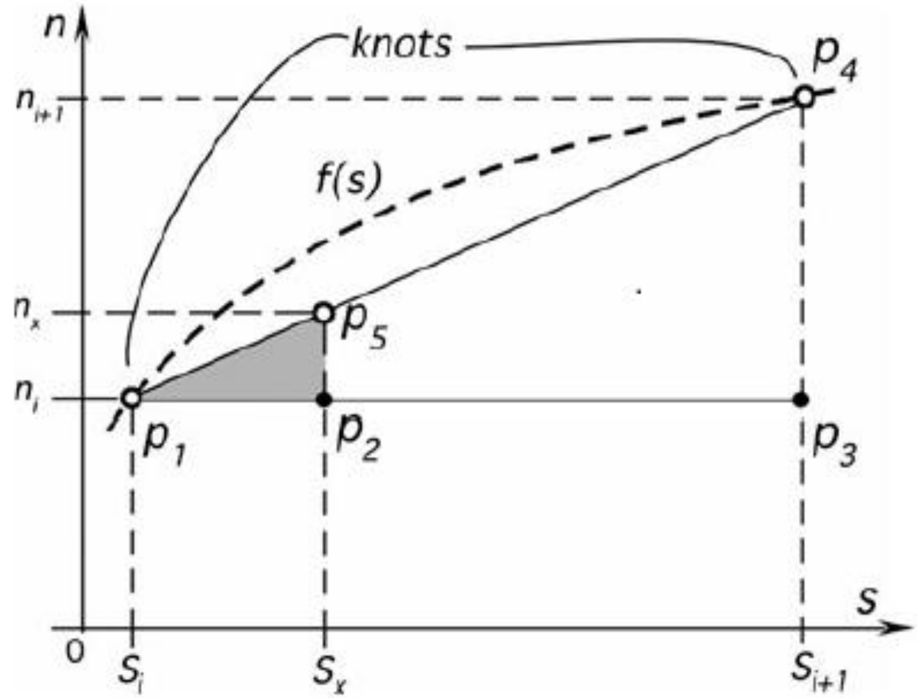
Computation from Linear Piecewise Approximation:

Consider triangles p1p2p3 and p1p5p2: Both triangles are similar, a linear equation is used for computing the unknown stimulus (s_x) from the measured value (n_x):

$$s_x = s_i + \frac{n_x - n_i}{n_{i+1} - n_i}(s_{i+1} - s_i)$$

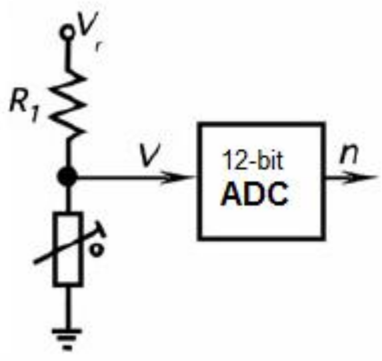
Look-up table

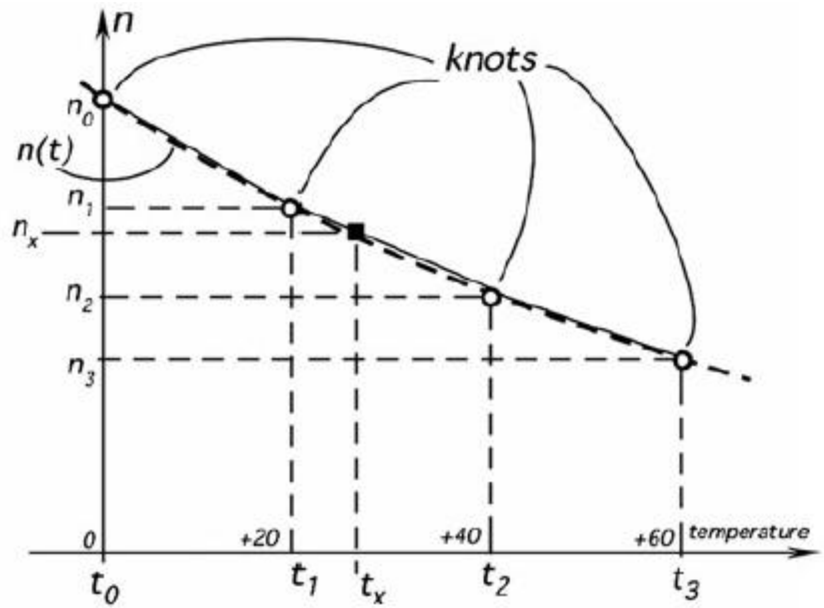
Knot	0	1	2	… i	… k
Output	n_0	n_1	n_2	… n_i	… n_k
Input	s_0	s_1	s_2	… s_i	… s_k



Example: Assume the thermistor is used to measure temperature from 0C to +60C. The output count from the ADC can be modeled by a nonlinear function n(T) of temperature:

$$n_x = N_0 \frac{R_0 e^{\beta(T^{-1} - T_0^{-1})}}{R_1 + R_0 e^{\beta(T^{-1} - T_0^{-1})}}$$





Where;
T is the measured temperature,
T_0 is the reference temperature,
R_0 is the resistance of the thermistor at T_0

- The inverted transfer function which enables us to compute analytically the input temperature is;

$$T_x = \left(\frac{1}{T_0} + \frac{1}{\beta} \ln \left(\frac{n_x}{N_0 - n_x} \frac{R_1}{R_0} \right) \right)^{-1}$$

Sensor Calibration:

- Calibrate the sensor at two temperatures Tc1 and Tc2 in order to find out values of the constants R_0 and β.
- Select two calibrating temperatures in the middle of the operating range as $\mathbf{T_0}$=Tc1= 293.15 K and Tc2= 313.15 K, which correspond to 20C and 40C.
- The thermistor sequentially is immersed into a liquid bath at these two temperatures and the ADC counts are registered as nc1=1863 and nc2=1078.
- By substituting these pairs into Equ.1, we find the values of $\mathbf{R_0}$=8.350 kΩ and **β**=3,895 K.
- The second equation can be used for computing temperature from any reasonable ADC count.

Example: A thermistor is used to measure some unknown temperature and receive counts $\mathbf{n_x}$= 1505.

