Digital System Design

Lecture 7

Boolean Algebra - I

Objectives:-

1. **Binary operators and their representations.**
2. **Relationships between Boolean expressions, Truth tables and Logic circuits.**
3. **Logic gates' postulates, laws and properties.**

1. Binary operators and their representations

- ***Boolean algebra*** is the basic mathematics needed for logic design of digital systems; Boolean algebra uses ***Boolean (logical) variables with two values (0 or 1).***

"Two- valued Boolean algebra"

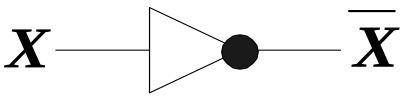
Basic operations:

- The basic operations of Boolean algebra are ***AND***, ***OR***, and ***NOT*** (*complement*).

a) NOT operation (*NOT Gate*):-

$$\overline{1} = 0; \overline{0} = 1$$

- The ***not*** operator is also called the ***complement*** or the ***inverse***:
- $\overline{x}$ is the complement of x.
- Output is ***opposite*** of input.
- ***Truth table***: truth table describes inputs and outputs in terms of 1_2 and 0_2 rather physical (voltage) levels.



Not gate representation

Truth table:

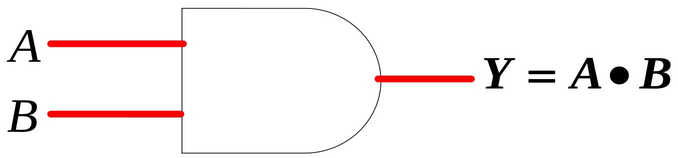
Input x	Output $\overline{x}$
0	**1**
1	**0**

1 – high
0 - low

b) AND operation (*AND gate*).

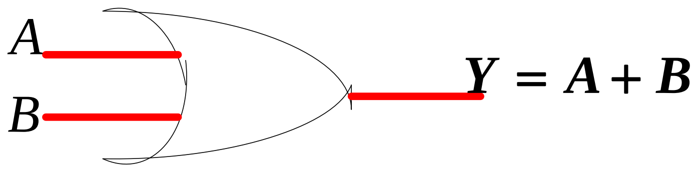
- The output is 1 only if all inputs are 1, if any of the input is 0, then the output is 0.
- The truth table of ***AND gate (2-inputs, 1-output)*** as the following:

Inputs		Output
A	B	$Y = A.B$
0	0	0
0	1	0
1	0	0
1	1	1



The AND operation is referred to as logical multiplication

c) OR operation (*OR gate*)



The output is 1 if

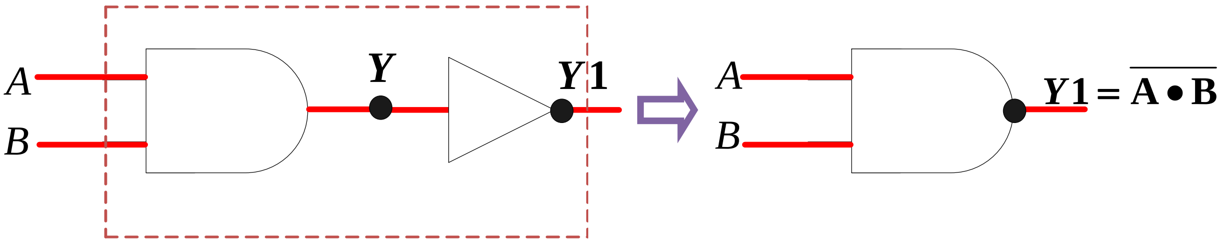
A is 1 or if B is 1

- The truth table of ***OR*** gate (***2-inputs, 1-output***)as the following:

Inputs		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

OR operation is sometimes referred to as "inclusive OR" or logical addition.

