①

Lect. 5

Relationships between pixels

a. Neighbors of a pixel:

1. $N_4(P)$: 4- neighbors of P.

A pixel P at coordinates (x, y) has 2 horizontal and 2 vertical neighbors with coordinates:
$(x+1, y)$, $(x-1, y)$, $(x, y+1)$ and $(x, y-1)$

This set of pixels, called 4- neighbors of P, is denoted by $N_4(P)$.

each pixel is a unit distance from $(P : f(x,y))$

y →, x ↓

	$f(x-1,y)$	
$f(x,y-1)$	P $f(x,y)$	$f(x,y+1)$
	$f(x+1,y)$	

$N_4(P)$

2. $N_D(P)$: four diagonal neighbors of P have coordinates:

$(x+1, y+1)$, $(x+1, y-1)$, $(x-1, y+1)$, and $(x-1, y-1)$, and are denoted by $N_D(P)$

y →, x ↓

$f(x-1,y-1)$		$f(x-1,y+1)$
	P $f(x,y)$	
$f(x+1,y-1)$		$f(x+1,y+1)$

$N_D(P)$

3. $N_8(P)$: 8- neighbors of P.

$N_4(P)$ and $N_D(P)$ together are called 8-neighbors of P, denoted by $N_8(P)$.

* Some of the neighbor locations in $N_4(P)$, $N_D(P)$ or $N_8(P)$ of P lie outside the digital image, if $P\ (f(x,y))$ is on the border of the image.

y →, x ↓

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

$N_8(P)$

②

Important Concepts: Adjacency, Connectivity, Regions and Boundaries.

Let V: a set of intensity values used to define adjacency and connectivity.

* In a binary Image $V=\{1\}$, if we are referring to adjacency of pixels with value 1.

In a Grayscale image, the idea is the same, but V typically contains more elements, for example $V=\{180, 181, 182, \dots, 200\}$.

if the possible intensity values 0 to 255, V set could be any subset of these 256 values.

types of adjacency.

1- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(P)$.

2. **8-adjacency**: Two pixels P & q with values from V are 8-adjacent if q is in the set $N_8(P)$.

important: the type of adjacency used must be specified

3. **m-adjacency = (mixed)**: Two pixels P & q with values from V are m-adjacent if:

* q is in $N_4(P)$ or
* q is in $N_D(P)$ and
* the set $N_4(P) \cap N_4(q)$ has no pixel whose values are from V (No intersection).

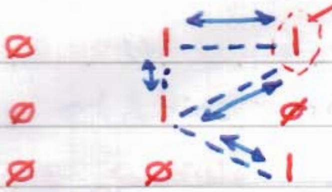
Mixed adjacency is a modification of 8-adjacency " introduced to eleminate the ambiguities that often arise when 8-adjacency is used. (eleminate multiple path connection)

Example: pixel arrangement as shown in figure ① for $V=\{1\}$.

0	1	1
0	1	0
0	0	1

figure ①

2 relations (X)



three pixels at the top (figure 2) show multiple (ambiguous) 8-adjacency. This ambiguity is removed by using m- adjacency. as shown in figure ③.

figure ②

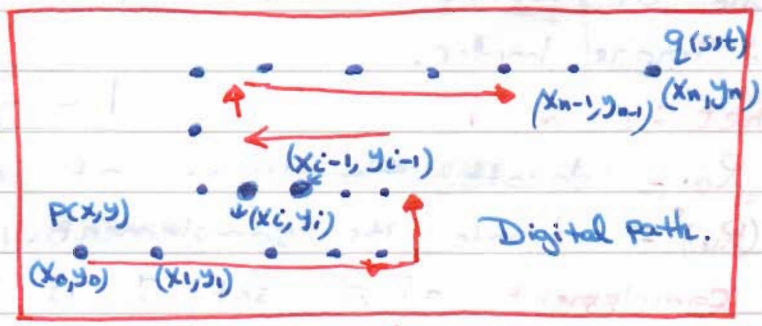
* Path :

A digital path (or curve) from pixel p with Coordinate (x, y) to pixel q with coordinate (s, t) is a Sequence of distinct pixels with coordinates $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ where

$(x_0, y_0) = (x, y)$, $(x_n, y_n) = (s, t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

n- the length of the path.

if $(x_0, y_0) = (x_n, y_n)$: the path is closed path.



entire image

We can define 4-, 8-, or m-paths depending on the type of adjacency specified.

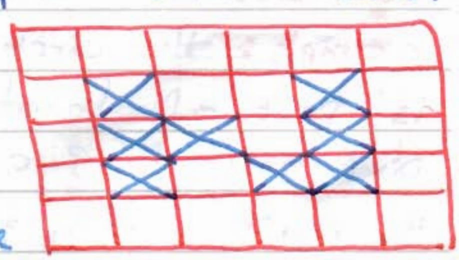
for example: in figure ② the paths between the top right and bottom right are 8-Paths, and the paths in figure ③ is m-path.

* Connectivity :

let S represent a subset of pixels in an image, Two pixels p and q are said to be connected in S if there exists a path between them.

* Region :

- Let R to be a sub set of pixels in an image, we call a R a region of the Image



Region

if R is a Connected set.

- Regions that are not adjacent are said to be <u>disjoint</u>.

