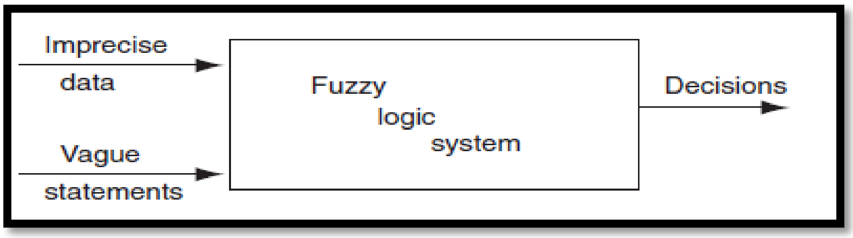
Neural Networks and Fuzzy Logic (630514)

Lecture 17

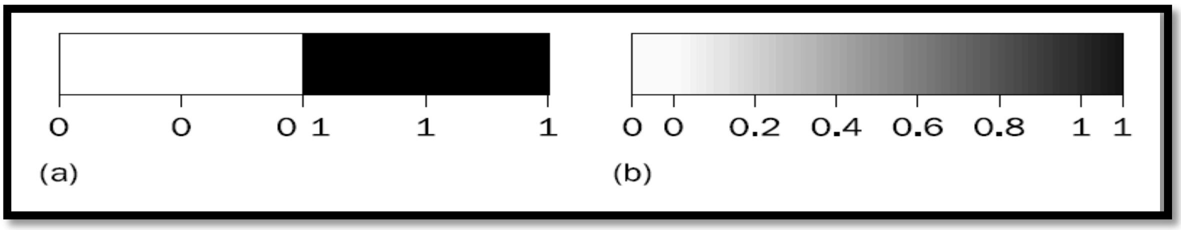
Fuzzy expert systems – Introduction

- **Fuzzy set theory.**
- **Build fuzzy expert systems.**

- The fuzzy theory provides a mechanism for representing linguistic constructs such as "***many***," "***low***," "***medium***," "***often***," "***few***."
- **Experts** usually rely on common sense when they solve problems. They also use ***vague*** and ***ambiguous*** terms.



- A fuzzy logic system which accepts imprecise data and vague statements such as **low**, **medium**, **high** and provides **decisions**.
- Fuzzy logic is based on the idea that all things admit of degrees. **Temperature, height, speed, distance, beauty**.
    - **The motor is running really hot.**
    - **Tom is a very tall man.**
    - **Electric cars are not very fast.**
    - **High-performance drives require very rapid dynamics and precise regulation.**
- How can we represent expert knowledge that uses vague and ambiguous terms in a computer? Can it be done at all?
    - **Fuzzy or multi-valued logic** was introduced in the 1930s by Jan Lukasiewicz; He studied the **mathematical representation of fuzziness based on such terms as tall, old and hot.** While classical logic operates with only two values 1 (true) and 0 (false), Lukasiewicz introduced logic that **extended the range of truth values to all real numbers in the interval between 0 and 1**. He used a number in this interval to represent the possibility that a given statement was true or false.
    - In 1965 **Lotfi Zadeh**, published his famous paper '**Fuzzy sets**'. In fact, Zadeh rediscovered fuzziness, identified and explored it.
- Boolean or conventional logic uses **sharp distinctions**.
- Fuzzy logic reflects how people **think**.



Range of logical values in Boolean and fuzzy logic:
(a) Boolean logic; (b) multivalued logic

- Fuzziness rests on **fuzzy set theory**, and **fuzzy logic** is just a small part of that theory.
- **Fuzzy logic** is determined as a set of mathematical principles for knowledge representation based on **degrees of membership** rather than on **crisp membership** of **classical binary logic**.
- Unlike two-valued Boolean logic, fuzzy logic is multi-valued. It deals with degrees of membership and **degrees of truth**. Fuzzy logic uses the continuum of logical values between 0 (**completely false**) and 1 (**completely true**).
- In the years to come **fuzzy computers** will employ both *fuzzy hardware* and *fuzzy software*, and they will be much closer in structure to the human brain than the present-day computers are.