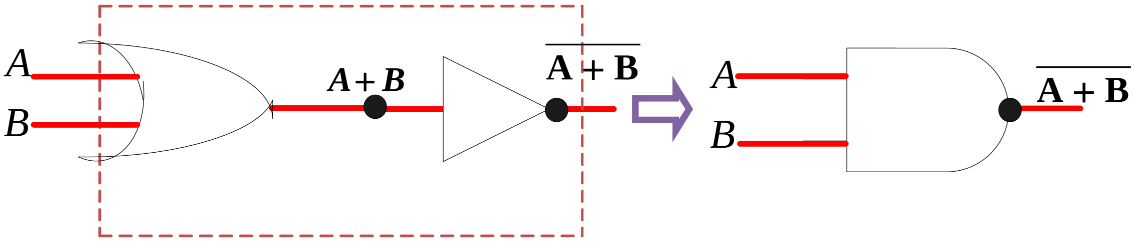
d) NAND gate: (*Not AND gate*)



➢ The truth table of ***NAND*** gate (***2-inputs, 1-output***) as the following:

inputs		outputs	
A	B	$Y = A.B$	$Y1 = \overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

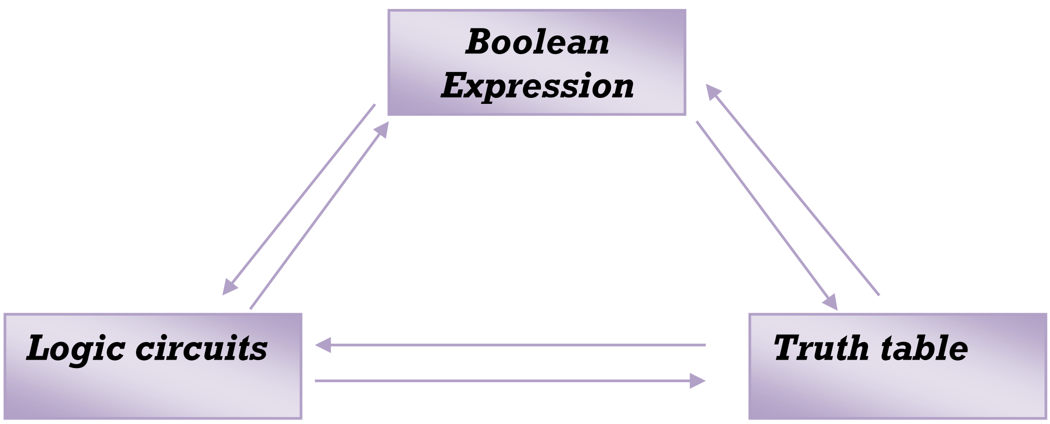
e) NOR gate (*Not OR gate*):



The truth table of ***NOR*** gate (***2-inputs, 1-output***) as the following:

inputs		outputs	
A	B	$A + B$	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

2. *Relationships between Boolean expression, truth tables and logic circuits*



- If one is given, we can get the other.

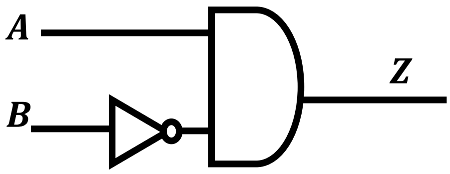
To draw a circuit from a Boolean expression:

- ✓ From the left, make an ***input line*** for each variable.
- ✓ Next, put a ***Not*** gate in for each variable, that appears negated in the expression.
- ✓ Still working, from left to right.

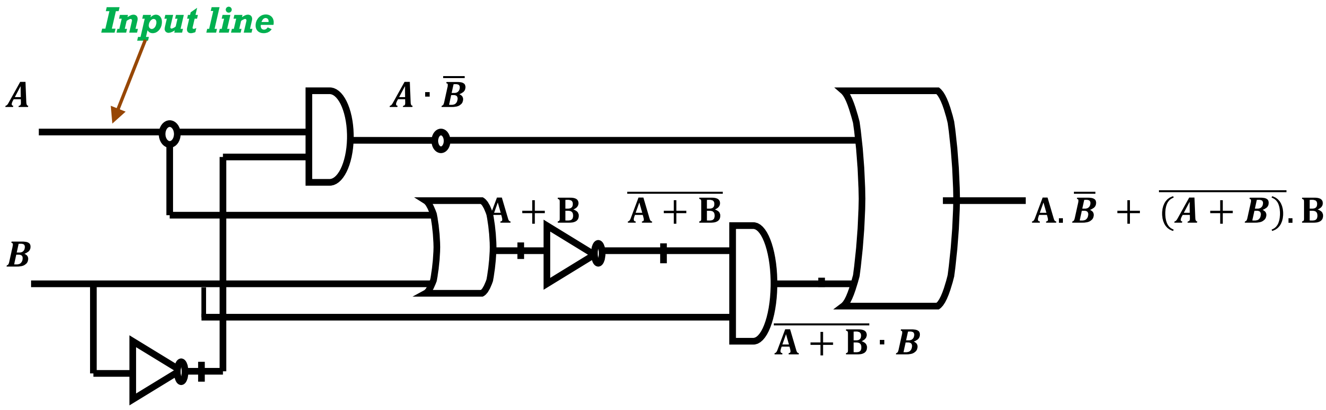
Precedence of operators:

1. Parenthesis
2. NOT
3. AND
4. OR

Example 1:- $Z = A\overline{B}$



Example 2:- $A.\overline{B} + \overline{(A+B)}.B$



3. Logic gate's postulates, laws and properties

- Postulates are used to deduce the rules, theorems and properties.

a) Postulates of Boolean algebra

Postulate	For OR Gate	For AND Gate
P1	$A + 0 = A$	$A \cdot 1 = A$
P2	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$
P3	$A + B = B + A$	$A \cdot B = B \cdot A$
P4	$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + B \cdot C = (A + B) \cdot (A + C)$
	Duality principle	

- ***Duality principle*** states that every algebraic expression is deducible if ***the operators and the identity elements are interchanged.***

Identity elements:

0 for or gate

1 for and gate

b) Boolean algebra theorems:

- There are six theorems of Boolean algebra:

Theorem	For OR Gate	For AND Gate
T1: Idempotent laws	$A + A = A$	$A \cdot A = A$
T2: operations with 0 and 1	$A + 1 = 1$	$A \cdot 0 = 0$
T3: associative laws	$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
T4: de Morgan laws (inversion law)	$\overline{A + B} = \overline{A}.\overline{B}$	$\overline{A.B} = \overline{A} + \overline{B}$
T5: Absorption laws	$A + A \cdot B = A$	$A \cdot (A + B) = A$
T6: involution law	$\overline{\overline{A}} = A$	

- To ***proof*** these theorems and other logic expressions, we can use ***two ways***:

[1] Truth table

Example 1: proof that $A + A \cdot B = A$

A	B	$A \cdot B$	$A + A \cdot B$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Example 2: verify the de Morgan's laws using a truth table.

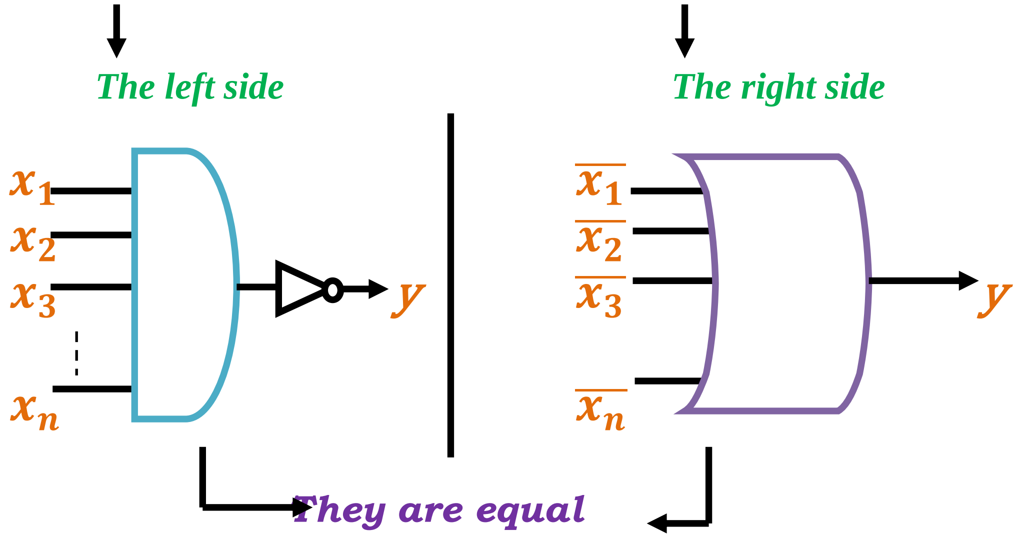
A	B	$\overline{A}$	$\overline{B}$	$A + B$	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$	$A \cdot B$	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	1	0	0	1	0	0	1	0	0
					$\overline{A + B} = \overline{A} \cdot \overline{B}$			$\overline{A \cdot B} = \overline{A} + \overline{B}$	

