Philadelphia University

Faculty of Engineering

Student Name:
Student Number:
Serial Number:

Final Exam, First Semester: 2019/2020
Dept. of Communication & Electronics Engineering

Course Title:	Probability and Random Variables	**Date:**	21/01/2020
Course No:	650364	**Time Allowed:**	2 hours
Lecturer:	Dr. Qadri Hamarsheh	**No. Of Pages:**	9

Instructions:

- **ALLOWED**: pens, calculators and drawing tools (**no red color**).
- **NOT ALLOWED**: Papers, literatures and any handouts. Otherwise, it will lead to the non-approval of your examination.
- **Shut down** Telephones, and other communication devices**.**

Please note:

- *This exam paper contains 6 questions totaling 40 marks.*

Basic notions: *The aims of the questions in this part are to evaluate the required minimal student knowledge and skills. Answers in the pass category represent the minimum understanding of basic concepts: Set definitions and operations, Joint and conditional Probability, Bayes' Theorem, Independent events, Random variable concept, Density function, Distribution functions, Expectation, Moments, Joint density and distribution functions, multiple random variables and Random processes.*

Question 1 ***Multiple Choice*** ***(10 marks)***

Identify the choice that best completes the statement or answers the question.

1) A box of **8 marbles** has **4 red**, **2 green**, and **2 blue** marbles. If you select one marble, what is the probability that it is a **red or blue** marble?

a)	**0.60**	**b)**	**0.75**
c)	**6.00**	**d)**	**0.80**

2) Compute $\binom{9}{4}$

a)	**84**	**b)**	**126**
c)	**3024**	**d)**	**15,120**

3) About the independent events **A** and **B** it is known that $\boldsymbol{P(A|B) = 0.2}$ and $\boldsymbol{P(B|A) = 0.5}$. Compute the probability $\boldsymbol{P(A \cup B)}$

a)	**0.5**	**b)**	**0.7**
c)	**0.1**	**d)**	**0.6**

4) In a sample of **10** telephones, **4** are defective. If **3** are selected at random and tested, what is the probability that all will be nondefective?

a)	**1/30**	**b)**	**8/125**
c)	**1/6**	**d)**	**27/125**

5) The corresponding **z** value (**standard normal value – Z score**) for a value of **9** if the **mean** of a variable is **12** and the standard deviation is **4**.

a)	**-0.75**	**b)**	**0.75**
c)	**0.5**	**d)**	**-0.5**

6) Which of the following is **NOT** required of a **binomial distribution**

a)	**Each trial has exactly two outcomes.**
b)	**There is a fixed number of trials.**
c)	**The probability of success remains fixed for all trials.**
d)	**There are more than 30 trials.**

7) Formula of **variance** of **uniform distribution** is as

a) $(b - a)^2 / 6$ **b)** $(b + a)^2 / 12$
c) $(b - a)^3 / 8$ **d)** $(b + a)^2 / 2$

8) The number of traffic accidents that occur on a particular stretch of road during a month follows a **Poisson distribution** with a **mean** of **7.9**. Find the probability of observing exactly **five** accidents on this stretch of road next month.

a) **0.095067** **b)** **1.727754**
c) **18.672798** **d)** **1.027438**

9) Consider the given discrete probability distribution. Find the probability that x equals **5**.

x	2	5	6	9
$P(x)$	0.09	?	0.23	0.21

a) **0.53** **b)** **2.65**
c) **0.47** **d)** **2.35**

10) If **X** is a discrete random variable and **f(x)** is the probability function of **X**, then the expected value of this random variable is equal to:

a) $\sum f(x)$ **b)** $\sum[x + f(x)]$
c) $\sum f(x) + x$ **d)** $\sum x f(x)$

Familiar and Unfamiliar Problems Solving: *The aim of the questions in this part is to evaluate that the student has some basic knowledge of the key aspects of the lecture material and can attempt to solve familiar and unfamiliar problems: Set definitions and operations, Joint and conditional Probability, Bayes' Theorem, Independent events, Random variable concept, Density function, Distribution functions, Expectation, Moments, Joint density and distribution functions, multiple random variables and Random processes*

Question 2 ***(5 marks)***

a) If the possible blood types are **A, B, AB**, and **O**, and each type can be **Rh+** or **Rh-**, draw a **tree diagram** and find all possible blood types. ***(2 marks)***

Solution

b) **Find** the **probability mass function (PDF)** of boys and girls in families with **3** children, assuming equal probabilities for boys and girls. Then **find** and **draw** the distribution function $\boldsymbol{F(x)}$ **(CDF)** for the random variable $\boldsymbol{X}$**.** ***(3 marks)***

Solution

Question 3 ***(5 marks)***

Consider two random variables **X** and **Y** with **joint PMF** given in the following table.

a) Find $\boldsymbol{P(X \leq 2, Y \leq 4)}$. ***(1 mark)***

b) Find the marginal **PMFs** of **X** and **Y**. ***(1.5 marks)***

c) Find $\boldsymbol{P(Y = 2 | X = 1)}$. ***(1.5 marks)***

d) Are **X** and **Y** independent? ***(1 mark)***

	$Y=2$	$Y=4$	$Y=5$
$X=1$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$
$X=2$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{8}$
$X=3$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$

Solution

Question 4 ***(8 marks)***

a) The **density function** of a random variable **X** is given by ***(5 marks)***

$$f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

i. The **expected value** of **X**. ***(1.5 marks)***

ii. The **expected value** of $E(3X^2 - 2X)$ ***(1.5 marks)***

iii. The **variance** and **standard deviation** of the random variable **X.** ***(2 marks)***

Solution

b) The **joint pdf** is given using the following equation: ***(3 marks)***

$$f_{xy}(x, y) = \begin{cases} k & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine $f_{x|y}(x|y)$ and $f_{y|x}(y|x)$

Solution

Question 5 ***(6 marks)***

The **joint density function** of two random variables ***X*** and ***Y*** is given by

$$f(x, y) = \begin{cases} xy/96 & 0 < x < 4,\ 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find

a) $E(X)$

b) $E(Y)$

c) **The correlation of X and Y**

d) $E(2X + 3Y)$.

Solution

Question 6 ***(6 marks)***

a) List any **4 (four)** properties of the **autocorrelation of a random process** $X(t)$ with necessary **equations.** ***(2 marks)***

Solution

b) Consider a random process $X(t)$ defined by ***(4 marks)***

$$X(t) = A\ \cos(\omega t + \Theta) \qquad -\infty < t < \infty$$

Where A and ω are constants and θ is a **uniform random** variable over $(-\pi, +\pi)$. Show that ***X(t)*** is **WSS** (**Wide-Sense Stationary Process**).

Solution

GOOD LUCK

Trigonometric Identities

$$
\begin{aligned}
\cos(x)\cos(y) &= (1/2)[\cos(x-y)+\cos(x+y)],\\
\sin(x)\sin(y) &= (1/2)[\cos(x-y)-\cos(x+y)],\\
\sin(x)\cos(y) &= (1/2)[\sin(x-y)+\sin(x+y)],\\
\cos(x\pm y) &= \cos(x)\cos(y)\mp\sin(x)\cos(y),\\
\sin(x\pm y) &= \sin(x)\cos(y)\pm\cos(x)\sin(y),\\
\cos(x\pm\pi/2) &= \mp\sin(x),\\
\sin(x\pm\pi/2) &= \pm\cos(x).
\end{aligned}
$$