Digital System Design

Lecture 3

Digital Number Systems I

Objectives:

1. **Understanding decimal, binary, octal and hexadecimal numbers.**
2. **Counting in decimal, binary, octal and hexadecimal systems.**
3. **Convert a number from one number system to another system.**
4. **Advantage of octal and hexadecimal systems.**

1. Understanding decimal, binary, octal and hexadecimal numbers

Decimal number systems:

- ✓ Decimal numbers are made of decimal digits:
  (0,1,2,3,4,5,6,7,8,9 --------10-base system)
- ✓ The decimal system is a "***positional-value system***" in which the value of a digit depends on its position.
  Examples:
- ❖ 453→4 hundreds, 5 tens and 3 units.
  - ✓ *4* is the most weight called "most significant digit" MSD.
  - ✓ *3 carries the last weight called "*least significant digit" LSD.
- ❖ number of items that a decimal number represent:

$$9261 = (9\times10^3)+(2\times10^2)+(6\times10^1)+(1\times10^0)$$

- ❖ The decimal fractions:

$$3267.317 = (3\times10^3)+(2\times10^2)+(6\times10^1)+(7\times10^0)+(3\times10^{-1})+(6\times10^{-2})+(1\times10^{-3})$$

- ✓ ***Decimal point*** used to separate the integer and fractional part of the number.
- ✓ ***Formal notation***→$(3267.317)_{10}$.
- ✓ Decimal position values of powers of (10).

Positional values "weights"

10^4	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	10^{-4}
↑	↑	↑	↑	↑		↑	↑	↑	↑
2	**7**	**7**	**8**	**3**	**.**	**2**	**3**	**4**	**5**
MSD									**LSD**

Binary numbers:

- **Base-2** system (**0 or 1**).
- We can represent any quantity that can be represented in decimal or other number systems using ***binary numbers***.
- Binary number is also ***positional–value system*** (power of **2**).

Example: 1101.011

2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}
↑	↑	↑	↑		↑	↑	↑
1	**1**	**0**	**1**	**.**	**0**	**1**	**1**
MSD							**LSD**

Notes:

- To find the equivalent of binary numbers in decimal system , we simply take the ***sum of products of each digit value (0,1)and its positional value***:

Example:$(1011.101)_2$

$= (1\times2^3) + (0\times2^2) + (1\times2^1) + (1\times2^0)+ (1\times2^{-1})+ (0\times2^{-2}) +(1\times2^{-3})$

$= 8 + 0 + 2 + 1 + \frac{1}{2} + 0 + \frac{1}{8} = 11.625_{10}$

In general, any number (decimal, binary, octal and hexadecimal) is simply the sum of products of each digit value and its positional value.

- In binary system, the term binary digit is often called ***bit***.

- Binary values at the output of digital system must be converted to decimal values for presentation to the outside world.
- Decimal values must be converted into the digital system.
- Group of 8 bits are called a ***byte***.

Octal Number System

- octal number system has a **base of 8 : (0,1,2,3,4,5,6,7)**

Examples:

- $(1101.011)_8$

8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
↑	↑	↑	↑		↑	↑	↑
1	1	0	1	.	0	1	1
MSD							LSD

- $(4327)_8$

$= (4\times8^3) + (3\times8^2) + (2\times8^1) + (7\times8^0)$

- 372.36_8

$= (3\times8^2) + (7\times8^1) + (2\times8^0) + (3\times8^{-1}) + (6\times8^{-2})$

Note: octal number don't use digits 8 or 9

Hexadecimal number system (16-base)

✓ Hexadecimal numbers are made of 16 digits, it uses the digits **0** through **9** plus the letters **A, B, C, D, E, F.**

Examples:

- $(A29)_{16}$

$= (10\times16^2) + (2\times16^1) + (9\times16^0) = (2601)_{10}$

- $(2c7.38)_{16}$

$= (2\times16^2) + (12\times16^1) + (7\times16^0) + (7\times16^0) + (3\times16^{-1}) + (8\times16^{-2})$

Note:

- ✓ For hex numbers the digits **10, 11, 12, 13, 14, 15** are represented by **a, b, c, d, e, f** as shown in the following table:

Number Systems			
Decimal	**Binary**	**Octal**	**Hex**
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

2. Counting in decimal ,binary, octal and hexadecimal systems

Decimal counting:

- *Start with 0 in the units position and take each digit in progression until reach 9.*
- *Add 1 to the next higher position and start over 0 in the first position.*
- *Continue process until the count 99.*
- *Add 1 to the third position and start over with 0 in the first position.*

Note: the largest number that can be represented using 8 bits is

$$2^n\text{-}1= 2^8\text{-}1= 255_{10} = 11111111_2$$

Counting in hexadecimal:

- ✓ For **n** hex digit positions, we can count for decimal **0** to $16^n - 1$, for a total of 16^n different values.
- ✓ The ***general representation*** for a number in the form:

$$a_4a_3a_2a_1a_0.\ a_{-1}a_{-2}a_{-3}$$

Using **r-base/radix** number system, in which the number of ***radix r*** can be written as

$$n_r = \dots + a_4 . r^4 + a_3 . r^3 + a_2 . r^2 + a_1 . r^1 + a_0 . r^0 + a_{-1} . r^{-1} + a_{-2} . r^{-2} + \dots$$

Numbering System	Radix
Decimal	r=10
Binary	r=2
Octal	r=8
Hex	r=16

Counting in binary system: (counting range)

- ✓ Using **n** bits, we can represent decimal numbers ranging from **0** to $2^n - 1$, a total of 2^n different numbers.

Examples:

- for n=4 bits

We can count from **0000** to $\mathbf{1111}_2$ (see table above) which is 0_{10} to 15_{10} (16 different numbers).

- How many bits are needed to represent decimal values ranging from 0 to 12500?

Answer:

- With **13** bits , we can count from **0** to 2^{13}-1 =**8191** (***not enough***)
- With **14** bits , we can count from **0** to 2^{14}-1 =**16.383** (***okay***)
  - What is the total range of decimal values that can be represented in 8 bits?

Answer:

For **N=8,** we can represent form **0** to 2^8-1 =**255.**