Advanced Measurement Systems & Sensors (0640732)

Lecture (4)

Physical Principles of Sensing

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- Since a sensor is a converter of generally nonelectrical effects into electrical signals, one and often several transformation steps are required before the electric output signal can be generated.
- These steps involve changes of types of energy where the final step must produce electrical signal of a desirable format.
- There are several physical effects resulting from a direct generation of electrical signals in response to nonelectrical influences and thus can be used in direct sensors.

Examples:

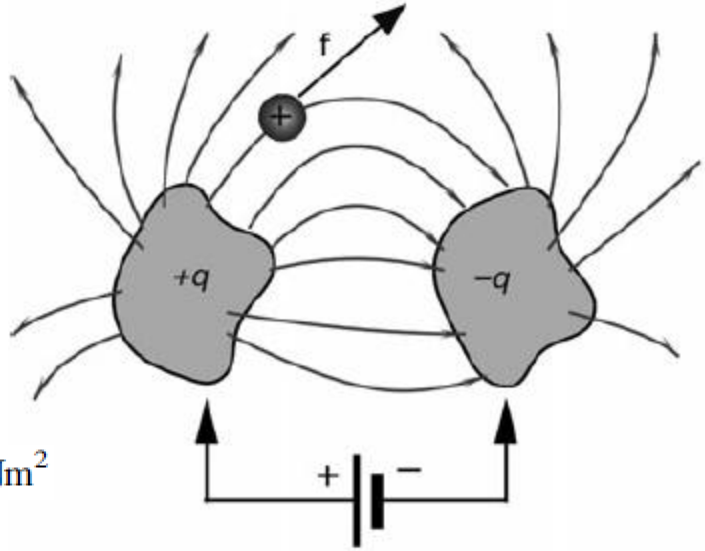
- ✓ Thermoelectric (Seebeck) effect,
- ✓ Magnetism,
- ✓ Piezoelectric effect,
- ✓ Photo effect.

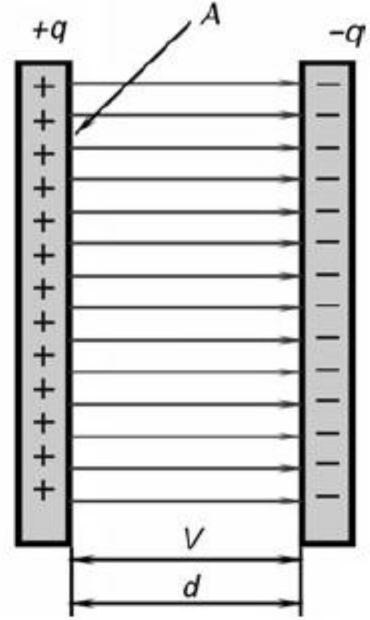
Capacitance:

$$\mathbf{E} = \frac{\mathbf{f}}{q_0}$$

$$C = \frac{q}{V} \qquad C = \frac{\varepsilon_0 A}{d}$$

where $\varepsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{C^2/Nm^2}$





A capacitor is a combination of plates which can hold an electric charge.
The +ve plate will repel the +ve test charge and the –ve will attract it, thus resulting in a combined push-pull force. Depending on the position of the test charge between the oppositely charged objects, the force will have a specific magnitude and direction which is characterized by vector f.

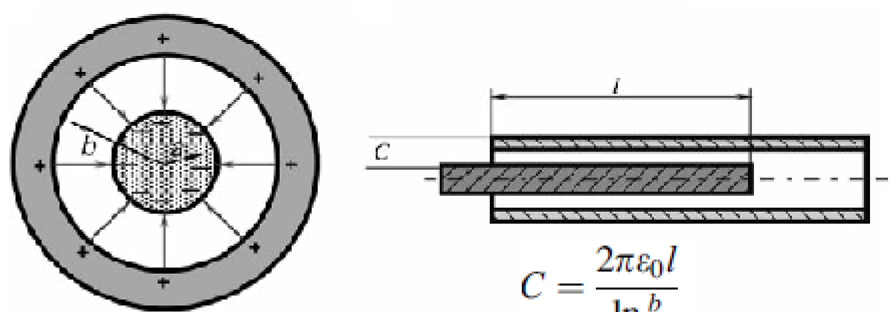
Capacitive Sensor:

The value of capacitance is the measure of a stimulus, so to change the capacitance, the stimulus needs to change one of the parameters that define the capacitance.

$$C = \frac{\varepsilon_0 A}{d}$$

- ➢ It represents a relationship between the plate area (A) and distance (d) between the plates.
- ➢ Varying one of them will change the capacitor's value, which can be measured quite accurately by an appropriate circuit.

A cylindrical capacitor consists of two coaxial cylinders of radii (a) and (b), and length (l).