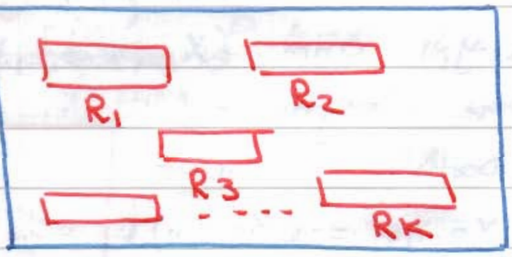
example: the two regions (of 1s) in figure, are are adjacent only if 8-adjacency is used.

1	1	1	R_i
1	Ø	1	
Ø	1	Ø	
Ø	Ø	1	R_j
1	1	1	
1	1	1	

* 4- path between the two regions does not exist, (So their union in not a Connected set).

* Boundary (border)

image contains K disjoint regions., R_K, $K = 1, 2, \ldots, K$, none of which touches the image border.



Let :

R_U - denote the union of all the K regions.

$(R_u)^c$ - denote its complement.

(Complement of a set S is the set of points that are not in S).

R_U - Called foreground; $(R_u)^c$ - called background of the image

Boundary (border or Contour) of a region R is the set of points that are adjacent to points in the Complement of R (another way: the border of a region is the set of pixels in the region that have at least one background neighbor)

(We must specify the connectivity being used to define adjacency).

example: the circled point (in figure) is part of boundary of the 1-valued pixel only if 8-adjacency between the region and background is used.

Ø	Ø	Ø	Ø	Ø
Ø	1	1	Ø	Ø
Ø	1	1	Ø	Ø
Ø	1	1	1	Ø
Ø	1	1	1	Ø
Ø	Ø	Ø	Ø	Ø

and it is not a member of boundary if 4-Connectivity is used.

As a rule: adjacency between points in a region and its background is defined in term of 8-Connectivity to handle like' Situations.

The preceding definition sometimes is referred to as the Inner Border of a region.

Outer Border: corresponding border in the background.

(Important: for development of border-following algorithms)

example: The inner boundary of the 1-valued region not form a closed path, but its outer boundary does.

∅	∅	∅
∅	1	∅
∅	1	∅
∅	1	∅
∅	1	∅
∅	∅	∅

* The boundary of a finite region forms a closed path.

"Distance Measures"

For pixels P, q and Z, with coordinates (X,y), (s,t) and (v,w), respectively, D is a distance function or metric if:

- $D(P,q) \geq \emptyset$, ($D(P,q) = \emptyset$ if $P = q$)
- $D(P,q) = D(q,P)$, and
- $D(P,z) \leq D(P,q) + D(q,z)$.

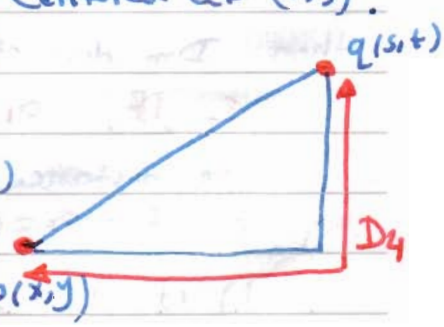
1) Euclidean Distance:

$$D_e(P,q) = \left[(x-s)^2 + (y-t)^2\right]^{\frac{1}{2}}$$

pixels having a distance less than p(x,y) or equal to some value r from (x,y) are the points contained in a disk of radius r Centered at (x,y).

2) D_4 distance (City-block distance)

$$D_4(P,q) = |x-s| + |y-t|$$



pixels having a D_4 distance from (x,y) less than or equal to some value r form a Diamond Centered at (x,y).

example ①: the pixels with distance $D_4 \leq 2$ from (x,y) form the following contours of constant distance

example ②:

The pixels with $D_4 = 1$ are the 4-neighbors of (x,y).

		2		
	2	1	2	
2	1	∅	1	2
	2	1	2	
		2		

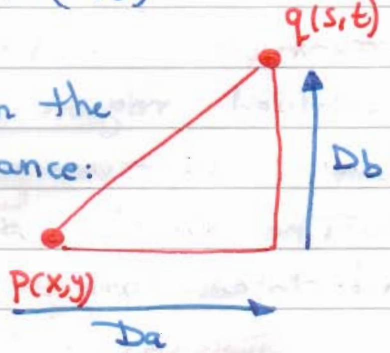
③ D_8 distance (Chess board distance)

- $D_8(P,Q) = \max(|x-s|, |y-t|)$
- Square - Centered at (x,y)
- $D_8 = 1$ are 8-neighbors of (x,y)

example:

D_8 distance ≤ 2 from (x,y) form the following contours of constant distance:

2	2	2	2	2
2	1	1	1	2
2	1	∅	1	2
2	1	1	1	2
2	2	2	2	2



$D_8 = \max(D_a, D_b)$

* D_m distance: is defined as the shortest m-path between the points.

- the distance between pixels depends only on the values of pixels.

example: Consider the following arrangement of pixels

P_3	P_4
P_1	P_2
P	

and assume that P, P_2, and P_4 have value 1 and that P_1 and P_3 can have a value of 0 or 1:

suppose, that we consider adjacency of pixels valued 1 ($V = \{1\}$)

a. if P_1 & P_3 are ∅:

then D_m distance = 2.

b. if $P_1 = 1$ and $P_3 = ∅$

m-distance = 3;

c) if $P_1 = ∅$; and $P_3 = 1$ (m-distance = 3)

d) if $P_1 = P_3 = 1$; m-distance = 4

path = $P P_1 P_2 P_3 P_4$