SUMMARY

1. **Speed of Sound:**
   
   $$c = \sqrt{kRT}$$
   $$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

   Isentropic Process (Adiabatic reversible)

2. **Mach Number:**

   $$M = \frac{V}{c}$$

   - $M < 1$: Subsonic flow
   - $M \approx 1$: Transonic flow
   - $M > 1$: Supersonic flow

3. **Total Temperature:**

   $$T_t = T \left(1 + \frac{k - 1}{2} M^2\right)$$

   (Adiabatic Process)

4. **Total Pressure:**

   $$p_t = p \left(1 + \frac{k - 1}{2} M^2\right)^{k/(k-1)}$$

5. **Total Density:**

   $$p_t = p \left(1 + \frac{k - 1}{2} M^2\right)^{k/(k-1)}$$
5. Kinetic Pressure: 

\[ q = \frac{1}{2} p V^2 \quad q = \frac{k}{2} p M^2 \]

6. Static Pressure Ratio across A Normal Shock Waves:

\[ \frac{p_2}{p_1} = \frac{1 + k M_1^2}{(1 + k M_2^2)} \]

7. Static Temperature Ratio across A Normal Shock Waves:

\[ \frac{T_2}{T_1} = \frac{1 + [(k-1)/2] M_1^2}{1 + [(k-1)/2] M_2^2} \]

8. Mach Numbers Upstream & Downstream of a Normal Shock Waves:

\[ \frac{M_1}{1 + k M_1^2} \left(1 + \frac{k-1}{2} M_1^2\right)^{1/2} = \frac{M_2}{1 + k M_2^2} \left(1 + \frac{k-1}{2} M_2^2\right)^{1/2} \]

8. Mach Numbers Downstream of a Normal Shock Waves:

\[ M_2^2 = \frac{(k-1) M_1^2 + 2}{2 k M_2^2 - (k-1)} \]
9. **Area Ratio in a Laval Nozzle:**

\[
\frac{A}{A^*} = \frac{1}{M} \left\{ \frac{1 + [(k-1)/2]M^2}{(k+1)/2} \right\}^{(k+1)/2(k-1)}
\]

10. **Mass Flow Rate Through a Laval Nozzle:**

\[
\dot{m} = 0.685 \frac{p_t A^*}{\sqrt{RT_t}}
\]

11. A Laval nozzle is classified by comparing the pressure at the exit, \(p_e\), for supersonic flow in the nozzle with the back (ambient) pressure, \(p_b\):

- \(p_e/p_b > 1\) underexpanded
- \(p_e/p_b = 1\) ideally expanded
- \(p_e/p_b < 1\) overexpanded

12. **Critical Pressure Ratio**

\[
\frac{p^*}{p_t} = \left( \frac{2}{k+1} \right)^{k/(k-1)}
\]

\[
\frac{p^*}{p_t} = 0.528
\]
13. **Mach No. Distribution along a Pipe with Friction**

\[
\frac{1 - M^2}{kM^2} + \frac{k + 1}{2k} \ln \left[ \frac{(k + 1)M^2}{2 + (k - 1)M^2} \right] = \frac{f(x_\ast - x_M)}{D}
\]

where \(x_M\) is the distance corresponding to a Mach number \(M\).

14. **Variation of Mach No. with Pressure along a Pipe with Friction:**

\[
\frac{p_M}{p_\ast} = \frac{1}{M} \left[ \frac{k + 1}{2 + (k - 1)M^2} \right]^{1/2}
\]

15. **Mach No. Distribution along a Pipe with Friction for Isothermal:**

\[
\frac{f(x_T - x_M)}{D} = \ln(DM^2) + \frac{(1 - kM^2)}{kM^2}
\]

16. **Variation of Mach No. with Pressure for Isothermal Process:**

\[
\frac{p_M}{p_T} = \frac{1}{\sqrt{kM}}
\]
Problem (12.4)

PROBLEM 12.4

Situation: A sound wave travels in helium and another in nitrogen both at 20 °C.

Find: Difference in speed of sound.

ANALYSIS

Speed of sound

\[ c_{He} = \sqrt{(kR)_{He}T} \]
\[ = \sqrt{1.66 \times 2077 \times 293} \]
\[ = 1005 \text{ m/s} \]

\[ c_{N_2} = \sqrt{(kR)_{N_2}T} \]
\[ = \sqrt{1.40 \times 297 \times 293} \]
\[ = 349 \text{ m/s} \]

\[ c_{He} - c_{N_2} = 656 \text{ m/s} \]
Problem (12.5)

PROBLEM 12.5

Situation: A sound wave travels in an ideal gas.

Find: Speed of sound for **an isothermal process**.

