Advanced Measurement Systems & Sensors
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Lecture (8)

Velocity and Acceleration Measurements

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Introduction:

- The measure of velocity depends on the scale of an object. For example, speed of a large object may be very efficiently determined by GPS.
- When the position of a vehicle is determined with a periodic rate, computation of its velocity is no problem.

- Acceleration is a dynamic characteristic of an object.
- Acceleration (a) can be obtained via inertial force (F) on a mass (m) subjected to acceleration (a) of the moving object:

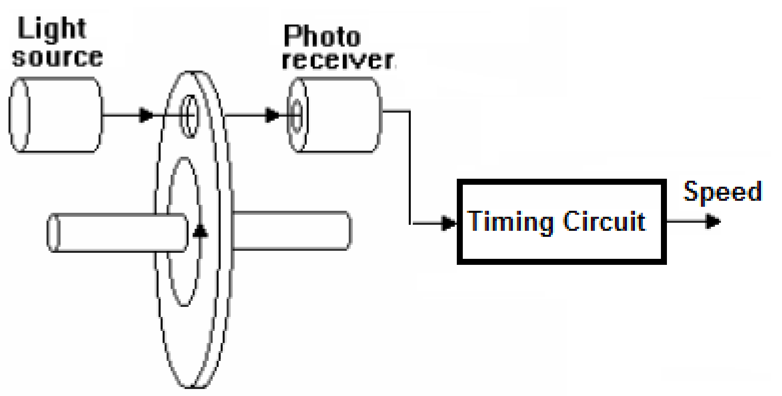
$$F = -\,m\,a$$

- The inertial force can be measured either through strain (if deformation is min) or through the deformation of elastic element.

SPEED TRANSDUCERS

Speed transducers are widely used for measuring the output speed of a rotating object. There are many types
using different principles and most of them produce an electrical output.

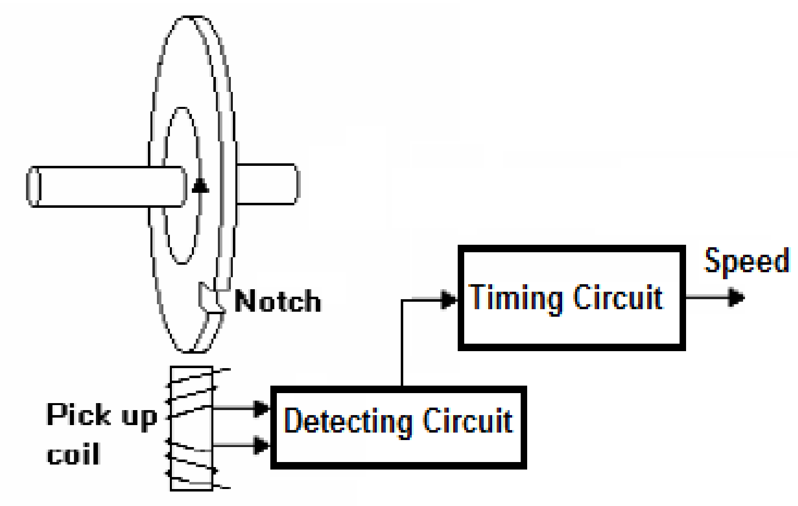
1. Optical Devices:



- These use a light beam and a light sensitive cell. The beam is either reflected or interrupted so that pulses are produced for each revolution.
- The pulses are then counted over a fixed time and the speed obtained.
- Electronic processing is required to time the pulses and turn the result into an analogue or digital signal.

2. Magnetic Devices:

- These use an inductive coil placed near to the rotating body. A small magnet on the body generates a pulse every time it passes the coil.
- If the body is made of ferrous material, it will work without a magnet.
- A discontinuity in the surface such as a notch will cause a change in the magnetic field and generate a pulse.
- The pulses must be processed to produce a digital output.



3. Tachometers:

- A **tachometer** is an instrument measuring the rotation speed of a shaft or a disk. It usually displays the revolution per minute (rpm) on a calibrated analogue dial, but digital displays are increasingly common.

There are two types, A.C. and D.C Tachometers;

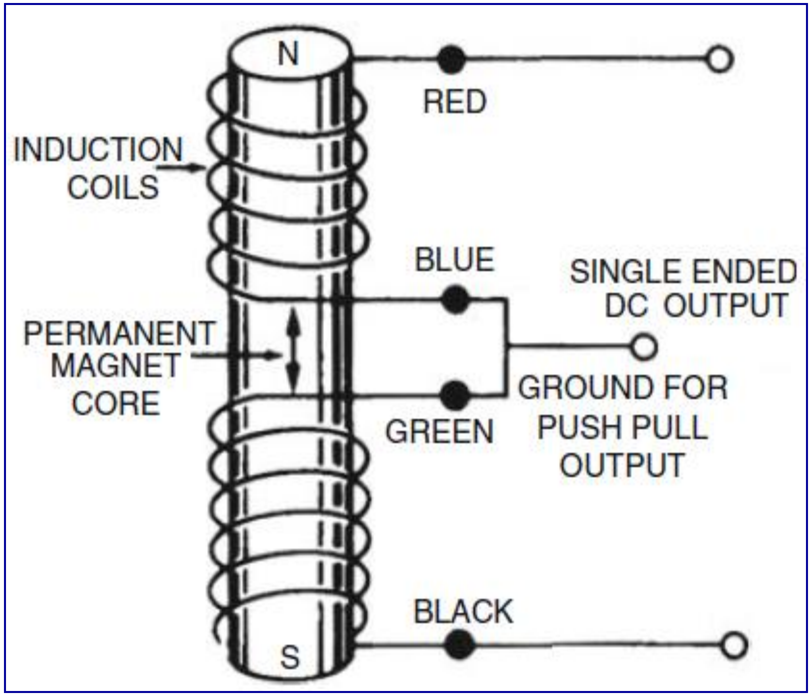
- The A.C. type generates a sinusoidal output. The frequency of the voltage represents the speed of rotation. The frequency must be counted and processed.
- The D.C. type generates a voltage directly proportional to the speed.
- Both types must be coupled to the rotating body.
- Very often the tachometer is built into electric motors to measure their speed.

