Engineering Measurements

Chapter Two

Analysis of experimental data

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Analysis of experimental data

Introduction

- In engineering practice, the experimental work has a high importance because it is the connection between the real case and the simulate one.

- when performing the experiment, engineers try hard to achieve certain level of validity. Validity means how much we can trust the data obtained from an experiment. The level of trust in data is called *the level of confidence*

- when performing the experiment – even with highly calibrated accurate measurement devices, error will enter to the experiment without knocking.

- Error mean in its simplest definitions: the deviation from the true. In this manner, error is the opposite of accuracy. Knowing the sources of errors in the experiment allow the experimenter to eliminate it and enhance the results.

- The analysis of error is a fundamental process done on the experimental data to reduce the gap between the true and measured values.

Analysis of experimental data

Errors

Experimental data

- Single sample
- Multi sample

The single sample experiment means performing the experiment and obtain one or more data results by using the same apparatus

The single sample experimental measurements is – in many time – avoided because it does not indicates the uncertainty in the experiment .

The multisample experimental measurements allow to the examiner to study the results statistically which insure more confidence in the decision made depending on the obtained results.

Analysis of experimental data

Uncertainty

Uncertainty means how much we are not confident (uncertain) about the experimental data that we have. The main source of uncertainty is the error. In some cases, we can say that we are uncertain in the magnitude of error.

- Error
  - Instrument error
  - Systematic or bias error
  - Random error

- Instrument error: errors in the measurement instrumentation
- Systematic error: fixed shift in multi-readings taken by the same device
- Random errors come from the operator (human errors), noise, ..., etc.

Analysis of experimental data

Uncertainty analysis

Kline and McClintock establish a method to calculate the uncertainty. They define the uncertainty as a range where the true value lies in. for example, if a temperature measurement was

$$T = 103C^{o} \pm 1^{o}C$$

- The ± sign means that the experimenter is not sure about the results and he/she define the range where the true value lies . In our example, the experimenter implies that the true value lies between 102C^{o} and 104C^{o}.

Analysis of experimental data

Uncertainty analysis

- If a set of measurements is made and we wish to calculate the total uncertainty for the total set, we can follow these procedures:
  - represent the results in terms of R. R is a function in independent variables: $x_1, x_2, \ldots, x_n$ (i.e. $R = R(x_1, x_2, \ldots, x_n.)$)
  - Define the uncertainties for each independent variable. For example, $w_1, w_2, \ldots, w_n$.
  - Assume w_R is the total uncertainty :

$$w_R = \sqrt{\left(\frac{\partial R}{\partial x_1} w_1\right)^2 + \left(\frac{\partial R}{\partial x_2} w_2\right)^2 + \ldots + \left(\frac{\partial R}{\partial x_n} w_n\right)^2}$$

Analysis of experimental data

Uncertainty analysis

- For product functions

$$R = \left(x_1^{a_1}\right)\left(x_2^{a_2}\right)\ldots\left(x_n^{a_n}\right) \Rightarrow \frac{w_R}{R} = \sqrt{\sum_{i=1}^{n}\left(\frac{a_i w_{x_i}}{x_i}\right)^2}$$

- For additive functions

$$R = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \Rightarrow w_R = \sqrt{\sum_{i=1}^{n}\left(a_i w_{x_i}\right)^2}$$

Engineering measurements

Analysis of experimental data

Uncertainty Example [1]

- The resistance of a copper wire is given as:

$$R = R_o\left[1+\alpha\left(T-20\right)\right]$$

Where: **R_o = 6Ω ± 0.3%** is the resistance at a reference temperature (20 °C), **α = 0.004 °C^{-1} ± 1%** is the temperature coefficient resistance and the temperature of the wire **T = 30°C ± 1°C.**

calculate the wire resistance and its uncertainty.

Solution

The nominal resistance: R =(6)[1+(0.004)(30-20)] = 6.24Ω

Analysis of experimental data

Uncertainty Example [1]

Solution

The total uncertainty can be calculated using the general form

$$w_R = \sqrt{\left(\frac{\partial R}{\partial x_1} w_1\right)^2 + \left(\frac{\partial R}{\partial x_2} w_2\right)^2 + \ldots + \left(\frac{\partial R}{\partial x_n} w_n\right)^2}$$

$$\frac{\partial R}{\partial R_o} = 1 + \alpha(T - 20) = 1 + 0.004(30 - 20) = 1.04$$

$$\frac{\partial R}{\partial \alpha} = R_o(T - 20) = 6(30 - 20) = 60$$

$$\frac{\partial R}{\partial T} = R_o \alpha = (6)(0.004) = 0.024$$

$$w_{R_o} = (6)(0.003) = 0.018\Omega$$

$$w_\alpha = (0.004)(0.01) = 4x10^{-5} C^{-1}$$

$$w_T = 1 \ C$$

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Uncertainty Example [1]