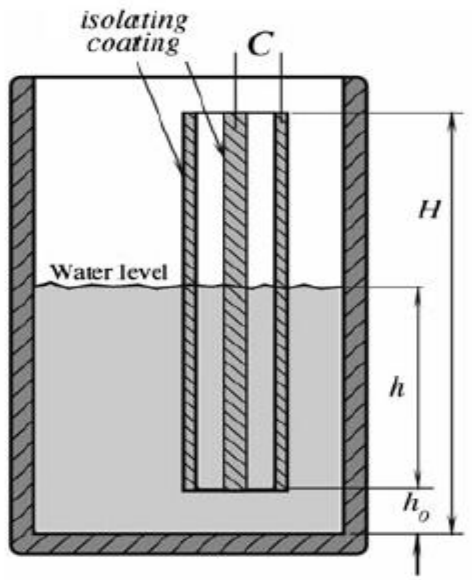
$$C = \frac{2\pi\varepsilon_0 l}{\ln\frac{b}{a}}$$

➢ The capacitance of such a sensor is in a linear relationship with the displacement (l).

Capacitive Water Level Sensor:

The sensor is fabricated in form of a coaxial capacitor where the surface of each conductor is coated with a thin isolating layer to prevent an electric short circuit through water.

The sensor is immersed in a water tank. When the level increases, water fills more and more space between the sensor's coaxial conductors, thus changing the average dielectric constant between the conductors and then changing the sensor's capacitance.

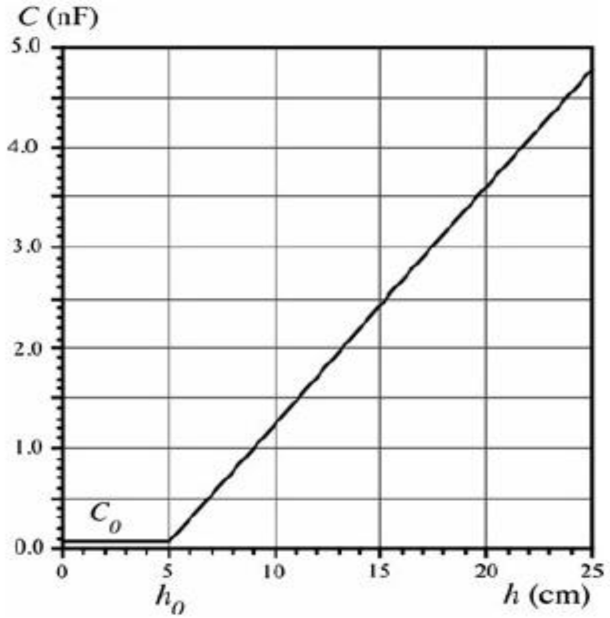


$$C = \frac{\kappa \varepsilon_0 A}{d} = \varepsilon_0 \kappa G$$

Total capacitance of the coaxial sensor is

$$C_h = C_1 + C_2 = \varepsilon_0 G_1 + \varepsilon_0 \kappa G_2$$

$$C_h = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}[H - h(1-\kappa)]$$



Induction:

Faraday's law of induction says that the induced voltage, or (e.m.f.), is equal to the rate at which the magnetic flux through the circuit changes.

$$e = -\frac{d\Phi_B}{dt}$$

If varying magnetic flux is applied to a solenoid, e.m.f. appears in every turn and all these e.m. f.s must be added;

$$V = -N\frac{d\Phi_B}{dt} = -N\frac{d(BA)}{dt}$$

Thus, the induced voltage depends on:

- moving the source of the magnetic field (magnet, coil, wire, etc.),
- varying the current in the coil or wire which produces the magnetic field,
- changing the orientation of the magnetic source with respect to the pick-up circuit,
- changing the geometry of a pick-up circuit, for instance, by stretching it or squeezing, or changing the number of turns in a coil.

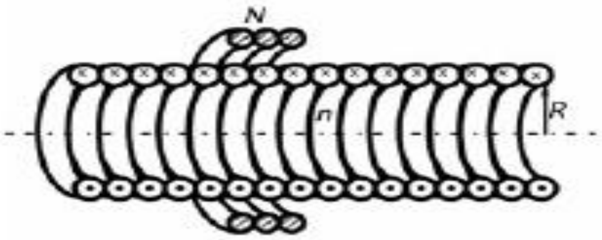
Mutual Inductance:

If two coils are brought in the vicinity of one another, and one coil conducts electric current, the magnetic field produced by that coil interacts with electrons in the second coil and induces (v_2) as an e.m.f.;

$$v_2 = -M_{21}\frac{di_1}{dt}$$

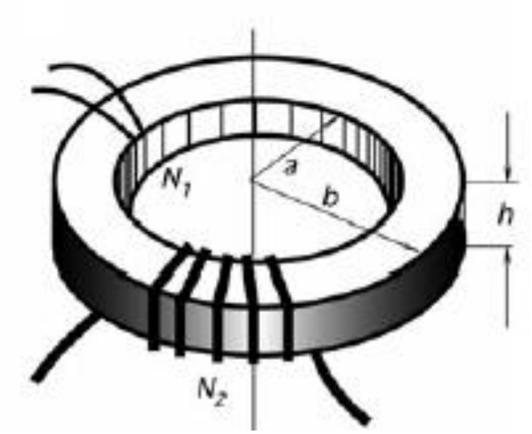
where M_{21} is the coefficient of mutual inductance between two coils.

- By varying the mutual inductance, a useful sensor can be designed. For example, a displacement sensor may have two coils, where one coil is stationary and the other is moving, causing the induced voltage to change.