Knot	0	1	2	3
Counts	2,819	1,863	1,078	593
Temperature (°C)	0	20	40	60

$$t_x = t_1 + \frac{n_x - n_1}{n_2 - n_i}(t_2 - t_1) = 20 + \frac{1505 - 1863}{1078 - 1863}(40 - 20) = 29.12$$

To check how far this computed temperature 29.12C deviates from that computed from a “true” temperature, we plug the same n_x=1,505 into second equation and compute t_x =28.22C.

Iterative Computation of Stimulus (Newton Method):

Numerical iterative methods for finding roots of this typically nonlinear equation can be used for calculating the unknown stimulus (s) without the knowledge of the inverse transfer function.

If a sensor transfer function is f(s), the Newton method prescribes computing for any measured output value (S) the following sequence of the stimuli values which after several steps converges to the sought input s.

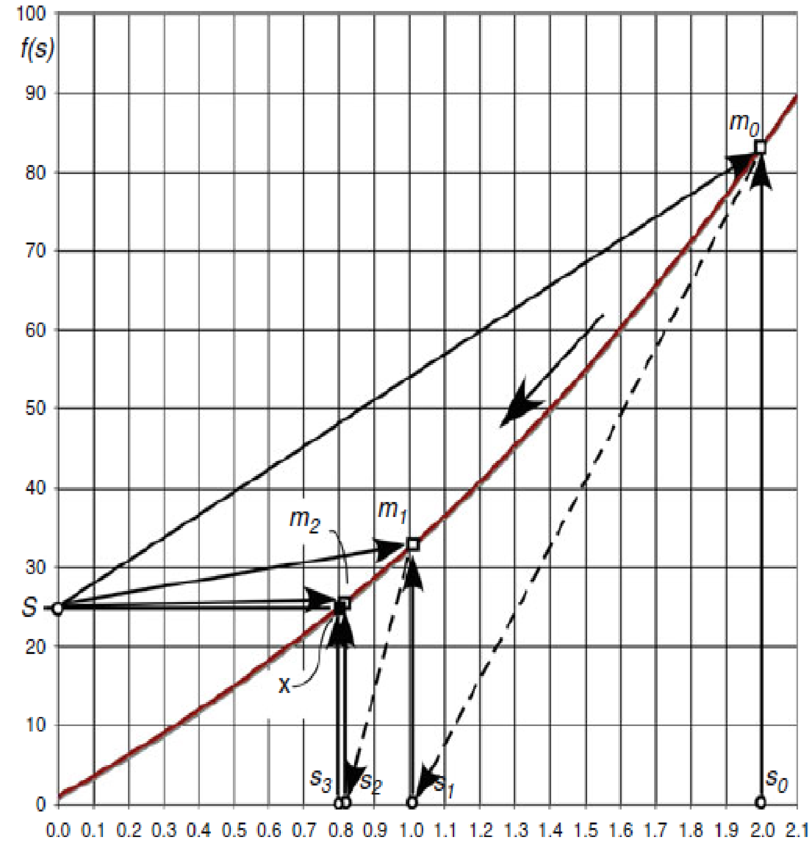
$$s_{i+1} = s_i - \frac{f(s_i) - S}{f'(s_i)}$$

Example: If a 3rd degree polynomial with coefficients a=1.5, b=5, c=25, d=1 is used to illustrate the Newton method.

$$f(s) = as^3 + bs^2 + cs + d$$

$$s_{i+1} = s_i - \frac{as_i^3 + bs_i^2 + cs_i + d - S}{3as_i^2 + 2bs_i + c} = \frac{2as_i^3 + bs_i^2 - d + S}{3as_i^2 + 2bs_i + c}$$

- If the measured sensor's response is S=22 and our initial guess of the stimulus is s_0=2, then;



$$s_1 = \frac{2 \cdot 1.5 \cdot 2^3 + 5 \cdot 2^2 - 1 + 22}{3 \cdot 1.5 \cdot 2^2 + 2 \cdot 5 \cdot 2 + 25} = 1.032$$

$$s_2 = \frac{2 \cdot 1.5 \cdot 1.032^3 + 5 \cdot 1.032^2 - 1 + 22}{3 \cdot 1.5 \cdot 1.032^2 + 2 \cdot 5 \cdot 1.032 + 25} = 0.738$$

$$s_3 = \frac{2 \cdot 1.5 \cdot 0.738^3 + 5 \cdot 0.738^2 - 1 + 22}{3 \cdot 1.5 \cdot 0.738^2 + 2 \cdot 5 \cdot 0.738 + 25} = 0.716$$

$$s_4 = \frac{2 \cdot 1.5 \cdot 0.716^3 + 5 \cdot 0.716^2 - 1 + 22}{3 \cdot 1.5 \cdot 0.716^2 + 2 \cdot 5 \cdot 0.716 + 25} = 0.716$$

- At step 4, the Newton algorithm stops and the stimulus value is deemed to be s=0.716.
- To check accuracy of this solution, plug this number into polynomial Equa and obtain f(s)= S=22.014, which is within the resolution error (0.06%) of the actually measured response S=22.

4. Full Scale Input (Span):

Full scale input represents the highest possible input value, which can be applied to the sensor without causing unacceptably large inaccuracy.

A decibel scale is used with nonlinear response characteristic, represents low level signals with high resolution while compressing the high level numbers.

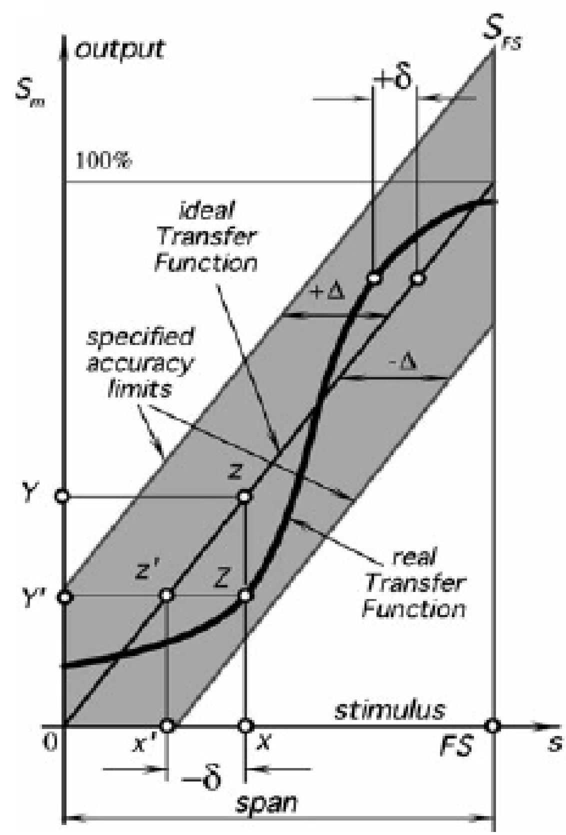
5. Full-Scale Output:

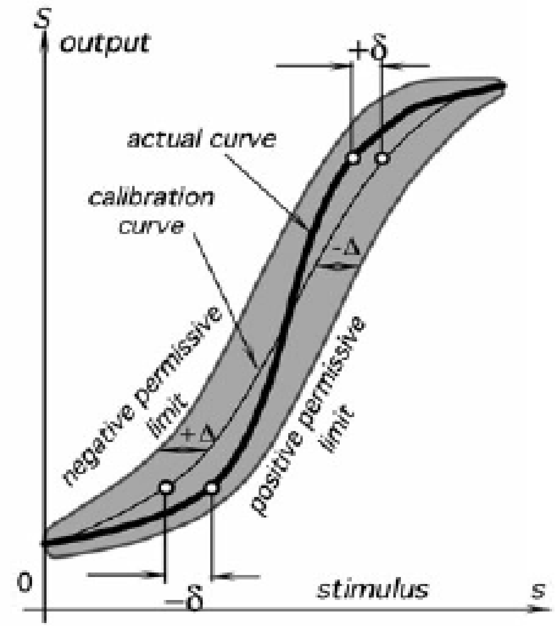
Full-scale output is the algebraic difference between the electrical output signals measured with maximum input stimulus and the lowest input stimulus applied.

6. Accuracy:

A very important characteristic of a sensor is accuracy, which really means inaccuracy. Inaccuracy is measured as a highest deviation of a value represented by the sensor from the ideal or true value of a stimulus at its input.

- The deviation is a difference between the value, which is computed from the output voltage, and the actual input value.
- All runs of the real transfer functions must fall within the limits of a specified accuracy. These permissive limits differ from the ideal transfer function line by ±Δ. The real functions deviate from the ideal by ±δ, where δ≤Δ.





Inaccuracy rating may be represented;

- **Directly** in terms of measured value (Δ): It is used when error is independent on the input signal magnitude. For example, it can be stated as 0.15C for a temperature sensor.
- **In % of the input full scale:** Itis useful for a sensor with a linear transfer function.
- **In % of the measured signal:** It is useful for a sensor with a highly nonlinear transfer function.
- **In terms of the output signal:** It is useful for sensors with a digital output format so the error can be expressed, for example, in units of LSB.

Absolute & Relative Errors:

Accuracy is the capacity of a measuring instrument to give RESULTS close to the TRUE VALUE of the measured quantity.

$$\text{ABSOLUTE ERROR} = \text{RESULT} - \text{TRUE VALUE}$$

$$\text{RELATIVE ERROR} = \frac{\text{ABSOLUTE ERROR}}{\text{TRUE VALUE}}$$

7. Calibration Error:

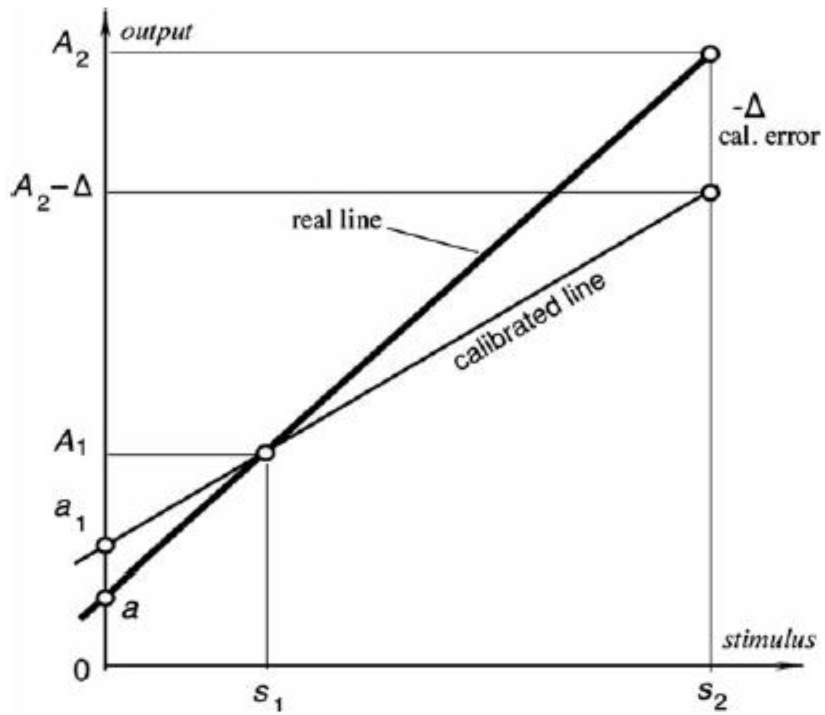
- It is inaccuracy permitted by a manufacturer when a sensor is calibrated in the factory.
- This error is of a systematic nature and is added to all possible real transfer functions. It shifts the accuracy of transduction for each stimulus point by a constant.
- This error is not necessarily uniform over the range and may change depending on the type of error in calibration.

Example: Two-point calibration of a real linear transfer function.

Determine the slope (b) and the intercept (a) of the function using two stimuli, s1 & s2.

