# Student Name: <br> Student Number: <br> Serial Number: 

Final Exam, First Semester: 2019/2020
Dept. of Communication \& Electronics Engineering

| Course Title: | Signals and Systems | Date: | $30 / 01 / 2020$ |
| :--- | :--- | :--- | :--- |
| Course No: | $650320+640543$ | Time Allowed: | 2 hours |
| Lecturer: | Dr. Qadri Hamarsheh | No. Of Pages: | 8 |

## Instructions:

- ALLOWED: pens, calculators and drawing tools (no red color).
- NOT ALLOWED: Papers, literatures and any handouts. Otherwise, it will lead to the non-approval of your examination.
- Shut down Telephones, and other communication devices.


## Please note:

- This exam paper contains 7 questions totaling 40 marks
- Write your name and your matriculation number on every page of the solution sheets.
- All solutions together with solution methods (explanatory statement) must be inserted in the labelled position on the solution sheets.
- You can submit your exam after the first hour.

Basic Notions: The aim of the questions in this part is to evaluate the required minimal student knowledge and skills. Answers in the pass category represent the minimum understanding of basic concepts: Mathematical Models of LTI continuous and discrete systems, Convolution, Linear Constant-Coefficient Difference and Differential Equation, Laplace transform, Fourier Series, Fourier Transform and their computation methods and representations.

## Question1 Multiple choices (circle the most appropriate one):

(10 marks)

1) The fundamental period of the sinusoidal DT signal: $\boldsymbol{x}[\boldsymbol{n}]=\boldsymbol{\operatorname { s i n }}\left(\frac{\pi n}{12}+\frac{\pi}{4}\right)$ is
a) 4
b) 8
c) 12
d) 24
2) Let $\boldsymbol{\delta}(\boldsymbol{t})$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \boldsymbol{\delta}(\boldsymbol{t}) \boldsymbol{\operatorname { c o s }}\left(\frac{3 t}{2}\right) d \boldsymbol{t}$ is
a) 0
b) $\quad 1$
c) -1
d) $\pi / 2$
3) If $\boldsymbol{x}[\boldsymbol{n}]=\left\{\begin{array}{cc}\mathbf{1}, & \mathbf{0} \leq \boldsymbol{n} \leq \mathbf{4} \\ \mathbf{0}, & \text { Otherwise }\end{array}\right\}$, describe $\boldsymbol{x}[\boldsymbol{n}]$ as superposition of two step functions.
a) $\quad x[n]=u[n]-u[n-5]$
b) $\quad x[n]=u[n-5]-u[n]$
c) $\quad x[n]=u[n-5]+u[n]$
d) $\quad x[n]=u[n]+u[n-5]$
4) Which one of the following systems is causal?
a) $\quad y(t)=x(t)+x(t-3)+x\left(t^{2}\right)$
b) $\quad y(n)=x(n+2)$
c) $y(n)=x\left(2 n^{2}\right)$
d) $y(t)=x(t-1)+x(t-2)$
5) A discrete linear time-invariant system is Bounded Input, Bounded Output (BIBO) stable if

| а) | $\sum_{k=-\infty}^{\infty}\|y[k]\|<\infty$ |
| :---: | :--- |
| b) | $\sum_{k=-\infty}^{\infty}\|h[k]\|<\infty$ |
| c) | $\sum_{k=-\infty}^{\infty}\|y[k]\| \leq \sum_{k=-\infty}^{\infty}\|h[k]\|$ |
| d) | $\sum_{k=-\infty}^{\infty}\|y[k]\| \leq \sum_{k=-\infty}^{\infty}\|x[k]\|$ |

6) The step response of a system can be written as

| $\mathbf{a )}$ | $s(t)=u(t) h(t)$ |
| :---: | :--- |
| $\mathbf{b})$ | $s(t)=u(t) * h(t)=\int_{-\infty}^{\infty} \delta(t) h(t-\tau) d \tau$ |
| $\mathbf{c )}$ | $s(t)=u(t) * h(t)=\int_{-\infty}^{\infty} u(\tau) h(t-\tau) d \tau$ |
| $\mathbf{d )}$ | None of above |

7) Determine the convolution sum of two sequences $\mathbf{x}(\mathbf{n})=\{\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{2}\}$ and $\mathbf{h}(\mathbf{n})=\{\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}\}$
a) $\quad \mathrm{y}(\mathrm{n})=\{3,8,8,12,9,4,4\}$
b) $\quad \mathrm{y}(\mathrm{n})=\{3,8,8,1,9,4,4\}$
c) $\quad \mathrm{y}(\mathrm{n})=\{3,8,3,12,9,4,4\}$
d) $y(n)=\{3,8,8,12,9,1,4\}$
8) The Fourier series of an odd periodic function, contains only
a) Sine terms
b) Cosine terms
c) Odd harmonics
d) Even harmonics
9) If $\mathbf{x}(\mathbf{n})=\mathbf{A} \mathbf{e}^{\mathbf{j} \omega \mathbf{n}}$ is the input of an $\mathbf{L T I}$ system and $\mathbf{h ( n )}$ is the response of the system, then what is the output $\mathbf{y}(\mathbf{n})$ of the system?
a) $\quad H(-\omega) x(n)$
b) $\quad-H(\omega) x(n)$
c) $\quad H(\omega) x(n)$
d) None of the mentioned
10)If $\boldsymbol{Y}(\boldsymbol{s})$ is the Laplace-transform of the output function, $\boldsymbol{X}(\boldsymbol{s})$ is the Laplace-transform of the input function and $\boldsymbol{H}(\boldsymbol{s})$ is the Laplace-transform of system function of the LTI system, then $\boldsymbol{H}(\boldsymbol{s})=$ ?
a) $\quad X(s) / Y(s)$
b) $\quad Y(s) / X(s)$
c) $\quad Y(s) \cdot X(s)$
d) None of the mentioned

