Sugeno Fuzzy Models

- The main difference between **Mamdani** and **Sugeno** is that the Sugeno output membership functions are either **linear or constant**.

- Mamdani-style inference requires finding the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.

- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A *singleton*, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

- The **Sugeno Fuzzy model** (also known as the **TSK fuzzy model**) was proposed by Takagi, Sugeno, and Kang. A typical fuzzy rule in a Sugeno fuzzy model has the form:

  \[
  \text{if } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z = f(x, y)
  \]

  - Where **A** and **B** are fuzzy sets in the antecedent, while \(z = f(x, y)\) is a crisp function in the consequent.
  - Usually \(f(x, y)\) is a polynomial of the input variables \(x\) and \(y\), but it can be any function.
  - When \(f(x, y)\) is a first-order polynomial, the resulting fuzzy inference system is called a **first-order Sugeno fuzzy model**, for example \(z = ax + by + c\).
  - When \(f\) is a constant, we then have a **zero-order Sugeno fuzzy model**, \((z = ax + by + c \text{ where } (a = b = 0)) \) which can be viewed either as a special case of the Mamdani Fuzzy inference system in which each rule's consequent is specified by a **fuzzy singleton**.
• Each rule weights its output level, \( z_i \), by the firing strength of the rule, \( w_i \). For example, for an **AND** rule with **Input 1 = x** and **Input 2 = y**, the firing strength is

\[
w_i = \text{AndMethod}(F_1(x), F_2(y))
\]

Where \( F_1(x), F_2(y) \) are the membership functions for **Inputs 1 and 2**.

• The final output of the system is the weighted average of all rule outputs, computed as

\[
\text{Final Output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}
\]

Where \( N \) is the number of rules

• A Sugeno rule operates as shown in the following diagram.

• Figure shows the fuzzy reasoning procedure for a first-order Sugeno fuzzy model. Since each rule has a **crisp output**, the overall output is obtained via **weighted average**, thus avoiding the time-consuming process of defuzzification required in a Mamdani model.
1st order Sugeno Fuzzy Model

- **Example 1**: Fuzzy and nonfuzzy rule set – a comparison

\[
\begin{align*}
&\text{If } X \text{ is small then } Y = 0.1X + 6.4 \\
&\text{If } X \text{ is medium then } Y = -0.5X + 4 \\
&\text{If } X \text{ is large then } Y = X - 2
\end{align*}
\]

- If “small”, “medium”, and “large” are **nonfuzzy** sets with membership functions shown Figure (a), the overall input-output curve is **piecewise linear**, as shown in Figure (b).
- If “small”, “medium”, and “large” are **nonfuzzy** sets with membership functions shown Figure (c), the overall input-output curve becomes a smoother one, as shown in Figure (d).
**Example 2:** Two-input single-output Sugeno fuzzy model

\[
\begin{align*}
\text{If } X \text{ is small and } Y \text{ is small then } z &= -x + y + 1 \\
\text{If } X \text{ is small and } Y \text{ is large then } z &= -y + 3 \\
\text{If } X \text{ is large and } Y \text{ is small then } z &= -x + 3 \\
\text{If } X \text{ is large and } Y \text{ is large then } z &= x + y + 2
\end{align*}
\]

- Figure (a) plots the membership functions of input \( X \) and \( Y \), and Figure (b) is the resulting input-output surface. The surface is complex; the surface is composed of four planes, each of which is specified by output equation of a fuzzy rule.

**Example 3:**

**Sugeno-style rule evaluation**

\[
\begin{align*}
\text{Rule 1: IF } x \text{ is } A_3 (0.0) \text{ OR } y \text{ is } B_1 (0.1) \text{ THEN } z &\text{ is } k_1 (0.1) \\
\text{Rule 2: IF } x \text{ is } A_2 (0.2) \text{ AND } y \text{ is } B_2 (0.7) \text{ THEN } z &\text{ is } k_2 (0.2) \\
\text{Rule 3: IF } x \text{ is } A_1 (0.5) \text{ THEN } z &\text{ is } k_3 (0.5)
\end{align*}
\]
Sugeno-style aggregation of the rule outputs

Weighted average (WA):

\[
WA = \frac{\mu(k_1) \times k_1 + \mu(k_2) \times k_2 + \mu(k_3) \times k_3}{\mu(k_1) + \mu(k_2) + \mu(k_3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65
\]

Sugeno-style defuzzification

How to make a decision on which method to apply – Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.