$$\delta_a = a_1 - a = \frac{\Delta}{s_2 - s_1}$$

$$\delta_b = -\frac{\Delta}{s_2 - s_1}$$

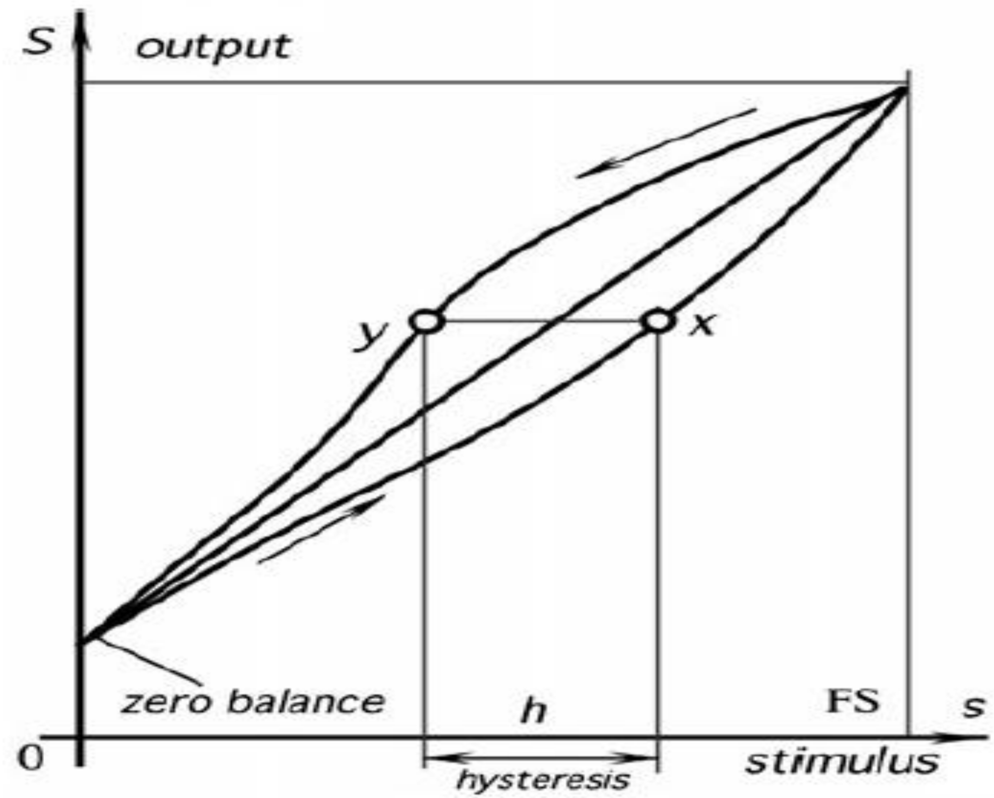


8. Hysteresis:

A hysteresis error is a deviation of the sensor's output at a specified point of the input signal when it is approached from the opposite directions.

Example: A displacement sensor;

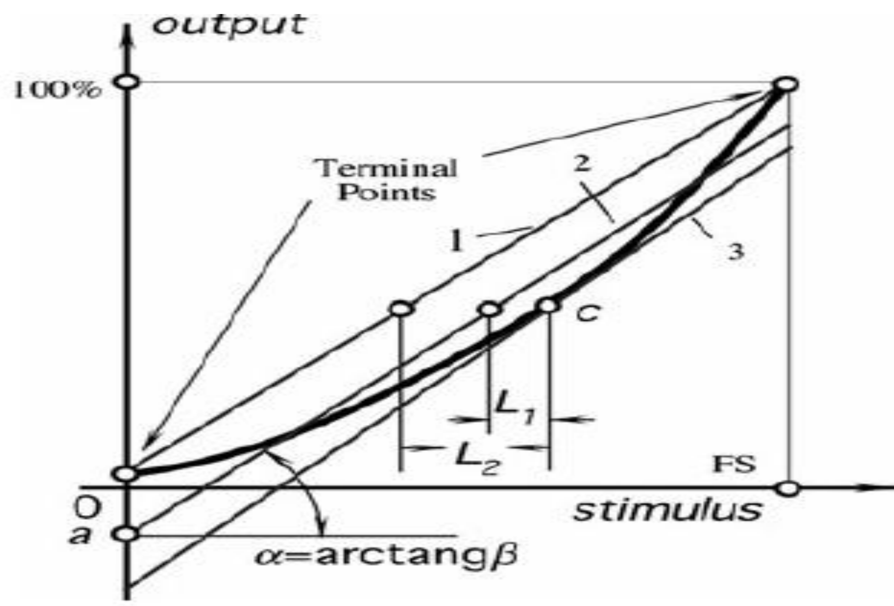
- When the object moves from left to right at a certain point produces voltage, which differs by 20 mV from that when the object moves from right to left.
- If sensitivity of the sensor is 10 mV/mm, the hysteresis error is 2 mm.

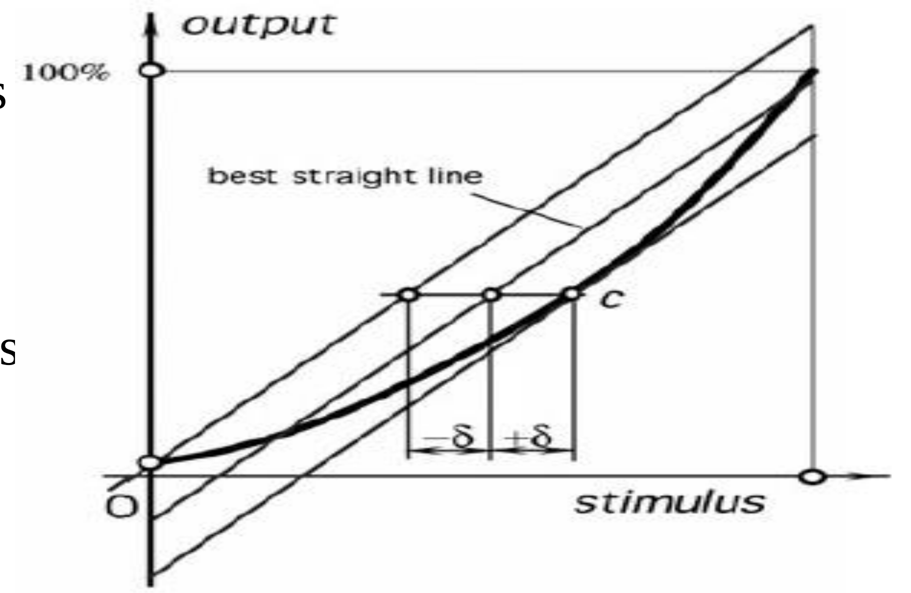


9. Nonlinearity:

- Nonlinearity error is specified for sensors whose transfer function may be approximated by a straight line.
- A nonlinearity is a maximum deviation (L) of a real transfer function from the approximation straight line.
- There are several ways to specify
- nonlinearity, depending how the line is superimposed on the transfer function;