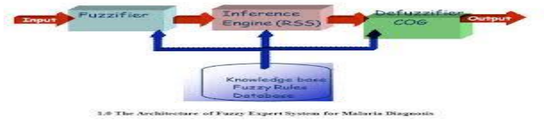
Fuzzy sets

- The concept of a set is fundamental to mathematics.
- Let **X** be a **classical (crisp)** set and **x** an element. Then the element **x** either belongs to **X** ($x \in X$) or does not belong to **X** ($x \notin X$). That is, classical set theory imposes a **sharp boundary** on this set and gives each member of the set the value of **1**, and all members that are not within the set a value of **0**.
- The classical example in the fuzzy set theory is **tall men**. The elements of the fuzzy set 'tall men' are all men, but their degrees of membership depend on their height, as shown in Table:

Name	Height, cm	Degree of membership	
		Crisp	Fuzzy
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

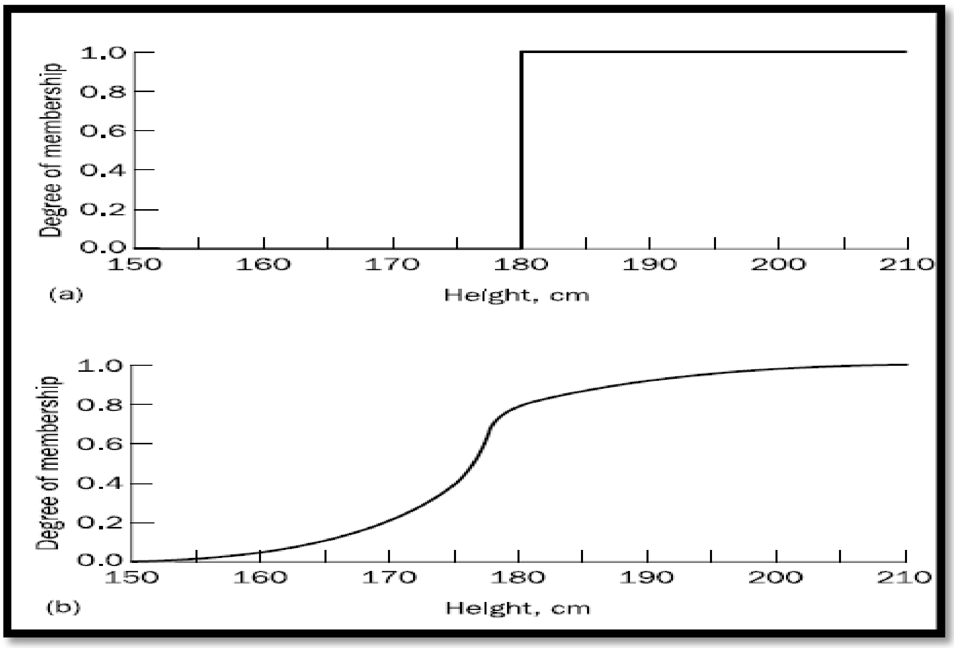
Degree of membership of 'tall men'

- It can be seen that the crisp set asks the question, '**Is the man tall**?' and draws a line at, say, 180 cm. Tall men are above this height and not tall men below.
- The fuzzy set asks, '**How tall is the man**?' The answer is the partial membership in the fuzzy set, for example, Tom is 0.82 tall.



- We might consider a few other sets such as '**very short men**', '**short men**', '**average men**' and '**very tall men**'.

- A fuzzy set is capable of providing a **graceful transition** across a boundary, as shown in Figure.



Crisp (a) and fuzzy (b) sets of 'tall men'

- **Horizontal axis** represents the **universe of discourse** – the range of all possible values applicable to a chosen variable.
- The **vertical axis** represents the membership value of the fuzzy set.

- A fuzzy set can be simply defined as a set with fuzzy boundaries.
- Let **X** be the **universe of discourse** and its elements be denoted as **x**. In classical set theory, crisp set **A** of **X** is defined as function $f_A(x)$ called the characteristic function of A

$$f_A(x) : X \rightarrow 0, 1,$$

Where

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

- In the fuzzy theory, fuzzy set **A** of universe **X** is defined by function $\mu_A(x)$called the membership function of set **A.**