- For a coil having N turns, which is placed around a long solenoid, with n turns per unit length, the mutual inductance is;

$$M = \mu_0 \pi R^2 n N$$



- For a coil placed around a toroid, the mutual inductance is defined through numbers of turns, N_1 and N_2

$$M = \frac{\mu_0 N_1 N_2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Resistance:

Specific Resistivity (ρ)**:**

It is a characteristic of a device. It depends on both the material and the geometry of the resistor. Material itself can be characterized by a specific resistivity (ρ), which is defined as;

$$\rho = \frac{E}{j}$$

where current density j= i/a (a is the area of the material cross section).

- To find the resistance of a conductor the following formula may be used:

$$R = \rho \frac{l}{a}$$

  where a is the cross-sectional area and l is the length of the conductor.

- To design a resistive sensor, you should find ways of modulating either the specific resistivity or the geometry factor of the resistor.

Resistance:

Temperature Sensitivity:

Specific resistivity or conductivity of a material changes with temperature (t) and in a relatively narrow range may be linearly approximated through temperature coefficient of resistance (α);

$$\rho = \rho_0\left(1 + \alpha\frac{t - t_0}{t_0}\right)$$

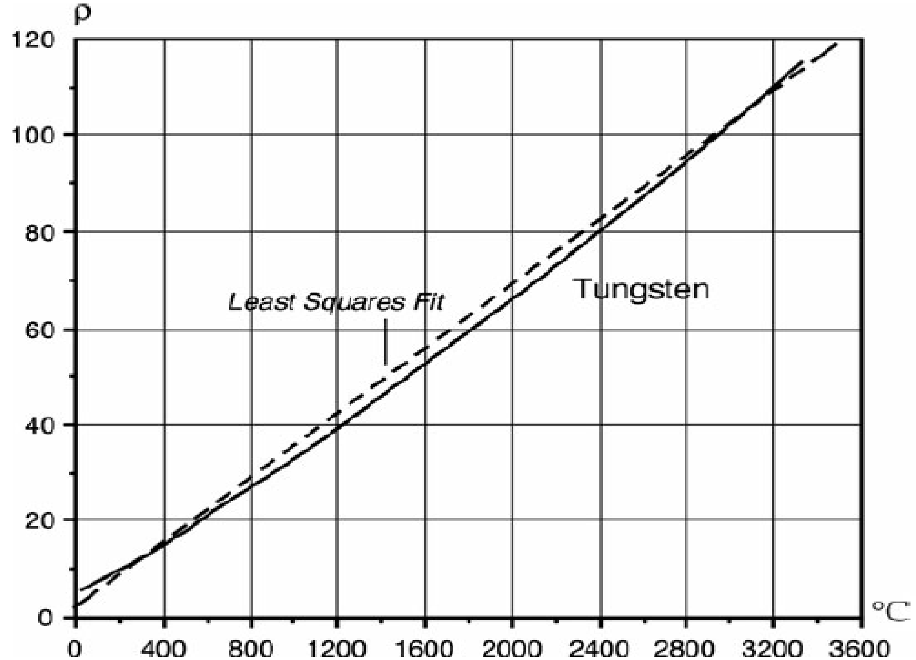
where ρ_0 is the specific resistivity at reference temperature (t_0), commonly either 0 or 25 °C. In a broader range, resistivity is a nonlinear function of temperature.

Resistivity of tungsten may be modeled by a best fit straight line.

For a broader temperature range, tungsten resistivity may be found from 2nd order equ.;

$$\rho = 4.45 + 0.0269t + 1.914 \times 10^{-6}t^2$$

where t is temperature in °C and ρ is in Ω·m.



Specific resistivity of tungsten as function of temperature

Temperature Sensors:

In sensor technologies, it is desirable to have a “bad” resistor whose temperature coefficient of resistivity (α) is high and predictable ($\alpha \geq 10^{-5}$).

A strong (α) allows to fabricate a temperature sensor known as a **thermistor** (thermal and resistor) and the **Resistive Temperature Detector** (RTD).

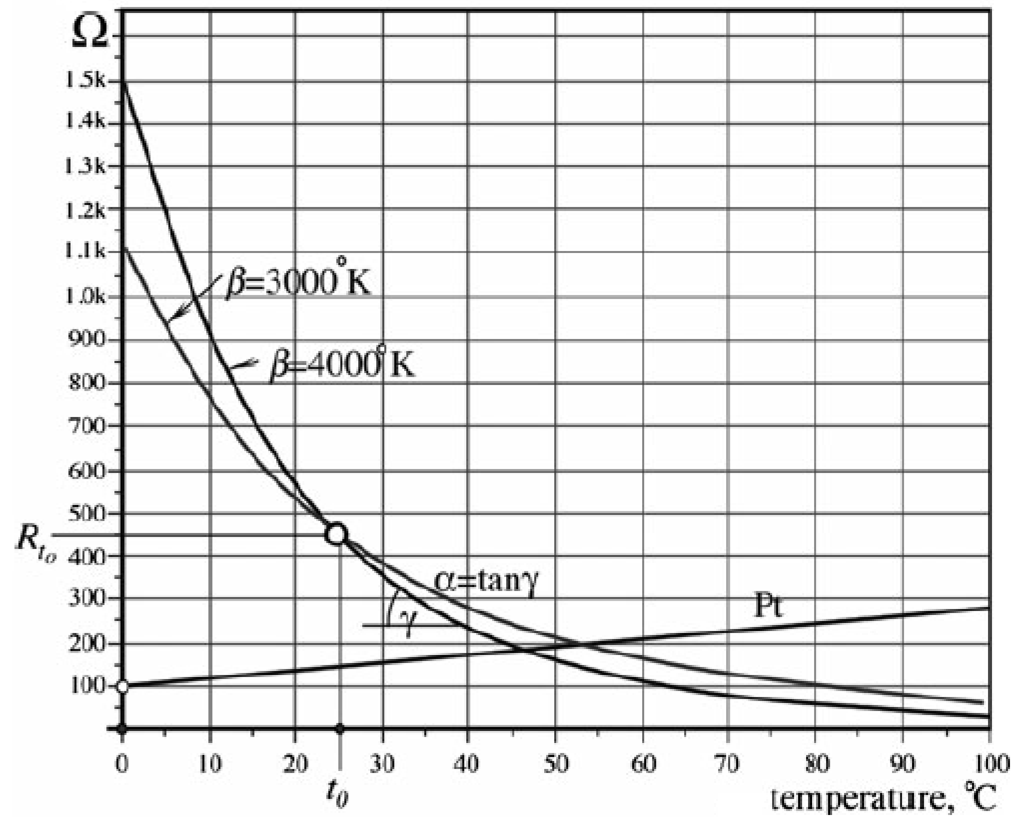
The most popular RTD is a platinum (Pt) temperature sensor, which operates over a temperature range from about 200°C to over 600°C.