Electromagnetic Velocity Sensor:

- Moving a magnet through a coil of wire will induce a voltage in the coil according to Faraday's law. This voltage is proportional to the magnet's velocity and the field strength .

$$v = -\frac{d(n\Phi_B)}{dt}$$

- The north pole of the magnet induces a current in one coil, while the south pole induces a current in the other coil. The two coils are connected in a series opposite direction to obtain an output proportional to the magnet's velocity.
- The output voltage of the coil is directly proportional to the magnet's relative velocity over its working range.
- This design is very similar to an LVDT position sensor, except that LVDT is an active sensor with a moving ferromagnetic core, while the velocity sensor is a passive device with a moving permanent magnet.



Sonar: **SO**und **N**avigation **A**nd **R**anging

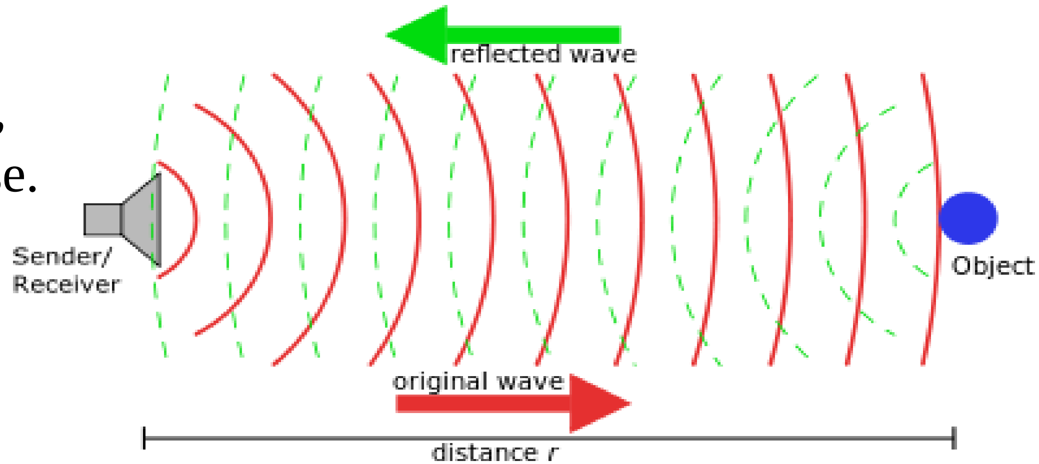
It is a technique that uses SOUND propagation to navigate, communicate with, or detect objects on or under the surface of the water.

Sonar are used as a means of acoustic location and measurement of the echo of objects in the water. There are two types of sonar technology;

Passive sonar: is essentially listening for the sound made by objects.

Active sonar: is emitting pulses of sounds and listening for echoes.

- It uses a sound transmitter and a receiver. It creates a pulse of sound, and then listens for echo of the pulse.
- This pulse of sound is created electronically using a sonar project (signal generator), power amplifier and electro-acoustic transducer.



- To measure the distance to an object, the time from transmission of a pulse to reception is measured and converted into a range by knowing the speed of sound.

SODAR: SOnic Detection And Ranging

It is a meteorological instrument used to measure the scattering of sound waves by atmospheric turbulence.

SODAR systems are used to measure wind speed at various heights above the ground.

SODAR systems are like RADAR (RAdio Detection And Ranging) and LIDAR (LIght raDAR) systems except that sound waves rather than radio or light waves are used for detection.

- The horizontal components of the wind velocity are calculated from the measured Doppler shifts and the specified tilt angle from the vertical.
- The vertical range: (0.2 to 2) km and is a function of frequency, power output, atmospheric stability, and the noise environment.
- Operating frequencies range: from less than 1000 Hz to over 4000 Hz, with power levels up to several hundred watts.

Accelerometer:

It is in general used for a device which measures linear acceleration.

Acceleration	m/s^2	Acceleration of a moving object whose velocity varies by 1 m/s in 1 second.
Angular acceleration	rad/s^2	Angular acceleration of a moving object which is rotating around a fixed axis with an angular velocity that varies by 1 radian per second, in 1 second.

Absolute Accelerometer:

It measures the inertial force exerted on the seismic mass. It is attached to the measured object and does not need a reference.

Relative Accelerometer:

It measures the distance between the measured object and reference point. The reference point should be stable or moving with constant speed.