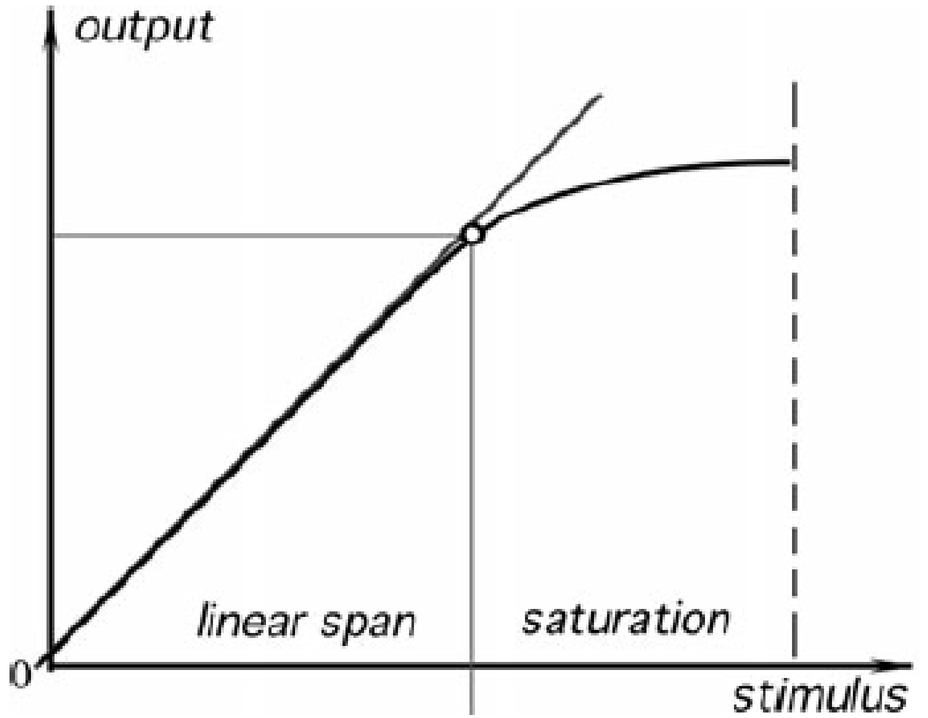
1. Use terminal points to determine output values at the smallest and highest stimulus values and to draw a straight line through these two points.
2. Use best straight line, which is a line midway between two parallel straight lines closest together and enveloping all output values on a real transfer function.





10. Saturation:

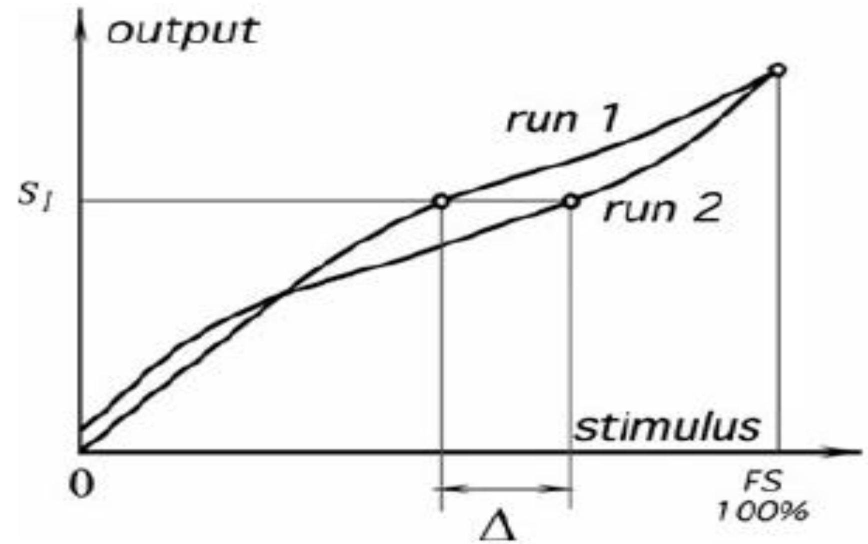
The sensor exhibits a span-end nonlinearity or saturation if further increase in stimulus does not produce a desirable output.



11. Repeatability:

Repeatability error is caused by the inability of a sensor to represent the same value under presumably identical conditions. (The same output signal S1 corresponds to two different input Signals).

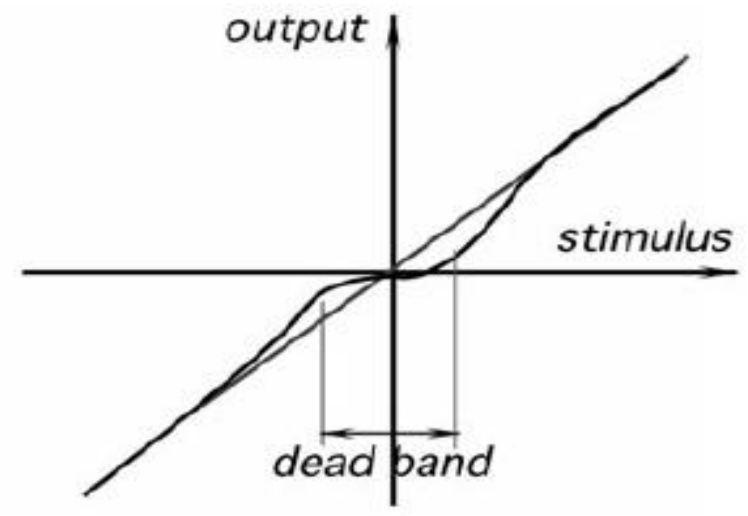
It is usually represented as % of FS:



$$\delta_r = \frac{\Delta}{FS} 100\%$$

12. Dead Band:

It is insensitivity of a sensor in a specific range of the input signals. In that range, the output may remain near a certain value (often zero) over an entire dead band zone.