- For a calibrating resistance R_0 at 0°C, use best fit straight line;

$$R = R_0(1 + 36.79 \times 10^{-4} t)$$

- For a better accuracy use a 2nd order polynomial which gives accuracy better than 0.01°C

$$R = R_0(1 + 39.08 \times 10^{-4} t - 5.8 \times 10^{-7} t^2)$$



Example:

If a Pt RTD sensor at a reference temperature 0°C has resistivity R_0=100 Ω, at +150°C;

The linear approximation gives;

$$R = 100 \cdot \left(1.0036 + 36.79 \times 10^{-4} \cdot 150\right) = 155.55\Omega$$

The 2nd order approximation gives;

$$R = 100\left(1 + 39.08 \times 10^{-4} \cdot 150 - 5.8 \times 10^{-7} \cdot 150^2\right) = 157.32\ \Omega$$

➢The difference is 1.76 Ω, which is equivalent to an error at °150C of approximately 4.8C (nearly 3%).

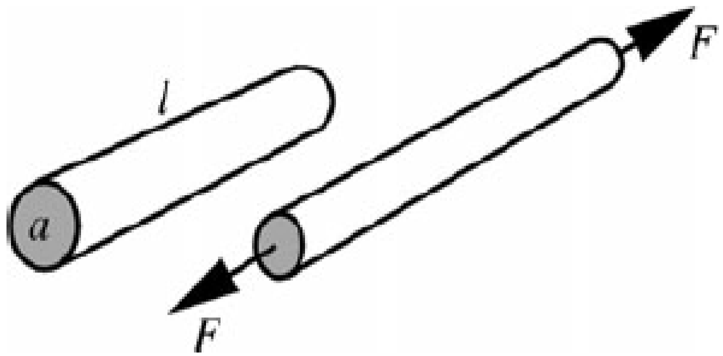
Resistance:

Strain Sensitivity: ⟹ Piezoresistive effect:

Electrical resistance changes when the material is mechanically deformed.

The applied stress (σ);

$$\sigma = \frac{F}{a} = E\frac{dl}{l}$$



where;

E is Young's modulus of the material,

F is the applied force, a is the cross-sectional area.

The ratio (***dℓ/ℓ =e***) is called strain, which is a normalized deformation of the material.

- When a cylindrical conductor (wire) is stretched by applied force (F), the material Volume (v) stays constant, while the length increases and the cross sectional area becomes smaller;

$$R = \frac{\rho}{v}l^2$$

- Then the sensitivity of resistance can be calculated;

$$\frac{dR}{dl} = 2\frac{\rho}{v}l = S_e e$$

where Se is known as the gauge factor or sensitivity of the strain gauge element.

For metallic wires it ranges from 2 to 6.

For the semiconductor gauges is between 40 and 200.

Resistance:

Moisture Sensitivity:

By selecting material for a resistor, one can control its specific resistivity. A moisture-dependent resistor can be fabricated of a hygroscopic material whose specific resistivity is strongly influenced by concentration of the absorbed water molecules. This is the basis for the resistive humidity sensors (**hygristors)**.