Relative accelerometers are mainly used to measure vibrations from a distant stable point (e.g. by laser vibrometers).

Applications of Accelerometer:

- They are components of inertial navigation systems for aircraft and missiles.
- They are used to detect and monitor vibration in rotating machinery.
- They are used in tablet computers and digital cameras so that images on screens are always displayed upright.
- They are used for flight stabilization.

Acceleration Sensors:

They are frequently used to determine the speed and the position of various vehicles, such as planes, ships, cars, robots, etc.

They can be classified according to the physical principle they use:

- **Direct measurement** of a force (piezoelectric sensor, sensor with force balance).
- **Indirect measurement**, by means of displacement or deformation of a sensing element.

The inertial force can be measured either through strain (if deformation is minimum) or through the deformation of elastic element.

The different families of accelerometers:

Secondary Measurand	Types of Accelerometers
force → strain	– piezoelectric accelerometers – piezoresistive accelerometers – resonators
force → displacement	– potentiometric accelerometers – capacitive accelerometers – inductive accelerometers – servo controlled accelerometers – optical accelerometers

Elastic sensing elements:

If a force is applied to a spring, then the amount of extension or compression of the spring is approximately proportional to the applied force.

Elastic elements are also commonly used for measuring torque, pressure and acceleration, which are related to force by the equations:

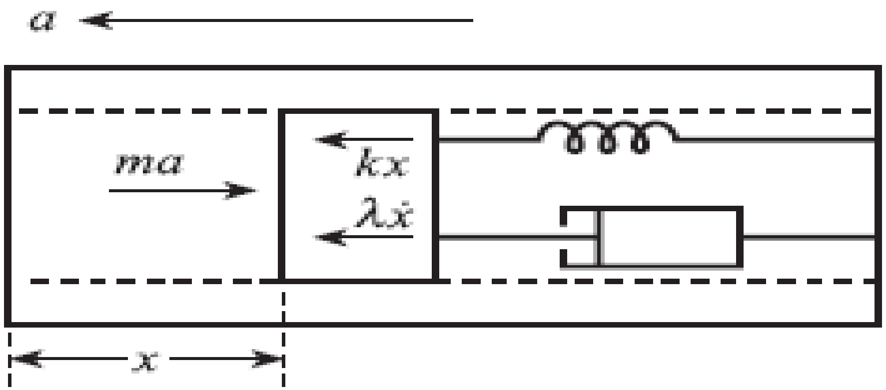
$$\text{torque} = \text{force} \times \text{distance}$$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{acceleration} = \frac{\text{force}}{\text{mass}}$$

- In a measurement system an elastic element will be followed by a suitable secondary displacement sensor, such as; potentiometer, strain gauge or LVDT, which converts displacement into an electrical signal.
- Elastic sensing elements have associated mass and damping (resistance) as well as spring characteristics.
- The dynamics of a mass–spring–damper force sensor has a 2nd order T.F.;

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + 1}$$

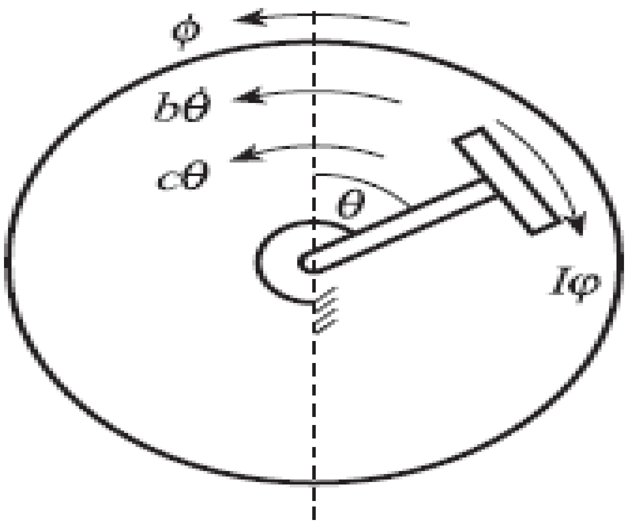


(a) Linear accelerometer

$$m\ddot{x} + \lambda\dot{x} + kx = ma$$

$$\frac{\Delta\bar{x}}{\Delta\bar{a}}(s) = \frac{\frac{1}{\omega_n^2}}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

$$\omega_n = \sqrt{\frac{k}{m}},\ \xi = \frac{\lambda}{2\sqrt{km}},\ K = \frac{1}{\omega_n^2}$$