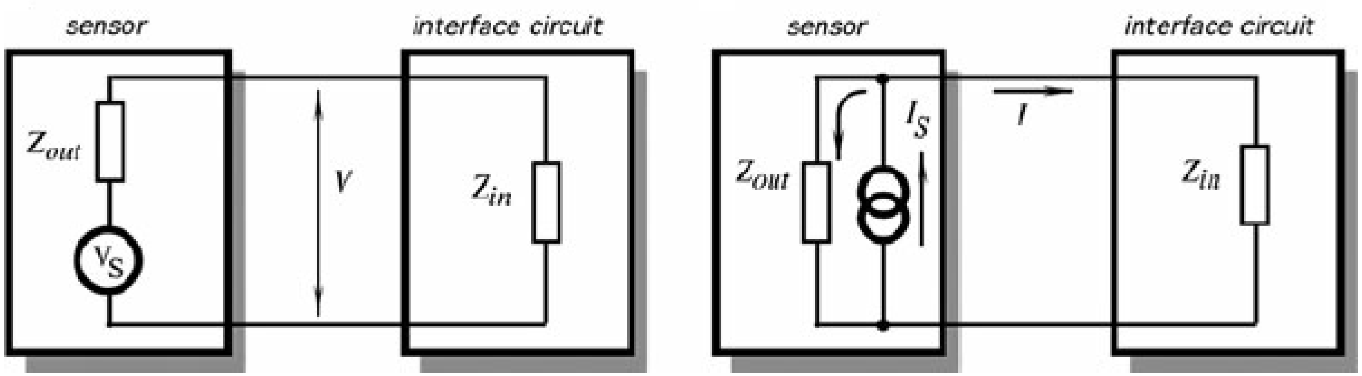


13. Resolution:

Resolution describes smallest increments of stimulus, which can be sensed.
The resolution of digital output format sensors is given by the number of bits in the data word.

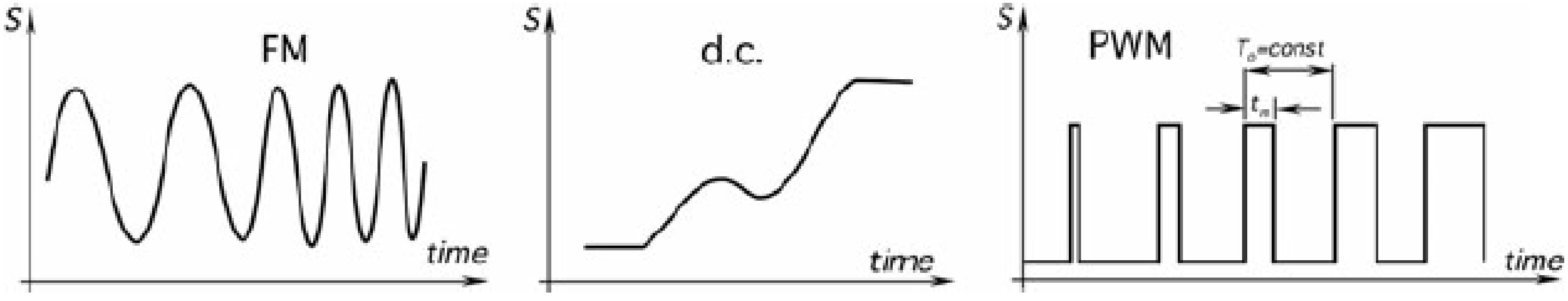
14. Output Impedance (Z_{out}):

Output impedance is important to know to better interface a sensor with the electronic circuit. The output impedance is connected to the input impedance (Z_{in}) of the circuit either in parallel (voltage connection) or in series (current connection).



15. Sensor Output Format:

- Output format is a set of the output electrical characteristics that is produced by the sensor.
- The characteristics may include voltage, current, charge, frequency, amplitude, phase, polarity, shape of a signal, time delay, and digital code.



16. Sensor Excitation:

- Excitation is the electrical signal needed for operation of an active sensor.
- Excitation is specified as a range of voltage and/or current.

17. Sensor Dynamic Characteristics:

A sensor is characterized with a time-dependent characteristic.

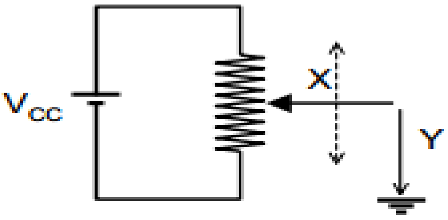
Zero-order Sensor:

- ➢ It is characterized by a transfer function that is time independent.
- ➢ Such a sensor does not incorporate any energy storage devices, like a capacitor.
- ➢ A zero-order sensor responds instantaneously. Such a sensor does not need any dynamic characteristics to be specified.

- Input and output are related by an equation of the type

$$y(t) = k \cdot x(t) \Rightarrow \frac{Y(s)}{X(s)} = k$$

- Zero-order is the desirable response of a sensor
  - No delays
  - Infinite bandwidth
  - The sensor only changes the amplitude of the input signal
- Zero-order systems do not include energy-storing elements
- Example of a zero-order sensor
  - A potentiometer used to measure linear and rotary displacements
    - This model would not work for fast-varying displacements



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17. Sensor Dynamic Characteristics:

First-order Sensor:

- A sensor that incorporates one energy storage component is specified by a first-order differential equation .
- An example: a temperature sensor where energy storage is thermal capacity.

- Inputs and outputs related by a first-order differential equation

$$a_1 \frac{dy}{dt} + a_0 y(t) = x(t) \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{a_1 s + a_0} = \frac{k}{\tau s + 1}$$

  - First-order sensors have one element that stores energy and one that dissipates it
  - Step response
    - $y(t) = Ak(1-e^{-t/\tau})$
      - A is the amplitude of the step
      - k ($=1/a_0$) is the static gain, which determines the static response
      - τ ($=a_1/a_0$) is the time constant, which determines the dynamic response
  - Ramp response
    - $y(t) = Akt - Ak\tau u(t) + Ak\tau e^{-t/\tau}$
  - Frequency response
    - Better described by the amplitude and phase shift plots