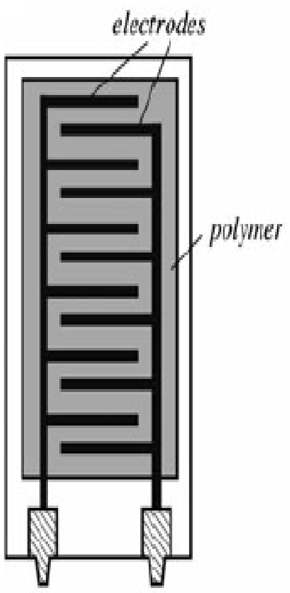
Hygristor Sensor:

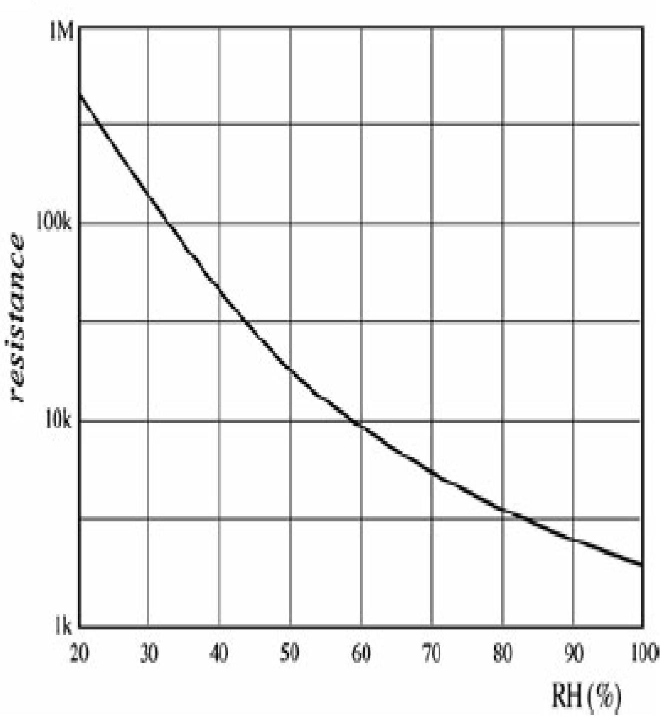
It is comprised of a ceramic substrate that has two silk-screen printed electrodes. The electrodes are metal conductors, covered by hygroscopic semiconductive gel which forms a matrix to hold the conductive particles. Then a resistor is formed between two electrodes.

The response time: 10 to 30 s.

The resistance range: 1 KΩ to 100M Ω.

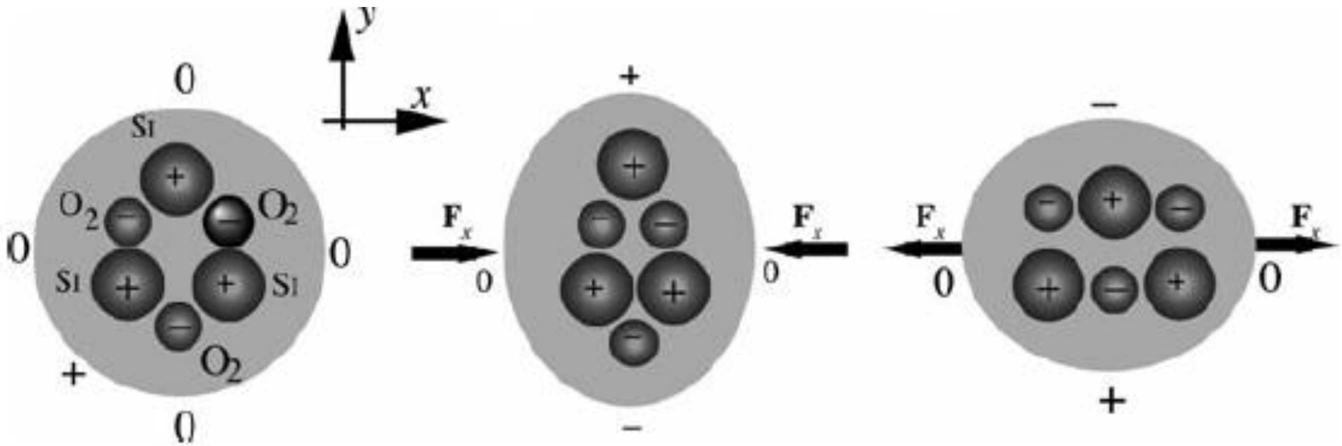
The hygristors are active sensors require an excitation signal to produce an electrical output.





Piezoelectric Effect:

The word **piezo** comes from the Greek, it means "**to press**". The piezoelectric effect is generation of electric charge by a crystalline material subjected to stress.
The effect exists in natural crystals, such as quartz (SiO_2), and poled (human-made ceramics) and some polymers, such as PVDF.



A crystalline material can develop electric charge on its surface in response to a mechanical deformation.