Familiar and Unfamiliar Problems Solving: The aim of the questions in this part is to evaluate that the student has some basic knowledge of the key aspects of the lecture material and can attempt to solve familiar and unfamiliar problems of Mathematical Models of LTI discrete systems, Convolution, Linear Constant-Coefficient Difference and Differential Equation, Laplace transform, Fourier Series, Fourier Transform and their computation methods and representations

## Question 2

a) In particular, a system may or may not be Linear, Time-invariant and Causal. Determine which of these properties hold which do not hold for each of the following signals. In each example, $\boldsymbol{y}(\boldsymbol{t})$ denotes the system output, and $\boldsymbol{x}(\boldsymbol{t})$ is the system input
(3 marks)

1) $y(t)=3 x(t) \cos (t)$
2) $y(t)=\int_{t}^{t+1} x(\lambda) d \lambda$

Hint: Put $\sqrt{ }$ if the system holds the property and $\times$ if not

| system | Linear | Time-invariant | Causal |
| :---: | :---: | :---: | :---: |
| $y(t)=3 x(t) \cos (t)$ |  |  |  |
| $y(t)=\int_{t}^{t+1} x(\lambda) d \lambda$ |  |  |  |

b) Consider the interconnection of four LTI systems shown in the figure below. Find the impulse response, $h[n]$ of the overall system from the properties of DT convolution.


## Solution

Calculate the convolution of the signal $\boldsymbol{x}[\boldsymbol{n}]$ with the impulse response, $\boldsymbol{h}[\boldsymbol{n}]$ (using the equation of convolution by hand) where:

$$
\begin{array}{ll}
\boldsymbol{x}[\boldsymbol{n}]=\{\mathbf{3}, \mathbf{4}, \mathbf{5}\}, & \boldsymbol{x}[\boldsymbol{n}] \text { has only non-zero values at } \boldsymbol{n}=\mathbf{0}, \mathbf{1}, \mathbf{2} \\
\boldsymbol{h}[\boldsymbol{n}]=\{\mathbf{2}, \mathbf{1}\}, & \boldsymbol{h}[\boldsymbol{n}] \text { is not zero at } \boldsymbol{n}=\mathbf{0}, \mathbf{1}
\end{array}
$$

Determine the Fourier Series representation (trigonometric form) for the function $\boldsymbol{f}(\boldsymbol{x})$, defined on $[-\mathbf{2}, \mathbf{2}]$, where

$$
f(x)= \begin{cases}-1, & -2 \leq x \leq 0 \\ 2, & 0<x \leq 2\end{cases}
$$

Solution
a) Evaluate the frequency-domain representations of the following signal using Fourier Transform:

$$
x(t)=e^{-4|t|}
$$

b) Use the Fourier transform tables and properties to obtain the Fourier transform of the following signal:

$$
x(t)=\sin (2 \pi t) e^{-x} x(t)
$$

## Solution

c) Consider a stable LTI system characterized by the differential equation

$$
\frac{d^{2} y(t)}{d t^{2}}+4 \frac{d y(t)}{d t}+3 y(t)=\frac{d x(t)}{d t}+2 x(t)
$$

Find the frequency response $\boldsymbol{H}(\boldsymbol{\omega})$ and impulse response $\boldsymbol{h}(\boldsymbol{t})$ of the system.

## Solution

## Question 6

Use the definition of the Laplace Transform; determine the Laplace transform of the following signal:

$$
x(t)=e^{a t} u(t-k)
$$

Sketch the pole-zero plot. What is the condition on for the Fourier Transform to exist?

## Solution

Write a complete MATLAB program to
a) Generate and plot in the same figure each of the following CT signals:

$$
\begin{array}{ll}
x_{1}(t)=5 e^{-0.2 t} \sin (2 \pi t), & -5 \leq t \leq 10 \\
x_{2}(t)=e^{(-4 \pi-0.5) t} u(t) & -1 \leq t \leq 15
\end{array}
$$

b) Compute the convolution $\boldsymbol{y}(\boldsymbol{t})=\boldsymbol{x}_{1}(\boldsymbol{t}) * \boldsymbol{x}_{\mathbf{2}}(\boldsymbol{t})$ and plot the result in the same figure.

Hints:

- Do not use the "for" loops in your code.
- Use suitable axis and titles in your code.


## Solution