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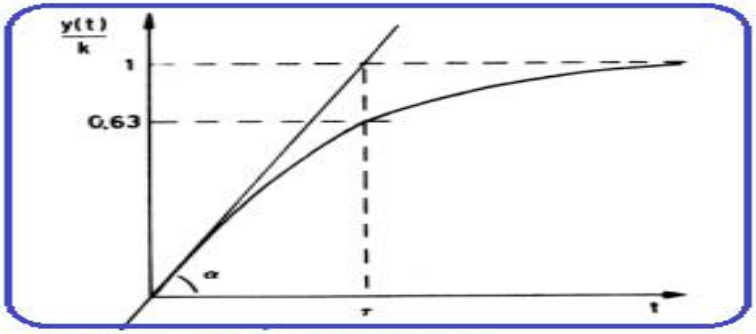
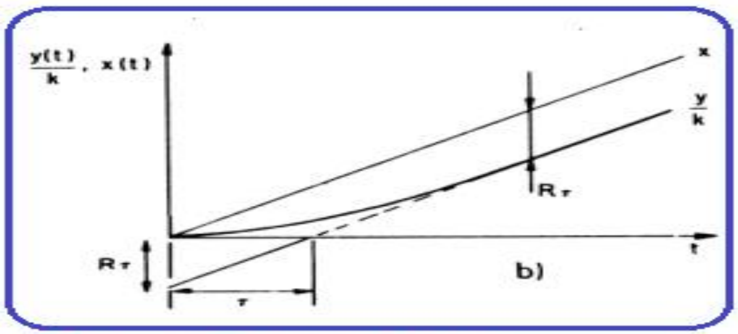
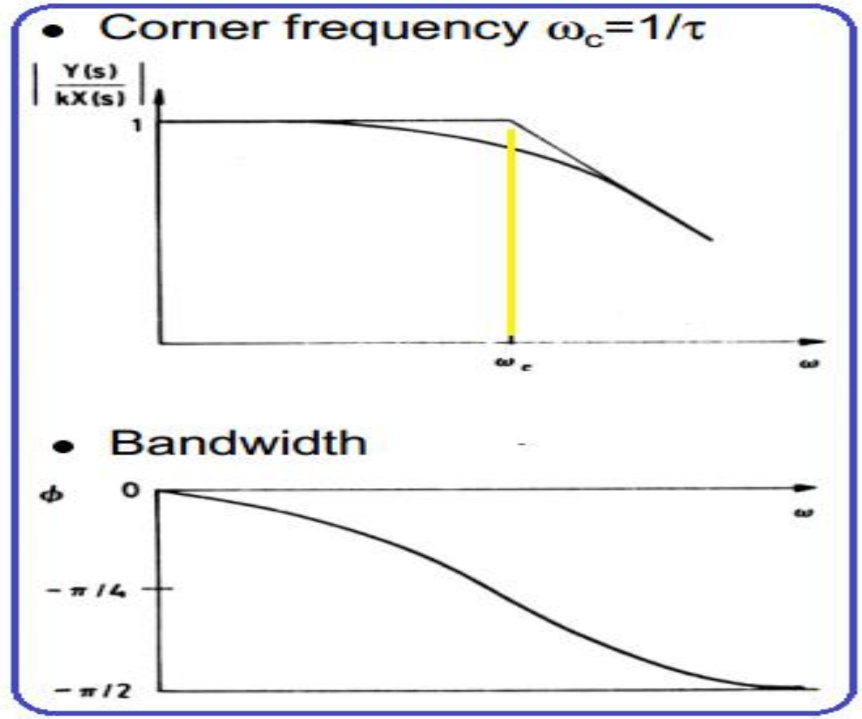
First-order sensor response:



Step response



Ramp response



Frequency response

A first order system response is as follows:

$$S = S_m(1 - e^{-t/\tau})$$

where S_m is steady-state output, and t is time.

Substituting $t = \tau$, we get;

$$\frac{S}{S_m} = 1 - \frac{1}{e} = 0.6321$$

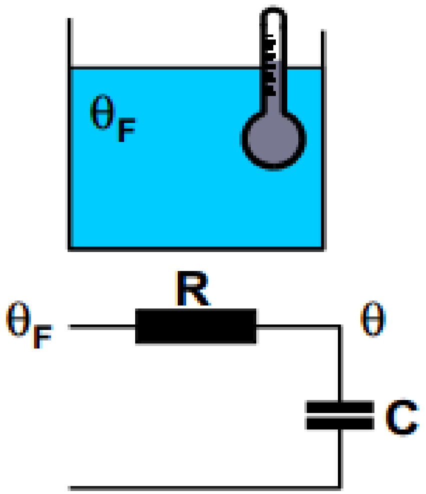
This means that after a time equal to one time constant, the response reaches about 63% of its steady-state level. Similarly, it can be shown that after two time constants, the height will be 86.5% and after three time constants it will be 95% of the level that would be reached at infinite time.

As a rule of thumb, a simple formula can be used to establish a connection between the cutoff frequency (f_c) and time constant (τ) in a first-order sensor:

$$f_c \approx \frac{0.159}{\tau}$$

Example of a first-order sensor

- **A mercury thermometer immersed into a fluid**
  - What type of input was applied to the sensor?
  - Parameters
    - C: thermal capacitance of the mercury
    - R: thermal resistance of the glass to heat transfer
    - θ_F: temperature of the fluid
    - $\theta(t)$: temperature of the thermometer
  - The equivalent circuit is an RC network



- **Derivation**
  - Heat flow through the glass $(\theta_F - \theta(t))/R$
  - Temperature of the thermometer rises as $\frac{d\theta(t)}{dt} = \frac{\theta_F - \theta(t)}{RC}$
  - Taking the Laplace transform

$$s\,\theta(s) = \frac{\theta_F(s) - \theta(s)}{RC} \Rightarrow (RCs+1)\,\theta(s) = \theta_F(s) \Rightarrow$$

$$\Rightarrow \theta(s) = \frac{\theta_F(s)}{(RCs+1)} \quad \Rightarrow \theta(t) = \theta_F\left(1 - e^{-t/RC}\right)$$

A second-order Sensor:

It incorporates two energy storage components, and represented by a second-order differential equation. The relationship between the input s(t) and output S(t) is;

$$b_2 \frac{d^2S(t)}{dt^2} + b_1 \frac{dS(t)}{dt} + b_0 S(t) = s(t)$$

We can express this second-order transfer function as;