(b) Angular accelerometer

$$I\ddot{\theta} + b\dot{\theta} + c\theta = I\phi$$

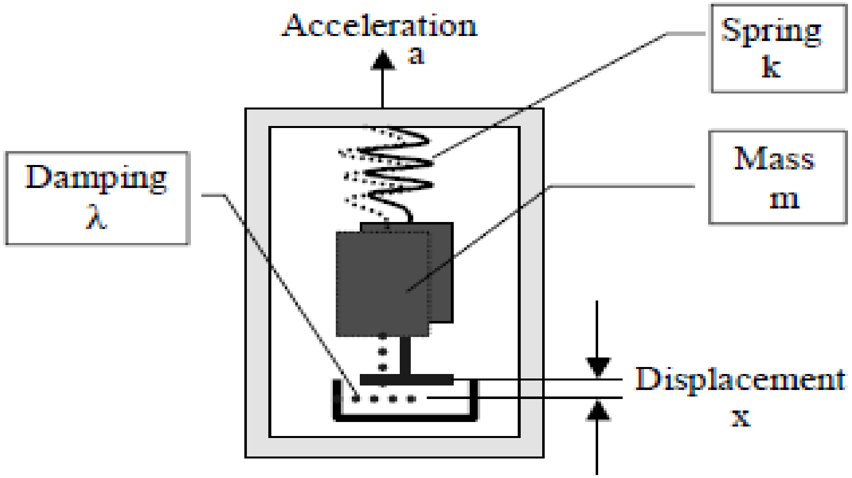
$$\frac{\Delta\bar{\theta}}{\Delta\bar{\phi}}(s) = \frac{\frac{1}{\omega_n^2}}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

$$\omega_n = \sqrt{\frac{c}{I}},\ \xi = \frac{b}{2\sqrt{cI}},\ K = \frac{1}{\omega_n^2}$$

- The equation of the movement is;

$$ma = m\left(\frac{d^2x}{dt^2}\right) + \lambda\left(\frac{dx}{dt}\right) + kx$$

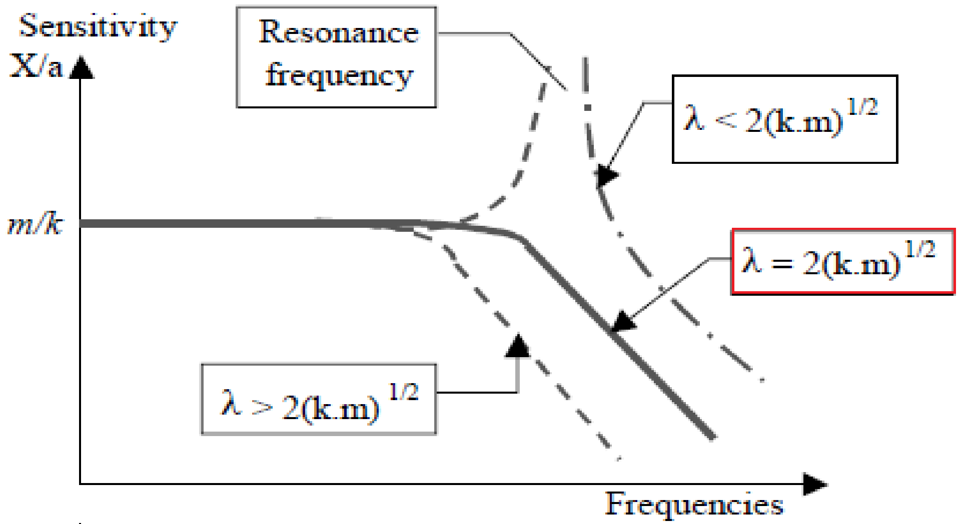
where: k = stiffness of spring,
λ = damping coefficient, t = time



- In a stable state, the relationship between (X) and (a) is: $\frac{x}{a} = \frac{m}{k}$

- The sensitivity of the accelerometer (x/a) is proportional to (m/k).
- The resonance frequency (f_r) of the system is;

$$f_r = \frac{1}{2\pi}\sqrt{(m/k)}$$



- The condition to obtain optimal freq. response and avoid deterioration of the accelerometer when resonance occurs is given; $\lambda \geq 2\sqrt{km}$

Application Ranges of Accelerators:

Depending on acceleration levels and frequency ranges;

1. Static and low-frequency acceleration:

- Ranges: Frequency: from DC to 50 Hz,
  Amplitude: from 0 to approx. 10 g.
- High precision is usually required.

FIELDS	APPLICATIONS
POSITION CONTROL	Military machines, cars, railways, stabilization of platforms
INCLINOMETRY	On-board instrumentation: aeronautics, vehicles, building machines, tools
INSTRUMENTATION	Test benches, vehicle tests, swell studies, process monitoring, transport
METROLOGY/CALIBRATION	Accelerometer calibration benches: centrifugal machines, control accessories on various measuring benches
NAVIGATION, GUIDANCE, PILOTING	Planes, boats, military and civil terrestrial vehicles. robots
SEISMICS	Vulcanology, oil research (geophones), monitoring for houses and road building
SECURITY SYSTEMS	Automobile, armament, nuclear power, home automation. aeronautics
MEDICAL	Measurement of tremor

2. Vibration:

Vibration frequencies range from 7 Hz to 10 kHz, with amplitudes up to 100 g.

FIELDS	APPLICATIONS
MODAL ANALYSIS	Research and Design: automotive, aeronautics, electric household appliances, house and road construction
CAR INDUSTRY	Engines, noise
METROLOGY	Calibration benches, test benches
STRUCTURE MONITORING	Industrial machines, tool monitoring, damping system monitoring, house construction
TRANSPORT	Railways, aeronautical, military

3. Shocks:

Sensors for measuring mechanical shocks should have a frequency range from 500 Hz to 100 kHz and a full-scale range up to 100,000 g.