Some Details:

- The duality principle is formed by ***replacing AND*** with ***OR***, ***OR*** with ***AND***, ***0*** with ***1***, ***1*** with ***0***, ***variables and complements are left unchanged***.
- de Morgan's laws Allow us to ***convert between types of gates***; we can generalize them to ***n*** variables:

$$\overline{(x_1 + x_2 + x_3 + \cdots + x_n)} = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} \cdot \cdots \cdot \overline{x_n}$$

$$\overline{(x_1 x_2 x_3 \ldots x_n)} = \overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots + \overline{x_n}$$



[2] Algebraically using basic theorems.

Example 1: verify that

a) $A + A \cdot B = A$
b) $A(A + B) = A$

Proof (a):

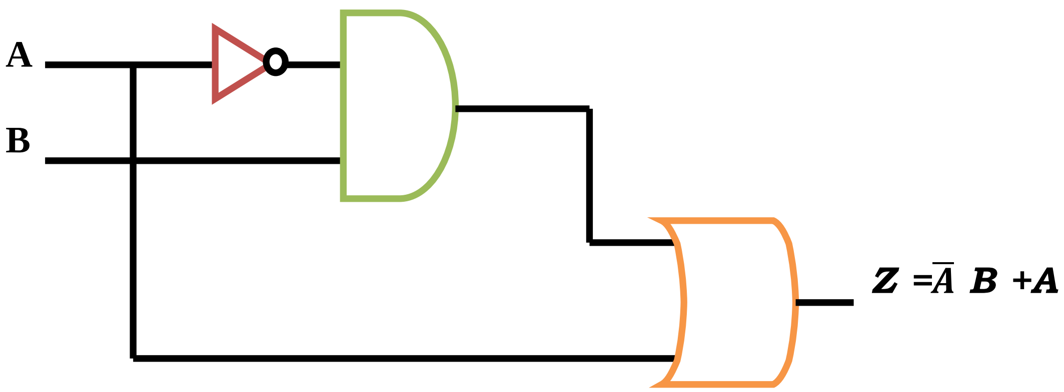
$$A + A \cdot B = A \cdot 1 + A \cdot B = A(1 + B) = A \cdot 1 = A$$

Proof (b):

$$A(A + B) = A$$ ***by duality.***

Example 2

a) $(A\overline{B} + 0)\, E + 1 = 1$
b) $(A\overline{B} + 0)\,\overline{(A\overline{B} + 0)} = 0$
c) An input ***A*** is inverted and applied to an ***AND*** gate. The other input is ***B***. the output of the ***AND*** gate is applied to an ***OR*** gate. ***A*** is the second input to ***OR*** gate. ***Draw the logic circuit and the truth table.***



A	B	Z
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

d) Proof the following Boolean expression "***Theorems***".

a) $X + XY = X$

Proof:

$$x + xy = x \cdot 1 + xy = x\,(1 + y) = x \cdot 1 = x$$

b) $X\,(X + Y) = X$

Proof:

$$x(x + y) = x.x + x.y = x + xy = x(1 + y) = x.1 = x$$

c) $X\,\overline{Y} + Y = X + Y$

Proof:

$$x\overline{y} + y = y + x\overline{y} = (y + x)\,(y + \overline{y}) = (y + x)\,.\,1 = y + x$$

Homework: proof that

a) $(x + y)(x + \overline{y}) = x$

b) $(x + \overline{y})y = xy$