Fuzzy expert systems – Introduction

- Fuzzy set theory.
- Build fuzzy expert systems.

- The fuzzy theory provides a mechanism for representing linguistic constructs such as “many,” “low,” “medium,” “often,” “few.”
- Experts usually rely on common sense when they solve problems. They also use vague and ambiguous terms.

- A fuzzy logic system which accepts imprecise data and vague statements such as low, medium, high and provides decisions.
- Fuzzy logic is based on the idea that all things admit of degrees. Temperature, height, speed, distance, beauty.
  - The motor is running really hot.
  - Tom is a very tall man.
  - Electric cars are not very fast.
  - High-performance drives require very rapid dynamics and precise regulation.
- How can we represent expert knowledge that uses vague and ambiguous terms in a computer? Can it be done at all?
  - Fuzzy or multi-valued logic was introduced in the 1930s by Jan Lukasiewicz; He studied the mathematical representation of fuzziness based on such terms as tall, old and hot. While classical logic operates with only two values 1 (true) and 0 (false), Lukasiewicz introduced logic that extended the range of truth values to all real numbers in the interval between 0 and 1. He used a number in this interval to represent the possibility that a given statement was true or false.
  - In 1965 Lotfi Zadeh, published his famous paper ‘Fuzzy sets’. In fact, Zadeh rediscovered fuzziness, identified and explored it.
- Boolean or conventional logic uses sharp distinctions.
- Fuzzy logic reflects how people think.
Fuzziness rests on **fuzzy set theory**, and **fuzzy logic** is just a small part of that theory.

**Fuzzy logic** is determined as a set of mathematical principles for knowledge representation based on **degrees of membership** rather than on **crisp membership** of classical binary logic.

Unlike two-valued Boolean logic, fuzzy logic is multi-valued. It deals with degrees of membership and **degrees of truth**. Fuzzy logic uses the continuum of logical values between 0 (**completely false**) and 1 (**completely true**).

In the years to come **fuzzy computers** will employ both **fuzzy hardware** and **fuzzy software**, and they will be much closer in structure to the human brain than the present-day computers are.

**Fuzzy sets**

The concept of a set is fundamental to mathematics.

Let $X$ be a **classical (crisp)** set and $x$ an element. Then the element $x$ either belongs to $X (x \in X)$ or does not belong to $X (x \notin X)$. That is, classical set theory imposes a **sharp boundary** on this set and gives each member of the set the value of 1, and all members that are not within the set a value of 0.

The classical example in the fuzzy set theory is **tall men**. The elements of the fuzzy set ‘tall men’ are all men, but their degrees of membership depend on their height, as shown in Table:

<table>
<thead>
<tr>
<th>Name</th>
<th>Height, cm</th>
<th>Crisp</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>208</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>Mark</td>
<td>205</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>John</td>
<td>198</td>
<td>1</td>
<td>0.98</td>
</tr>
<tr>
<td>Tom</td>
<td>181</td>
<td>1</td>
<td>0.92</td>
</tr>
<tr>
<td>David</td>
<td>179</td>
<td>0</td>
<td>0.78</td>
</tr>
<tr>
<td>Mike</td>
<td>172</td>
<td>0</td>
<td>0.24</td>
</tr>
<tr>
<td>Bob</td>
<td>167</td>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>Steven</td>
<td>158</td>
<td>0</td>
<td>0.06</td>
</tr>
<tr>
<td>Bill</td>
<td>155</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>Peter</td>
<td>152</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Degree of membership of ‘tall men’**

- It can be seen that the crisp set asks the question, ‘**Is the man tall?**’ and draws a line at, say, 180 cm. Tall men are above this height and not tall men below.
- The fuzzy set asks, ‘**How tall is the man?**’ The answer is the partial membership in the fuzzy set, for example, Tom is 0.82 tall.
We might consider a few other sets such as ‘very short men’, ‘short men’, ‘average men’ and ‘very tall men’.

- A fuzzy set is capable of providing a graceful transition across a boundary, as shown in Figure.

Crisp (a) and fuzzy (b) sets of ‘tall men’

- Horizontal axis represents the universe of discourse – the range of all possible values applicable to a chosen variable.
- The vertical axis represents the membership value of the fuzzy set.

- A fuzzy set can be simply defined as a set with fuzzy boundaries.
- Let $X$ be the universe of discourse and its elements be denoted as $x$. In classical set theory, crisp set $A$ of $X$ is defined as function $f_A(x)$ called the characteristic function of $A$

$$f_A(x) : X \rightarrow 0, 1,$$

Where

$$f_A(x) = \begin{cases} 
1, & \text{if } x \in A \\
0, & \text{if } x \not\in A
\end{cases}$$

- In the fuzzy theory, fuzzy set $A$ of universe $X$ is defined by function $\mu_A(x)$ called the membership function of set $A$.

$$\mu_A(x) : X \rightarrow [0, 1],$$

Where

$$\mu_A(x) = 1 \text{ if } x \text{ is totally in } A;$$
$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A;$$
$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A.$$
How to represent a fuzzy set in a computer?

- The membership function must be determined first. A number of methods learned can be used:
  - Knowledge of a single expert.
  - Acquire knowledge from multiple experts.
  - Using artificial neural networks which learn available system operation data and then derive the fuzzy sets automatically.

- After acquiring the knowledge for men’s heights, fuzzy set of tall, short and average men can be obtained.

![Crisp (a) and fuzzy (b) sets of short, average and tall men](image)

**Example:** a crisp set containing five elements

\[
X = \{x_1, x_2, x_3, x_4, x_5\}
\]

Let \(A\) be a crisp subset of \(X\):

\[
A = \{x_2, x_3\}
\]

Subset \(A\) can now be described by:

\[
A = \{(x_1, 0), (x_2, 1), (x_3, 1), (x_4, 0), (x_5, 0)\}
\]

or as a set of pairs:

\[
\{(x_i, \mu_A(x_i))\}
\]

Where \(\mu_A(x_i)\) is the membership function of element \(x_i\) in the subset \(A\).

- If \(X\) is the reference super set and \(A\) is a subset of \(X\), then \(A\) is said to be a fuzzy subset of \(X\) when:

\[
A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) : X \to [0, 1]\}
\]
Special case, when $X \to \{0, 1\}$ is used instead of $X \to [0, 1]$ the fuzzy subset $A$ becomes the crisp subset $A$.

Fuzzy subset $A$ of the finite reference super set $X$ can be expressed as:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\}$$

Or

$$A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\},$$

To represent a continuous fuzzy set in a computer, we need to express it as a function:

Typical functions:

- Sigmoid, Gaussian. These functions can represent the real data in fuzzy sets, but they also increase the time of computation.
- Therefore, in practice, most applications use linear fit functions similar to those shown in Figure in previous example. For example, the fuzzy set of tall men can be represented as a fit-vector,
  - tall men = (0/180, 0.5/185, 1/190) or tall men = (0/180, 1/190)
  - Fuzzy sets of short and average men:
    - short men = (1/160, 0.5/165, 0/170) or short men = (1/160, 0/170)
    - average men = (0/165, 1/175, 0/185)