FIELDS	APPLICATIONS
CAR INDUSTRY	Crash tests, airbags, engines
INSTRUMENTATION	Packaging, transported products
METROLOGY	Calibration and test benches
MILITARY	Various release systems (alarms, firing, protections, etc.)
SAFETY AND MONITORING SYSTEMS	Transport, cargo monitoring, structure and fatigue monitoring, house automation

4. Inclination:

It is the measurement of components of the gravitational acceleration. The required range is therefore ± 1 g, and sensors should measure from DC.

FIELDS	APPLICATIONS
BUILDING AND ROADS	Monitoring of structures (buildings, bridges), machines and crane safety
MILITARY	Gun mounting control, various vehicles, roll and pitch control
PETROLEUM DRILLING	Drill rig monitoring, drill path control
INSTRUMENTATION	Tools, machine tool monitoring, control processing
CAR INDUSTRY	Frame stabilization, robotics

Different uses and required accelerometers:

	USES	TYPES OF ACCELEROMETERS
1	– Acceleration of moving object having a certain mass such as aircraft, missiles, terrestrial or maritime vehicles – Frequencies (0-50 Hz) – Low acceleration	– Servo controlled accelerometers – Accelerometers with measurement of displacement (inductive, capacitive, with potentiometers, optics) – Strain gauge accelerometers – Accelerometers with contact or threshold, although generally of average precision, are included in this first sensor category intended for measurement of moving object center of gravity movement
2	– Vibratory acceleration – For rigid structures or significant masses – Around 100 Hz – Measuring a continuous or pseudo-continuous acceleration with a satisfactory damping	– Accelerometers with variable inductance – Metallic or generally piezoresistive strain gauges
3	– Vibratory acceleration – Around 10,000 Hz	– Piezoresistive or piezoelectric sensors
4	– Measurement of shocks – Pulsated accelerations – Up to 100,000 Hz	– Sensors with a bandwidth extending from low to high frequencies

Main models of accelerometers:

Principles of detection	Recommended range of frequencies (Hz)						
	0,1	1	10	100	1,000	10,000	100,000
With Foucault current							
With resonator							
Servo controlled (electrodynamic)							
Electromagnetic							
Electrostatic							
Optical							
Piezoelectric (quartz or ceramics)							
Piezotransistor							
Capacitive bridge							
Bridge of piezoresistive gauges							
Bridge of resistive gauges							

Main models of accelerometers:

1. Piezoelectric accelerometers:

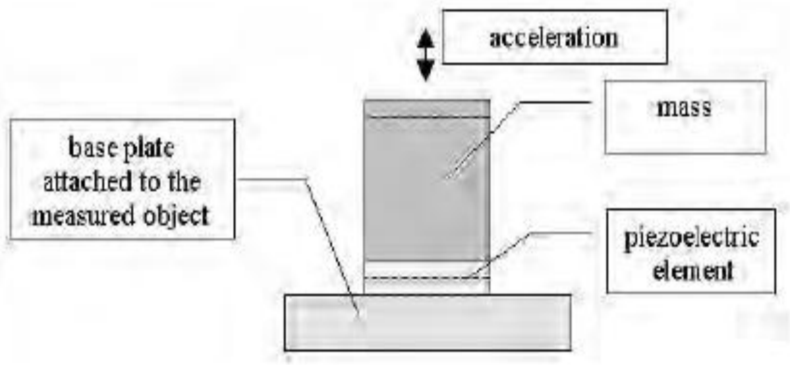
Piezoelectricity is defined as the electric polarization of certain crystals caused by a mechanical strain.

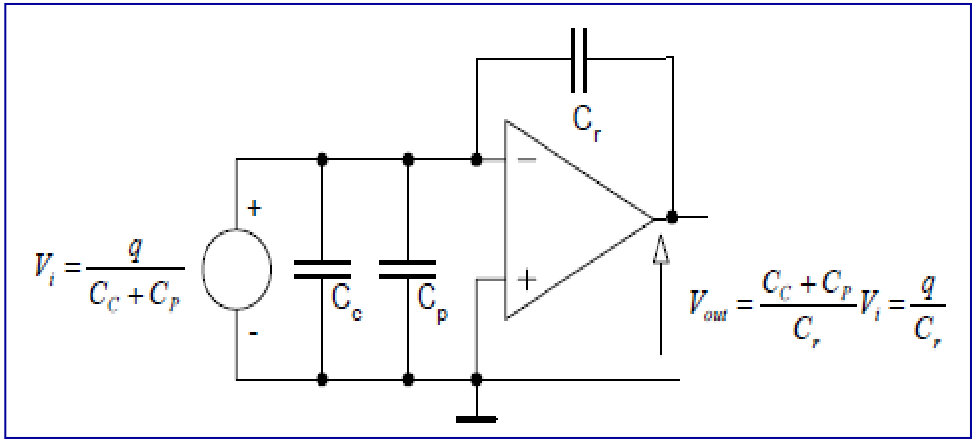
- The piezoelectric materials are sensitive to compressive linear stress and shear.
- The piezoelectric materials can be divided into two categories: crystals and artificially polarized ferroelectric ceramics containing barium Titanate and lead Zirconate.
- The choice of material depends on the working environment and the measurement to be carried out.