**Analysis**

\[ c^2 = \frac{\partial p}{\partial \rho}; \quad p = \rho RT \]

If isothermal, \( T = \text{const.} \)

\[ \therefore \frac{\partial p}{\partial \rho} = RT \]

\[ \therefore c^2 = RT \]

\[ c = \sqrt{RT} \]
**Problem (12.7)**

**PROBLEM 12.7**

Situation: An aircraft flying in air at Mach 1.5 is described in the problem statement.

Find: (a) Surface temperature.  
(b) Airspeed behind shock.

Properties: (a) From Table A.1 at \( M_1 = 1.5 \), \( \frac{T}{T_t} = 0.6897 \); \( M_2 = 0.7011 \), \( \frac{T_2}{T_1} = 1.320 \). (b) Air (Table A.2) \( k = 1.4 \) and \( R = 287 \) J/kg/K.

**ANALYSIS**

Total temperature will develop at exposed surface

\[
\frac{T}{T_t} = 0.6897
\]

\[
T_t = \frac{(273 - 30)}{0.6897} = 352.3 \text{ K} = 79.2 \text{ C}
\]

Temperature (behind shock)

*Given:* \( T_1 = 30 \) °C

\[
\frac{T_2}{T_1} = 1.320
\]

\[
T_2 = 1.320 \times (273.15 - 30) = 320.96 \text{ K}
\]
Problem (12.7)

Speed of sound (behind shock)

\[ c_2 = \sqrt{kRT_2} \]
\[ = \sqrt{(1.4)(287)} \times 320.96 \]
\[ = 359.1 \text{ m/s} \]

Mach number (behind shock)

\[ M_2 = \frac{V_2}{c_2} \]
\[ V_2 = c_2 M_2 \]
\[ = (359.1)(0.7011) \]
\[ = 251.77 \text{ m/s} \]

\[ V_2 = 252 \text{ m/s} = 906 \text{ km/h} \]
Problem (12.12)

Situation: An object immersed in airflow is described in the problem statement.

Find: (a) Pressure.  
(b) Temperature at stagnation point.

**ANALYSIS**

**Speed of sound**

\[ c = \sqrt{kRT} \]
\[ = \sqrt{(1.4)(287)(293)} \]
\[ = 343 \text{ m/s} \]

**Mach number**

\[ M = \frac{250}{343} \]
\[ = 0.729 \]

**Total properties**

**Temperature**

\[ T_t = T \left( 1 + \frac{k-1}{2} M^2 \right) \]
\[ T_t = (293)(1 + 0.2 \times (0.729)^2) \]
\[ = 293 \times 1.106 \]
\[ = 324 \text{ K} \]
\[ T_t = 51^\circ \text{C} \]

**Pressure**

\[ p_t = p \left( 1 + \frac{k-1}{2} M^2 \right)^{k/(k-1)} \]
\[ p_t = (200)(1.106)^{3.5} \]
\[ p_t = 284.6 \text{ kPa} \]

Given: \( T_s = 20^\circ \text{C} \)

\( P_s = 200 \text{ kPa} \)

\( V = 250 \text{ m/s} \)
PROBLEM 12.16

Situation: Hydrogen flow from a reservoir—additional details are provided in the problem statement.

Find: (a) Temperature.
(b) Pressure.
(c) Mach number.
(d) Mass flow rate.

Given: \( T_t = 20^\circ\text{C} = 293 \, \text{K} \), \( P_t = 500 \, \text{kPa} \)
\( d = 2 \, \text{cm} \), \( V = 250 \, \text{m/s} \)

Isentropic Flow

ANALYSIS

\[ T_t = 20^\circ\text{C} = 293 \, \text{K} \]
\[ P_t = 500 \, \text{kPa} \]
\[ c_p T + \frac{V^2}{2} = c_p T_0 \]
\[ T = T_t - \frac{V^2}{2c_p} \]
\[ = 293 - \frac{(250)^2}{2(14.223)} \]
\[ T = 290.8 \, \text{K} \]

\[ h_i = h + \frac{1}{2} V^2 \]
\[ h = C_p T \]

Speed of sound

\[ c = \sqrt{kRT} \]
\[ = \sqrt{(1.41)(4.127)(290.8)} \]
\[ = 1,301 \, \text{m/s} \]
Problem (12.16)

Speed of sound

\[ c = \sqrt{kRT} \]
\[ = \sqrt{(1.41)(4,127)(290.8)} \]
\[ = 1,301 \text{ m/s} \]

Mach number

\[ M = \frac{250}{1301} \]
\[ = 0.192 \]

Total properties (pressure)

\[ P_t = P \left(1 + \frac{k-1}{2} M^2 \right)^{\frac{K}{K-1}} \]
\[ \rho = \frac{500/[1 + (0.41/2) \times 0.192^2]^{(1.41/0.41)}}{\text{[487.2 kPa]}} \]

Ideal gas law

\[ \rho = \frac{p}{RT} \]
\[ = \frac{(487.2)(10^3)}{(4,127 \times 290.8)} \]
\[ = 0.406 \text{ kg/m}^3 \]

Flow rate equation

\[ \dot{m} = \rho AV \]
\[ = (0.406)(0.02)^2(\pi/4)(250) \]
\[ \dot{m} = 0.032 \text{ kg/s} \]
Problem (12.23)

Situation: A shock wave is described in the problem statement.