Solution

The total uncertainty can be calculated using the general form

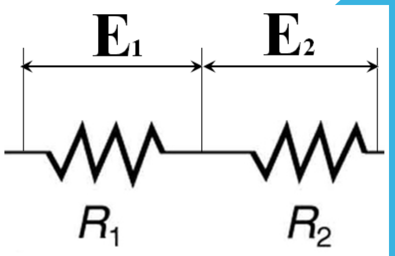
$$w_R = \sqrt{\left((1.04)(0.018)\right)^2 + \left((60)(4x10^{-5})\right)^2 \left((0.24)(1)\right)^2}$$

$$= 0.0305\Omega \text{ or } 0.49\%$$

Analysis of experimental data

Uncertainty Example [2]

- Two resistance are connected in series as shown in the figure



If the measurements of voltage drop across these R_1 and R_2 (E_1 and E_2 respectively) were found as

$$E_1 = 10V \pm 0.1V\ (1\%)$$

$$E_2 = 1.2V \pm 0.005V\ (0.467\%)$$

And the value of R_2 was found as **$R_2 = 0.0066\Omega \pm 0.25\%$**

Now , find the power dissipated in resistance in R_1 and its uncertainty.

Analysis of experimental data

Uncertainty Example [2]

Solution

The power dissipated (P) through a resistance R_1 is given as: ***P = $E_1 I$*** and the current passes through the resistance (I) is calculated as: ***I=E/R*** so the power dissipated through a resistance R_2 is calculated as:

$$P = \frac{E_1 E_2}{R_2} = \frac{(10)(1.2)}{0.0066} = 1818.2W \;---(1)$$

The relation between P and the drop in voltage E is a product relation, so:

$$R = \left(x_1^{a_1}\right)\left(x_2^{a_2}\right)\ldots\left(x_n^{a_n}\right) \Rightarrow \frac{w_R}{R} = \sqrt{\sum_{i=1}^{n}\left(\frac{a_i w_{x_i}}{x_i}\right)^2}$$

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Uncertainty Example [2]

Solution

$a_{E1} = 1$, $a_{E2} = 1$ and $a_{R2} = -1$

$$\frac{w_P}{P} = \sqrt{\left(\frac{a_{E1} w_{E1}}{E_1}\right)^2 + \left(\frac{a_{E2} w_{E2}}{E_2}\right)^2 \left(\frac{a_{R2} w_{R2}}{R_2}\right)^2} = 0.0111$$

Then

$$w_P = (0.0111)(1818.2) = 20.18$$

Analysis of experimental data

Uncertainty: further analysis

It is noted from the uncertainty calculations that: if there is a single uncertainty in one of the measured parameters or variables much larger than the uncertainties in the other parameters, the total uncertainty will be much affected by this uncertainty rather than the other uncertainties due to the presence of the square power. For example, assume you perform an experiment and the value of the term $(\partial R/\partial x)w_r$ were: 10.00, 0.50, 1.00, 0.75, then the total uncertainty (w_R) will equal :

$$w_R = \sqrt{10^2 + 0.5^2 + 1^2 + 0.75^2} \cong 10.09 \approx 10.00$$

As you can see, the total uncertainty tends to the site of the largest uncertainty in the experiment. This type of analysis is useful when performing the experiment because it permits to the experimenter to focus on reducing the largest uncertainty in the experiment which reduce the effort and time

E n g i n e e r i n g m e a s u r e m e n t s

Analysis of experimental data

Selection of measurement method and measuring instrument

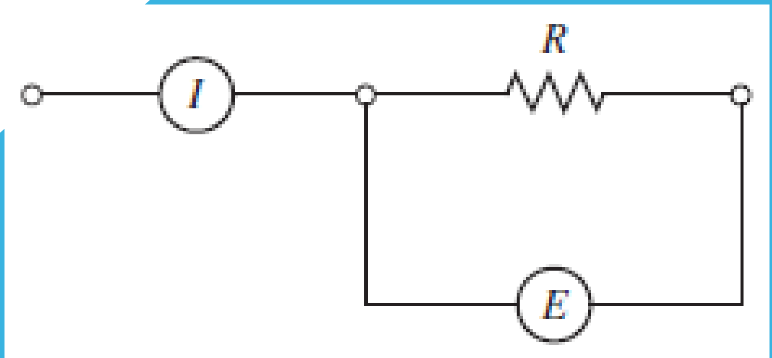
When an experiment includes many variables to measure and we have many options to perform the experiment, the method of measurement must be selected carefully. The selection of measurement method depends, mainly, on:

- the level total uncertainty desired
- Time and effort
- Cost
- Environmental conditions and technical difficulties

Analysis of experimental data

Example [3]: selection of measurement method

A resistor (R) has a nominal stated value of 10Ω ± 1%. The circuit shown in the figure was used to measure the current (I) passes through R and the voltage drop (E) across R. To find the power (P) dissipated in R, we have tow formulas:



1. $P=E^2/R$
2. $P=EI$

The first method implies a single measurement for (E) while the second needs another measurement for (I). If the measurements for I and E were:

E = 100 V± 1%
I = 10A ± 1%

Find the uncertainty in the power calculations using both methods

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Example [3]: selection of measurement method

Solution

Case 1:

$$\frac{\partial P}{\partial E}=\frac{2E}{R}$$
$$\frac{\partial P}{\partial R}=-\frac{E^2}{R^2}$$
$$\Rightarrow w_P=\sqrt{\left(\frac{2E}{R}\right)^2 w_E^2+\left(-\frac{E^2}{R^2}\right)^2 w_R^2}$$

Divide by P=E²/R

$$\frac{w_P}{P}=\sqrt{4\left(\frac{w_E}{E}\right)^2+\left(\frac{w_R}{R}\right)^2}=\sqrt{(4)(0.01)^2+(0.01)^2}=2.236\%$$

Case 2:

$$\frac{\partial P}{\partial I}=I$$
$$\frac{\partial P}{\partial I}=E$$
$$\Rightarrow \frac{w_P}{P}==\sqrt{(0.01)^2+(0.01)^2}=1.414\%$$

Analysis of experimental data

Example [3]: selection of measurement method

Comment

- You may see from the previous example that the uncertainty decreased when in the 2nd case although there were two uncertainties even there was one uncertainty in 1st case.

- This is not necessarily correct for all cases. However, we can conclude that the selection of method depends on the 4 factors we mentioned before. In our example, reducing the uncertainty means more effort, more time and more cost.

Analysis of experimental data

Statistical Analysis of Experimental Data

Fundamental concepts

- **Mean value** is average of a set of collected measurements and it is given by: $x_m = \frac{1}{n}\sum_{i=1}^{n} x_i$ where x_m is the arithmetic mean, n is the number of readings and x_i is the reading.

- **Deviation** is difference between the threading and the mean and given by : $d_i = x_i - x_m$. Note that the average of deviations equal zero

- **Absolute deviations average** is given by: $\left|\overline{d_i}\right| = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - x_m\right|$

Analysis of experimental data

Statistical Analysis of Experimental Data

Fundamental concepts

- **Standard deviation (σ) or root-mean-square deviation** is defined by:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - x_m\right)^2}$$

The variance is defined as σ^2. This is sometimes called the *population or biased standard deviation because it strictly applies only* when a large number of samples is taken to describe the population.

To obtain reliable estimates of standard deviation, it is desired to have at least 20 measurements. In many cases, the engineer is not able to have this number of readings. In such cases (small set of data), *unbiased* or *sample standard deviation* is used instead

Analysis of experimental data

Statistical Analysis of Experimental Data

Fundamental concepts

Unbiased or sample standard deviation is given by

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - x_m\right)^2}$$

- The median is defined as the value that divide the readings in half

Analysis of experimental data

Example [4]:

Find the mean reading, standard deviation, variance, and average of the absolute value of the deviation for the given data in the table below

Reading No.	Pressure, kPa
1	1.25
2	2.45
3	1.10
4	2.03
5	3.11
6	2.95
7	2.36
8	3.42
9	3.01
10	2.10

Analysis of experimental data

Example [4]:

Solution

1. x_m

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(23.78) = 2.378kPa$$

2. σ

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x_m)^2} = 0.7388$$

3. $\sigma^2 = 0.5458$ kPa^2

4. $|\overline{d_i}|$

$$|\overline{d_i}| = \frac{1}{n}\sum_{i=1}^{n}|x_i - x_m| = 0.61$$

E n g i n e e r i n g m e a s u r e m e n t s

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Example [4]: solve using table

No.	x_i	$d_i = x_i - x_m$	d_i^2	$\lvert d_i \rvert$
1	1.25	-1.128	1.272384	1.128
2	2.45	0.072	0.005184	0.072
3	1.10	-1.278	1.633284	1.278
4	2.03	-0.348	0.121104	0.348
5	3.11	0.732	0.535824	0.732
6	2.95	0.572	0.327184	0.572
7	2.36	-0.018	0.000324	0.018
8	3.42	1.042	1.085764	1.042
9	3.01	0.632	0.399424	0.632
10	2.10	-0.278	0.077284	0.278
Sum	23.78	------	5.45776	6.10