	Materials	Piezoelectric coefficients 10^{-12} C/N	Maximum temperature °C	Resonance frequency at 50 pC/g in Hz
Natural crystals	Quartz Tourmaline BGO	2.2 1.8 22	250 600 350	7,000 7,000 8,000
Ferroelectric crystals	Bism. titanate Zirconate Pb	20 280	500 260	15,000 25,000
Conclusion: natural crystals are less sensitive to temperature variations but have a lower piezoelectric coefficient.				

General principle:

The piezoelectric element is placed in such way that when the unit is in vibration, a mass applies a force proportional to acceleration to the piezoelectric element.



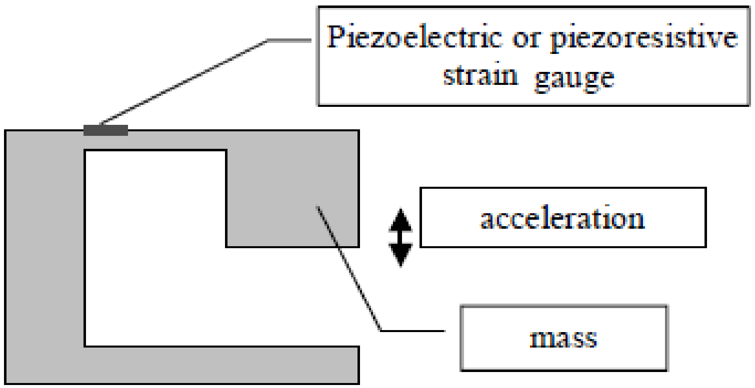


- The piezoelectric accelerometer has capacitive impedance, and generally it cannot be connected to the circuit having resistive input impedance.
- The discharge of the capacity would be too fast.

Piezoelectric Accelerometers	
Advantages	**Disadvantages**
– robust – compact – high reliability – generally very light – very large bandwidth (from a few Hz to several tens of kHz)	– operates only in dynamic mode (cannot measure constant acceleration) – detector output signal is high impedance, hence the need for a specific connection between the detector and the electronics signal processing which increases the measurement cost – high sensitivity to temperature

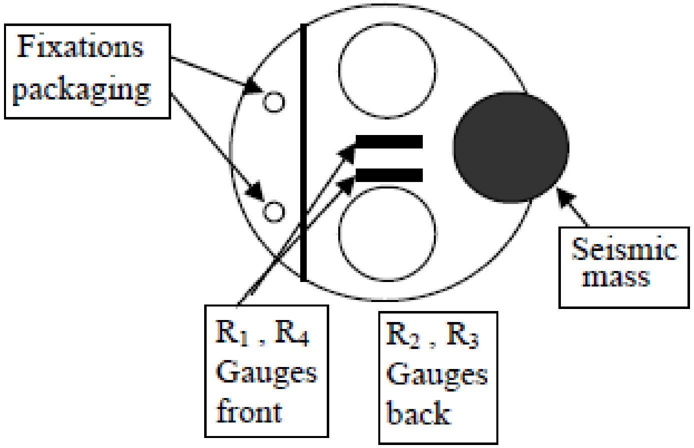
2. Piezoresistive Accelerometers:

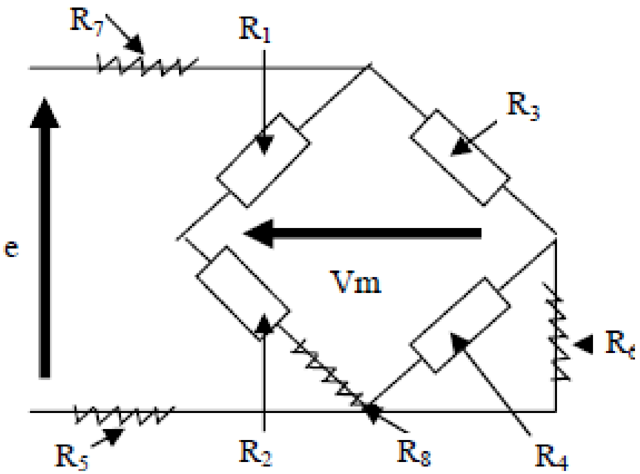
- A seismic mass is placed on an elastic return blade equipped with two or four piezoresistive gauges in a Wheatstone Bridge.
- The blade flexion is translated into gauged deformation. These gauges enable conversion of the acceleration into an electric quantity.



Silicon semiconductor strain gauges:

The resistivity variation depends on material, resistivity, doping level, type of doping agent and the crystallographic direction in which the material is machined, and the resistivity itself is given by the concentration of the doping agent.





Features and limits of these accelerometers

1. Sensitivity is defined by;

$$S = \frac{m}{a} = \frac{V_m}{\varepsilon}\frac{\varepsilon}{a} = S_1 * S_2$$

Where;

V_m = output voltage of the Wheatstone Bridge

ε = deformation

a = acceleration

S_1 = characterizes the response of the mechanical part of the accelerometer

S_2 = electric sensitivity of the Wheatstone Bridge formed by the 4 gauges.