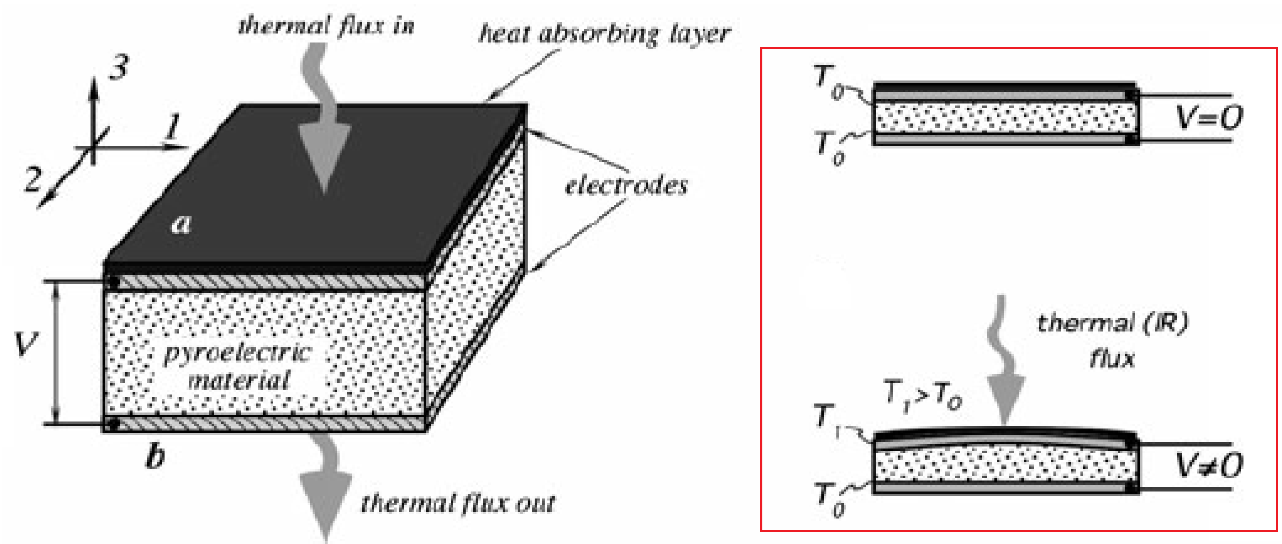
- ➢ When external force (Fx) is applied along the x-axis, the crystal develops an electric charge along the y-axis.
- ➢ If the crystal is stretched along the x-axis, a charge of opposite polarity is built along the y-axis, which is a result of a different deformation.

Pyroelectric Effect:

The pyroelectric effect is very closely related to the piezoelectric effect.

The pyroelectric materials are crystalline substances capable of generating an electrical charge in response to heat flow.

Like piezoelectrics, the pyroelectric materials are used in form of thin slices or films with electrodes deposited on the opposite sides to collect the thermally induced charges.

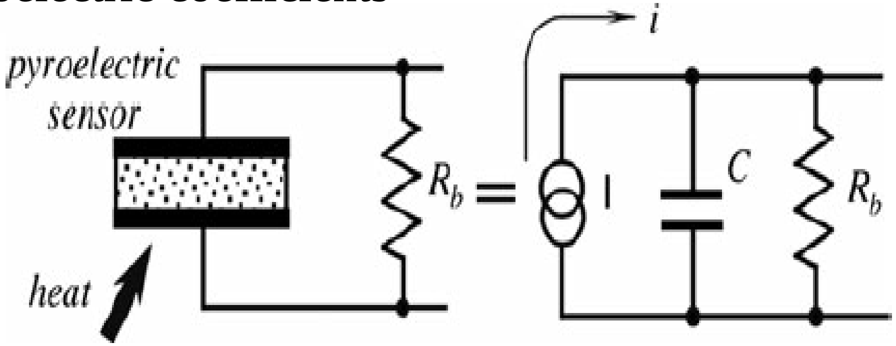


The pyroelectric sensor is essentially a capacitor, which can be electrically charged by flux of heat. The detector does not require any external electrical bias, it is a direct converter of heat into electricity. It needs only an appropriate electronic interface circuit to measure the charge.

The pyroelectric sensor output signal is either charge (current) or voltage, depending on the application.
Being a capacitor, the pyroelectric device is discharged when connected to a resistor (R_b). Electric current through the resistor and voltage across the resistor represent the heat flow induced charge. It can be characterized by two pyroelectric coefficients

$$P_Q = \frac{dP_s}{dT} \quad \text{Pyroelectric charge coefficient}$$

$$P_V = \frac{dE}{dT} \quad \text{Pyroelectric voltage coefficient}$$



where;
P_s is the spontaneous polarization (electric charge),
E is the electric field strength, and T is the temperature.

Both coefficients are related by way of the electric permitivity (ε_r) and dielectric constant (ε_0);

$$\frac{P_Q}{P_V} = \frac{dP_s}{dE} = \varepsilon_r \cdot \varepsilon_0$$

If a pyroelectric material is exposed to a heat source, its temperature rises by ΔT, then;

$$\boxed{\Delta Q = P_Q A \Delta T} \quad \text{and} \quad \boxed{\Delta V = P_V h \Delta T}$$

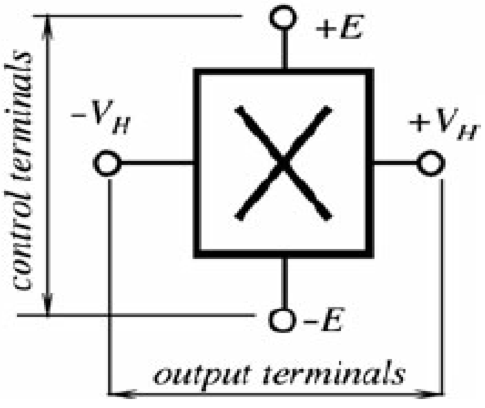
$$\boxed{C_e = \frac{\Delta Q}{\Delta V} = \varepsilon_r \varepsilon_0 \frac{A}{h}} \qquad \boxed{\Delta V = P_Q \frac{A}{C_e} \Delta T = P_Q \frac{\varepsilon_r \varepsilon_0}{h} \Delta T}$$

- ➢ It is clear that the peak output voltage is proportional to the sensor's temperature rise (ΔT) and pyroelectric charge coefficient (ε_r ε_0) and inversely proportional to its thickness (h).