Find: (a) The downstream Mach number.
(b) Static pressure.
(c) Static temperature.
(d) Density.

Properties: From Table A.2 $k = 1.31$ Methane

APPROACH

Apply the Normal shock wave equations to find Mach number, pressure, and temperature. Apply the ideal gas law to find density.

ANALYSIS

Normal shock wave Mach number

\[ M_1 = 3 \]

Given Upstream: $T_s = 20 \, ^\circ C$

\[ P_s = 100 \, kPa \]

\[
M_2^2 = \frac{[(k-1)M_1^2 + 2]}{2kM_1^2 - (k-1)} \\
= \frac{((0.31)(9) + 2)}{(2)(1.31)(9) - 0.31) = 0.2058 \]

\[ M_2 = 0.454 \]
Problem (12.23)

Pressure ratio

\[ P_1 = 100 \text{kPa} \]
\[ \frac{p_2}{p_1} = \frac{(1 + kM_1^2)/(1 + kM_2^2)}{(1 + 1.31 \times 9)/(1 + 1.31 \times 0.2058)} = 10.07 \]
\[ p_2 = 1,007 \text{ kPa, abs} \]

Temperature ratio

\[ T_1 = 20 \degree \text{C} \]
\[ \frac{T_2}{T_1} = \frac{[1 + ((k - 1)/2)M_1^2]/[1 + ((k - 1)/2)M_2^2]}{(293)(2.32)} = 2.32 \]
\[ T_2 = 680 \text{ K} = 407 \degree \text{C} \]

Ideal gas law

\[ \rho_2 = \frac{p_2}{(RT_2)} \]
\[ = \frac{(1,007)(10^3)/((518)(680))}{\rho_2 = 2.86 \text{ kg/m}^3} \]
**Problem (12.28)**

**PROBLEM 12.28**

**Situation:** A truncated nozzle is described in the problem statement.

**Find:** Mass flow rate

**ANALYSIS**

\[
A_T = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2 \\
\rho_t = 300 \text{ kPa}; \ T_t = 20^\circ = 293 \text{ K} \\
\rho_b = 90 \text{ kPa} \\
\frac{\rho_b}{\rho_t} = \frac{90}{300} = 0.3
\]

Because \( \frac{\rho_b}{\rho_t} < 0.528 \), sonic flow at exit. \[
\frac{p^*}{p_t} = 0.528
\]

Laval nozzle flow rate equation

\[
\dot{m} = 0.685 \rho_t A_\ast / \sqrt{RT_t} \\
= (0.685)(3 \times 10^5)(3 \times 10^{-4}) / \sqrt{(287)(293)} \\
\boxed{\dot{m} = 0.212 \text{ kg/s}}
\]
PROBLEM 12.36

Situation: The design of a Laval nozzle is described in the problem statement.

Find: The nozzle throat area.

Properties: From Table .2 \( k = 1.4; \ R = 297 \text{ J/kgK} \). \textbf{Nitrogen}

\[ \text{ANALYSIS} \]

Find Mach number

\[
M_e = \sqrt{\frac{2}{(k-1)}} \left[ \frac{p_t}{p_e} \right]^{(k-1)/k} \\
= \sqrt{5} \left[ \frac{1,000}{30} \right]^{0.286} - 1 \\
= 2.94
\]

Mach number-area ratio relationship

\[
\frac{A_e}{A_*} = \frac{1}{M_e} \left[ \left( 1 + \frac{(k-1)}{2} M_e^2 \right) \left( 1 + \frac{k+1}{2} \right) \right]^{(k+1)/(2(k-1))} \\
= \frac{1}{2.94} \left[ \left( 1 + \frac{0.2}{2.94} \right)^2 \right]^{3/2} \\
\frac{A_e}{A_*} = 4.00
\]

Flow rate equation for Laval nozzle

\[
\dot{m} = 0.685 p_t A_T \sqrt{RT_t} \\
A_T = \frac{\dot{m} \sqrt{RT_t}}{0.685 \times p_t} \\
= 5 \times \sqrt{\left( \frac{297}{(550)} \right) \left( \frac{10^6}{(0.685)} \right)} \\
= 0.00295 \text{ m}^2 \\
A_T = 29.5 \text{ cm}^2
\]