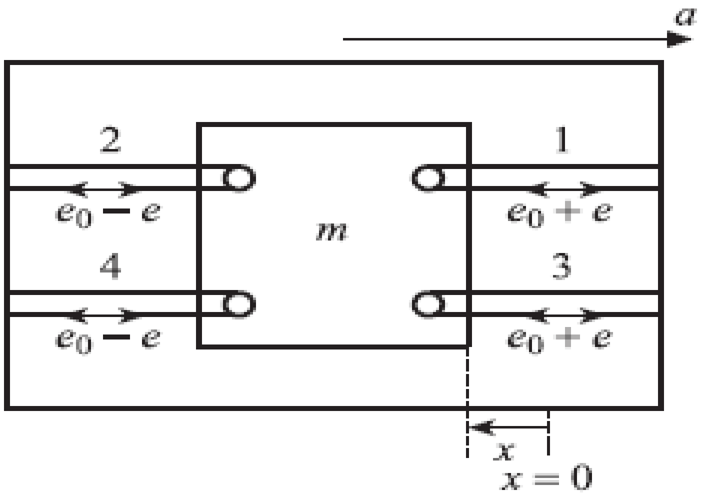
Note: The sensitivity varies from 1 to 25 mV/g according to the gauge.

Piezoresistive Accelerometers	
Advantages	**Disadvantages**
– high sensitivity – low cost – quite high bandwidth – possibility of obtaining a very high natural frequency (> 30 KHz) – simple data processing – possible miniaturization	– no significant linearity – high sensitivity to temperature – generally average performance – the lower the sensitivity, the higher the bandwidth

3. Strain gauge accelerometer:

A practical accelerometer uses four unbonded strain gauges is given.

The space between the seismic mass and casing is filled with liquid to provide damping. The unbonded strain gauges are stretched fine metal wires, which provide the spring restoring force as well as acting as secondary displacement sensors.



The gauges are prestressed, so that at zero acceleration each gauge experiences a tensile strain e_0 and has a resistance $R_0(1 + Ge_0)$. If the casing is given an acceleration (a), then the resultant displacement of the seismic mass (m) relative to the casing is;

$$x = \frac{m}{k} a = \frac{1}{\omega_n^2} a$$

where k is the effective stiffness of the strain gauges.

Gauges 1 and 3 increase in length from (L) to (L + x), and gauges 2 and 4 decrease in length from (L) to (L − x). The tensile strain in gauges 1 and 3 increases to $(e_0 + e)$, and that in gauges 2 and 4 decreases to $(e_0 - e)$, where:

$$e = \frac{x}{L} = \frac{a}{\omega_n^2 L}$$

The four gauges are connected into a deflection bridge circuit.
In order to ensure that all four gauges are kept in tension over the whole range of movement of the mass, the maximum acceleration induced strain is only one-half of the initial strain, i.e.

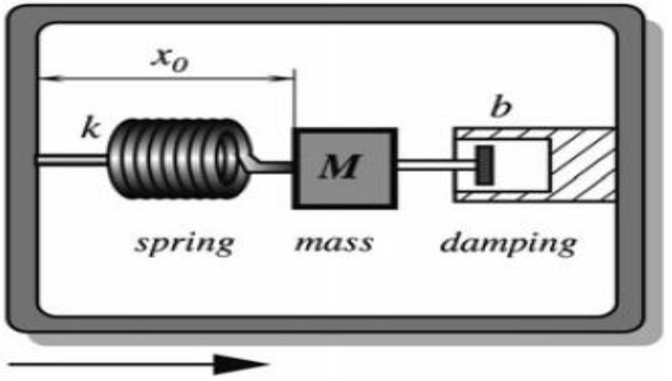
$$e_{\text{MAX}} = \frac{a_{\text{MAX}}}{\omega_n^2 L} = \frac{e_0}{2} \quad \text{or} \quad a_{\text{MAX}} = \frac{e_0 L \omega_n^2}{2}$$

Thus the acceleration input span is proportional to the square of the natural frequency.
A family of accelerometers of this type, using 350 Ω gauges, cover the ranges ±5 g to ±500 g with natural frequencies between 300 and 3000 Hz and a damping ratio of 0.7 ± 0.1

Accelerometer Characteristics:

An accelerometer can be specified as a single-degree of-freedom device, which has mass, a spring, and a frame structure with damping properties.

A mathematical model of an accelerometer is;



$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = M\frac{d^2y}{dt^2}$$

$$Ms^2X(s) + bsX(s) + kX(s) = -MA(s)$$

$$X(s) = -\frac{MA(s)}{Ms^2 + bs + k}$$

We introduce a conventional variable $\omega_0 = \sqrt{k/M}$ and $2\zeta\omega_o = b/M$, then