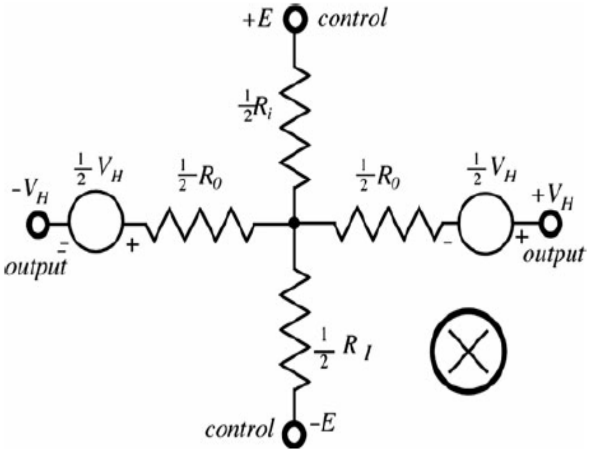
Hall Effect:

The Hall sensors are used to detect magnetic fields, position, and displacement of objects.

The effect is based on the interaction between moving electric carriers (electrons) and an external magnetic field.



A linear Hall effect sensor is usually packaged in a four-terminal housing. Control terminals for applying the control current and a resistance (R_i) between them. Differential output terminals for observing output voltage and a resistance (R_o) between them.



The sensor is specified by its resistances (R_i and R_o), across both pairs of terminals, the offset voltage at no magnetic field applied, the sensitivity and the temperature coefficient of sensitivity.

Thermoelectric Effects

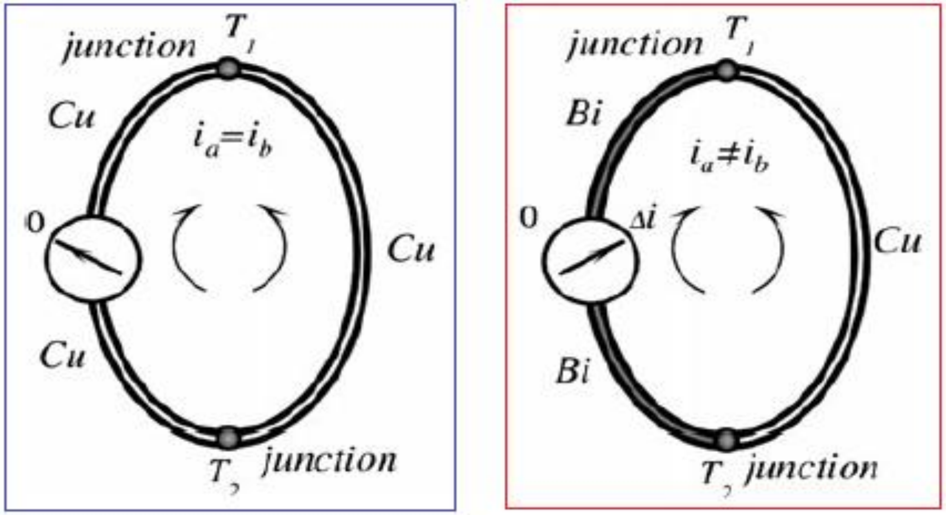
Seebeck Effect:

Thomas J. Seebeck (1770–1831) is an Estonian born and Berlin educated physician.
If one end of a conductor is placed into a cold place and the other end into a warm place, energy (heat) will flow from the warm to cold part. The intensity of the heat flow is proportional to the thermal conductivity of the conductor.

$$dV_a = \alpha_a dT$$

Where; dT is the temperature gradient, and α_a is the absolute Seebeck coefficient of the material.

- Joints of identical metals produce zero net current at any temperature difference, while joints of dissimilar metals produce net current Δi.



- Coefficient α_a is a unique property of a material. When a combination of two dissimilar materials (A and B) is used, then,

$$\alpha_{AB} = \alpha_A - \alpha_B$$

$$dV_{AB} = \alpha_{AB} dT$$

Example:

The voltage (as function of a temperature gradient for a T-type thermocouple with a high degree of accuracy) can be approximated by a 2nd order equation;

$$V_{AB} = a_0 + a_1 T + a_2 T^2 = -0.0543 + 4.094 \times 10^{-2} T + 2.874 \times 10^{-5} T^2$$

$$\alpha_T = \frac{dV_{AB}}{dT} = \alpha_1 + 2\alpha_2 = 4.094 \times 10^{-2} + 5.7481 \times 10^{-5} T$$

It is seen that;

- The coefficient is a linear function of temperature.
- Sometimes this coefficient α_{AB} is called the sensitivity of a thermocouple junction.
- The Seebeck coefficient does not depend on the nature of the junction: metals may be pressed together, welded, fused, twisted, etc.
- The Seebeck effect is a direct conversion of thermal energy into electric energy.