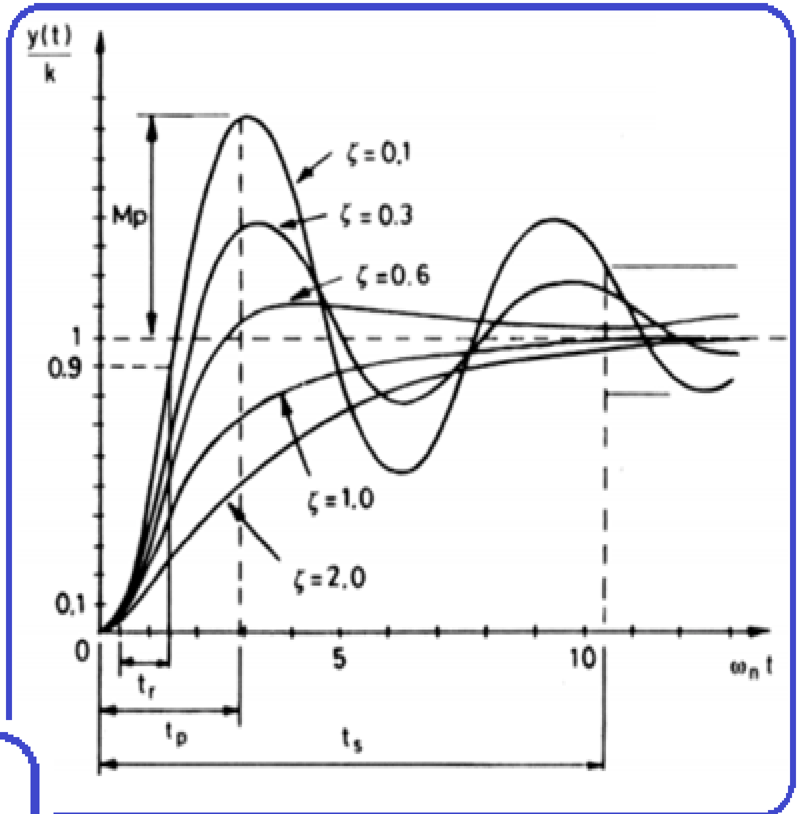
$$\frac{Y(s)}{X(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

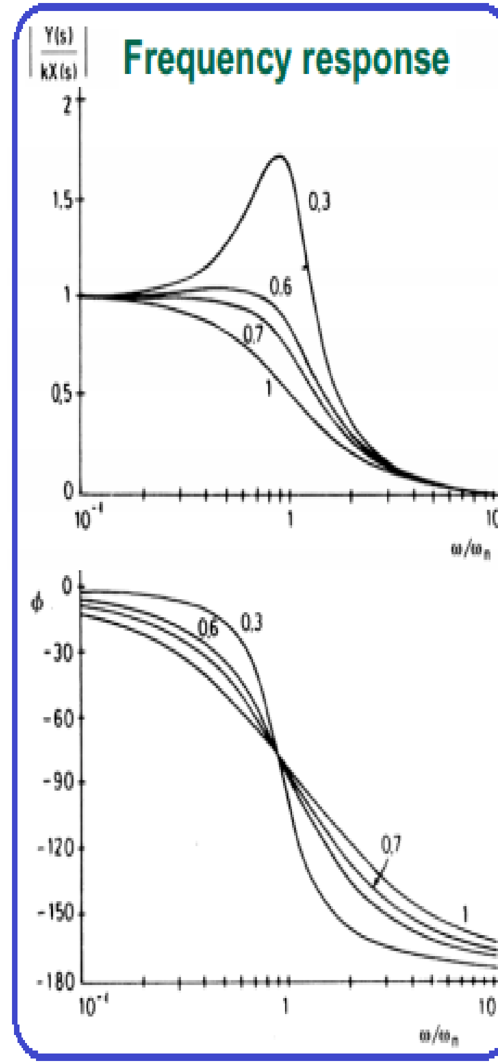
$$\text{with } k = \frac{1}{a_0},\ \zeta = \frac{a_1}{2\sqrt{a_0 a_1}},\ \omega_n = \sqrt{\frac{a_0}{a_2}}$$

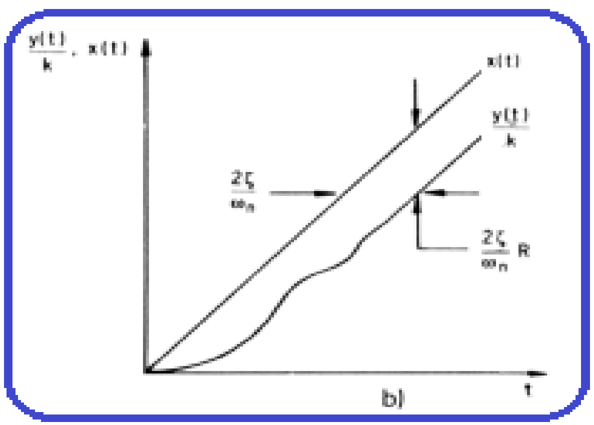
- Where
  - k is the static gain
  - ζ is known as the damping coefficient
  - ω_n is known as the natural frequency

Second-order step response

- **Response types**
  - Underdamped (ζ<1)
  - Critically damped (ζ=1)
  - Overdamped (ζ>1)
- **Response parameters**
  - Rise time (t_r)
  - Peak overshoot (M_p)
  - Time to peak (t_p)
  - Settling time (t_s)







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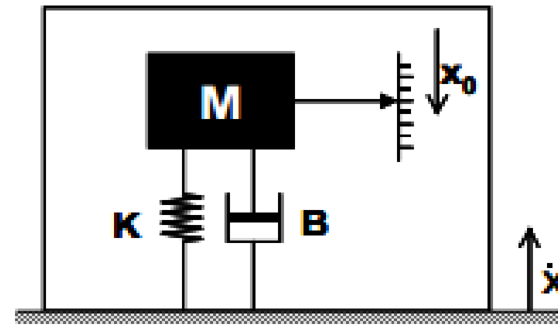
Example of second-order sensors

- **A thermometer covered for protection**
  - Adding the heat capacity and thermal resistance of the protection yields a second-order system with two real poles (overdamped)
- **Spring-mass-dampen accelerometer**
  - The armature suffers an acceleration
    - We will assume that this acceleration is orthogonal to the direction of gravity
  - x_0 is the displacement of the mass M with respect to the armature
  - The equilibrium equation is:



$$M(\ddot{x}_i - \ddot{x}_0) = Kx_0 + B\dot{x}_0$$

$$\Downarrow$$

$$Ms^2X_i(s) = X_0(s)\left[K + Bs + Ms^2\right]$$

$$\Downarrow$$

$$\frac{X_0(s)}{s^2X_i(s)} = \frac{M}{K}\frac{K/M}{s^2 + s(B/M) + K/M}$$

18. Reliability:

- It is the ability of a sensor to perform a required function under stated conditions for a stated period.
- It is expressed in statistical terms as a probability that the device will function without failure over a specified time or a number of uses.
- The procedure for predicting in-service reliability is the MTBF (mean-time between-
- failure) calculation.
- The qualification tests on sensors are performed at combinations of the worst possible conditions. One approach (suggested by MIL-STD-883) is 1000 h, loaded at maximum temperature. Three goals are behind the test:
  1. to establish MTBF;
  2. to identify first failure points that can then be strengthened by design changes;
  3. to identify the overall system practical life time.

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