$$X(s) = -\frac{A(s)}{s^2 + 2\xi\omega_o s + \omega_o^2}$$

Let us set

$$G(s) = -\frac{1}{s^2 + 2\xi\omega_o s + \omega_o^2}$$

$$X(s) = G(s)A(s)$$

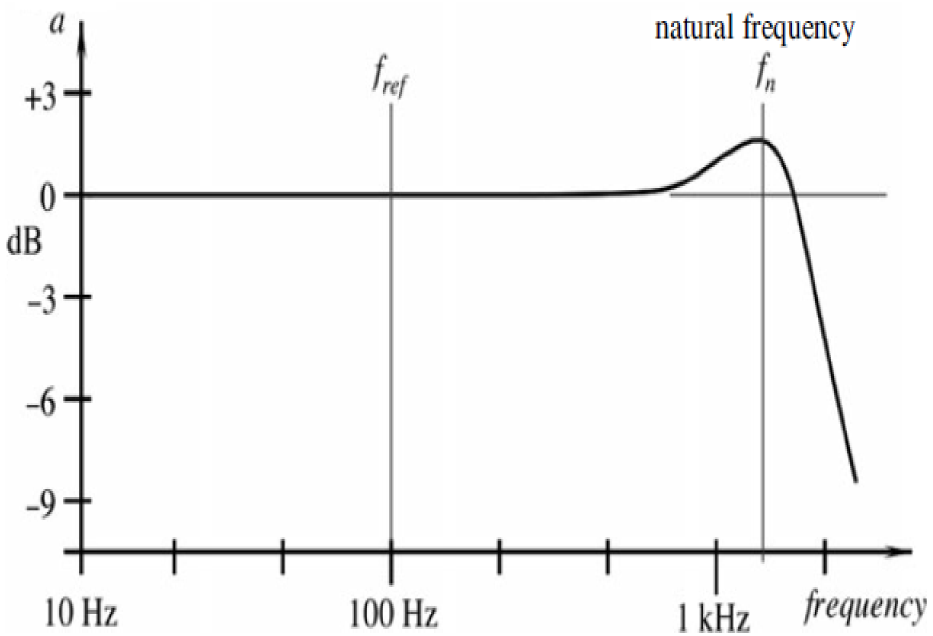
$$x(t) = \mathcal{L}^{-1}\{G(s)A(s)\} = \int_0^t g(t-\tau)\alpha(t)d\tau$$

$$x(t) = \int_0^t -\frac{1}{\omega}e^{-\xi\omega_0(t-\tau)} \sin\omega(t-\tau)\alpha(t)d\tau, \quad \text{underdamped mode } (\xi < 1)$$

$$x(t) = \int_0^t -\frac{1}{\omega}e^{-\xi\omega_0(t-\tau)} \sinh\omega(t-\tau)\alpha(t)d\tau, \quad \text{overdamped mode } (\xi > 1)$$

Accelerometer characteristics:

A correctly designed, installed, and calibrated accelerometer should have one clearly resonant (natural) frequency and a flat frequency response.



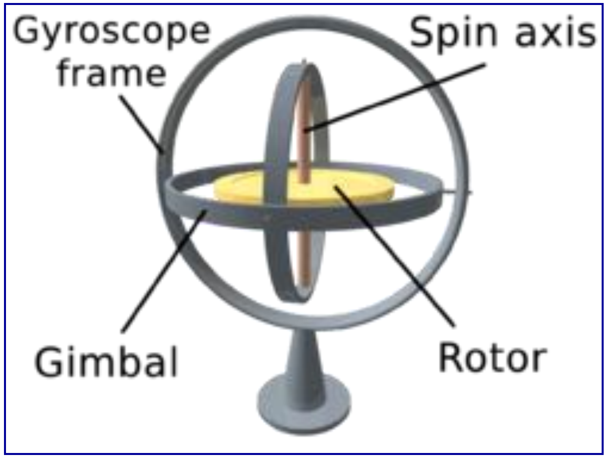
- **Frequency response:** is the outputs signal over a range of frequencies where the sensor should be operating.
- **Sensitivity:** is specified as 1 V/g , where g= 9.80665 m/s^2
- **Resonant frequency:**
  - ✓ Undamped sensor: clearly defined peak that can be 3–4 dB higher than the response at the reference frequency.
  - ✓ Critically damped device: the resonant may not be clearly visible; therefore, the phase shift is measured.
- **Linearity:** is specified over the dynamic range of the input signals.

Selection of Accelerometers:

The instrument mass is particularly important in choosing between the different types of accelerometer for a particular application. This should be very much less than that of the body whose motion is being measured, in order to avoid loading effects that affect the accuracy of the readings obtained. In this respect, instruments based on strain gauges are best.

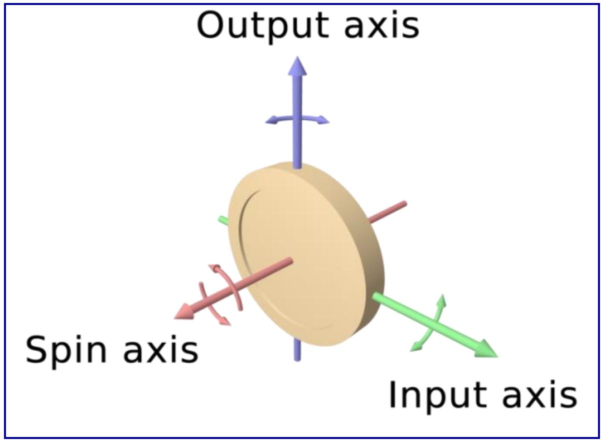
Gyroscope:

It is a spinning wheel in which the axis of rotation is free to assume any orientation. When rotating, the orientation of this axis is unaffected by rotation of the mounting, therefore, gyroscopes are useful for measuring or maintaining orientation.



Applications of gyroscopes include:

- Inertial navigation systems where magnetic compasses would not work, or would not be precise enough, or for the stabilization of flying vehicles.
- Gyroscopes can be used to construct gyrocompasses, which complement or replace magnetic compasses (in ships, aircraft and spacecraft, vehicles in general), to assist in stability or be used as part of an inertial guidance system.



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