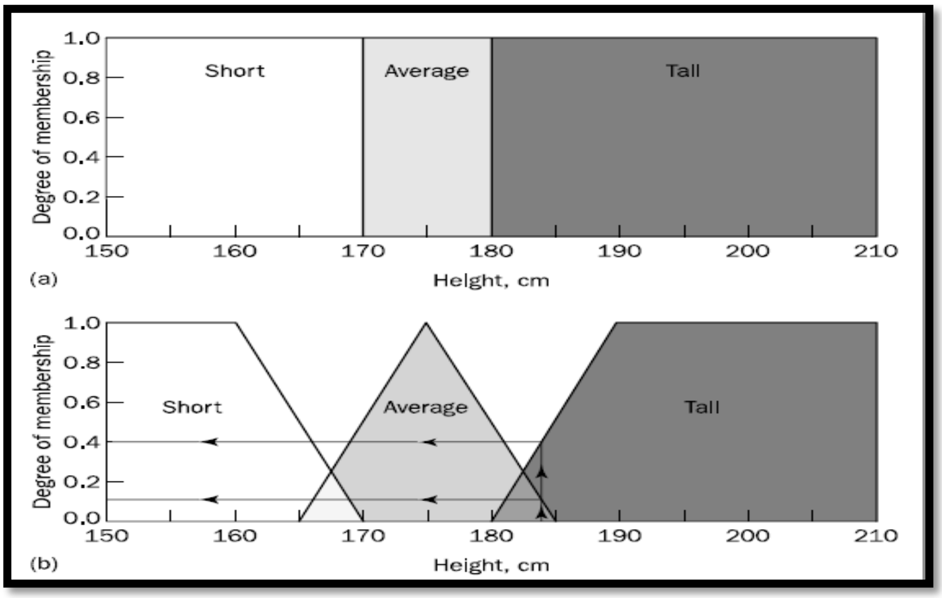
$$\mu_A(x) : X \rightarrow [0, 1],$$

Where

$$\mu_A(x) = 1 \text{ if } x \text{ is totally in } A;$$
$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A;$$
$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A.$$

How to represent a fuzzy set in a computer?

- The membership function must be determined first. A number of methods learned can be used:
  - Knowledge of a **single expert**.
  - Acquire knowledge from **multiple experts**.
  - Using artificial neural networks which learn available system operation data and then derive the fuzzy sets automatically.
- After acquiring the knowledge for men's heights, fuzzy set of **tall**, **short** and **average** men can be obtained.



Crisp (a) and fuzzy (b) sets of short, average and tall men

- **Example**: a crisp set containing five elements

$$X = \{x_1, x_2, x_3, x_4, x_5\}$$

Let **A** be a crisp subset of **X**:

$$A = \{x_2, x_3\}$$

Subset **A** can now be described by:

$$A = \{(x_1, 0), (x_2, 1), (x_3, 1), (x_4, 0), (x_5, 0)\}$$

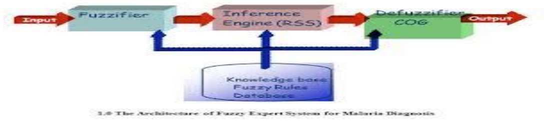
or as a set of pairs:

$$\{(x_i, \mu_A(x_i))\}$$

Where $\mu_A(x_i)$ is the membership function of element $\mathbf{x_i}$ in the subset **A**.

- If **X** is the reference super set and **A** is a subset of **X**, then **A** is said to be a fuzzy subset of **X** when:

$$A = \{(x, \mu_A(x))\} \qquad x \in X, \mu_A(x) : X \rightarrow [0, 1]$$



- Special case, when $X \rightarrow \{0,1\}$ is used instead of $X \rightarrow [0,1]$the fuzzy subset **A** becomes the crisp subset **A**.
- Fuzzy subset **A** of the finite reference super set **X** can be expressed as:

$$A = \{(x_1, \mu_A(x_1)\}, \{(x_2, \mu_A(x_2)\}, \ldots, \{(x_n, \mu_A(x_n)\}$$

Or

$$A = \{\mu_A(x_1)/x_1\}, \{\mu_A(x_2)/x_2\}, \ldots, \{\mu_A(x_n)/x_n\},$$

- To represent a continuous fuzzy set in a computer, we need to express it as a function
  - Typical functions:
    - Sigmoid, Gaussian. These functions can represent the real data in fuzzy sets, but they also increase the time of computation.
    - Therefore, in practice, most applications use linear fit functions similar to those shown in Figure in previous example. For example, the fuzzy set of tall men can be represented as a fit-vector,

tall men = (0/180, 0.5/185, 1/190) or
tall men = (0/180, 1/190)

- Fuzzy sets of short and average men:

short men = (1/160, 0.5/165, 0/170) or
short men = (1/160, 0/170)
average men = (0/165, 1/175, 0/185)