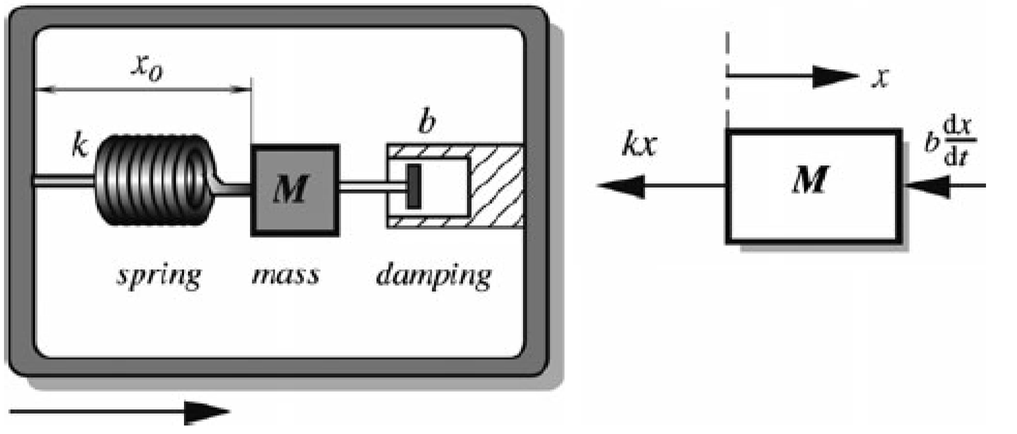
Dynamic Models of Sensor Elements:

- Mathematical modeling of a sensor is a powerful tool in assessing its performance.
- The modeling may address two issues: static and dynamic.
- The models usually deal with the sensor's transfer function.
- The dynamic models may have several independent variables, such as time.
- The mathematical models are formed by applying physical laws. A sensor is divided into simple elements and each element is considered separately. Once the equations describing the elements have been formulated, individual elements can be recombined to yield the mathematical model of the original sensor.

Mechanical	Thermal	Electrical	
Mass M $F = M\frac{d(v)}{dt}$	Capacitance C $Q = C\frac{dT}{dt}$	Inductor L $V = L\frac{di}{dt}$	Capacitor $i = C\frac{dV}{dt}$
Spring k $F = k\int v dt$	Capacitance C $T = \frac{1}{C}\int Q dt$	Capacitor C $V = \frac{1}{C}\int i dt$	Inductor L $i = \frac{1}{L}\int V dt$
Damper b $F = bv$	Resistance R $Q = \frac{1}{R}(T_2 - T_1)$	Resistor R $V = Ri$	Resistor R $i = \frac{1}{R}V$

Mechanical Elements:

Mechanical model of an accelerometer:



Applying Newton's 2nd law of motion gives;

$$Mf = -kx - b\frac{dx}{dt}$$

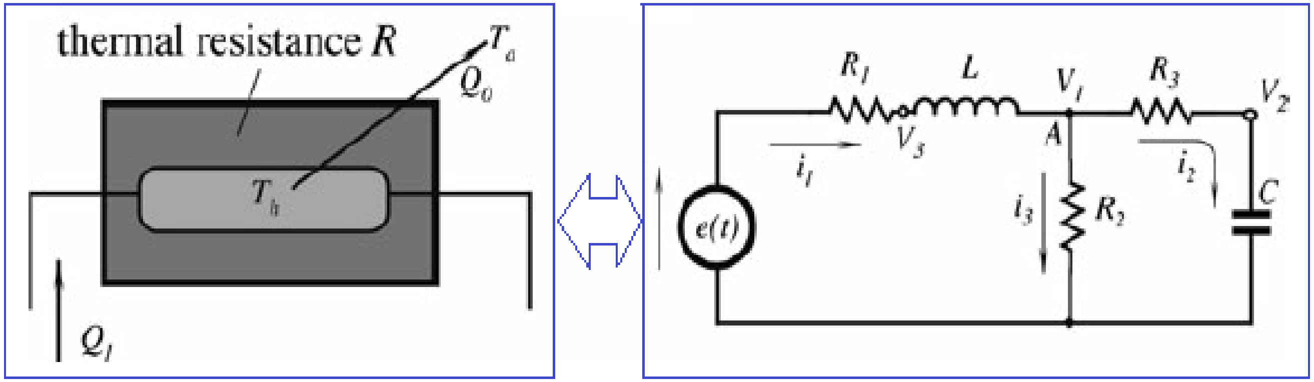
where; k is spring stiffness, x is displacement, b is the damping coefficient, and f is the acceleration of the mass (M) relative to the earth, and is given by;

$$f = \frac{d^2x}{dt^2} - \frac{d^2y}{dt^2}$$

$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = M\frac{d^2y}{dt^2}$$

The accelerometer output signal may have the oscillating shape. By selecting an appropriate damping coefficient (b), the output signal may be brought to a critically damped state which, in most cases, is a desirable response.

Thermal model of a heating element:



A heating element having temperature (T_h) is coated with insulation. The temperature of the surrounding air is (T_a). The rate of heat supply (Q_1) to the element, and the rate of heat loss (Q_o), then;

$$C\frac{dT_h}{dt} = Q_1 - Q_0 \quad \text{and} \quad Q_0 = \frac{T_h - T_a}{r}$$

$$\frac{dT_h}{dt} + \frac{T_h}{rC} = \frac{Q_1}{C} + \frac{T_a}{rC}$$

where C is the thermal capacity of a body, and r is the thermal resistance.

- ➢ This is a 1st DE, which is typical for thermal systems.
- ➢ A response of a simple thermal element may be characterized by a thermal time constant (τ=Cr), which is equal to time which takes to reach about 37% of the initial temperature gradient.

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