



Philadelphia University
Faculty of Engineering

Marking Scheme

Examination Paper

BSc CEE

Signals and Systems (650320+640543)

Second Exam

First semester

Date: 07/01/2020

Section 1

Weighting 20% of the module total

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Marking Scheme

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The presented exam questions are organized to overcome course material through 4 questions.
The *all questions* are compulsory requested to be answered.

Marking Assignments

Question 1 This question is attributed with 5 marks if answered properly; the answers are as following:

1) For a discrete linear time-invariant system, the **system response** can be written in terms of convolution as

a)	$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[k]$
b)	$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[n+k] x[n-k]$
c)	$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$
d)	$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n+k]$

2) If h_1, h_2 and h_3 are cascaded (**series connection**), and $h_1 = u(t)$, $h_2 = \exp(t)$ and $h_3 = \sin(t)$, find the **overall impulse response**

- a) $\sin(t) * \exp(t) * u(t)$
- b) $\sin(t) + \exp(t) + u(t)$
- c) $u(t) * \sin(t)$
- d) all of the mentioned

3) The linear time-invariant system with $h(t) = 4e^{-2t} u(t)$

- a) **Stable, causal, and memoryless**
- b) **Stable, but not causal**
- c) **Stable, causal, but has memory**
- d) **Not stable**

4) What is the equation $x(t) = \sum_{k=-\infty}^{\infty} D_k e^{jk\omega t}$ called?

- a) **Analysis equation**
- b) **Synthesis equation**
- c) **Frequency domain equation**
- d) **Discrete equation**

5) Choose the condition from below that is **not** a part of **Dirichlet's conditions**

a)	If it is continuous then there are a finite number of discontinuities in the period T
b)	It has a finite number of positive and negative maxima in the period T
c)	It has a finite average value over the period T
d)	It is a periodic signal

Question 2: This question is attributed with 6 marks if answered properly,

a) (3 marks)

Solution

a) $h[n] = \cos\left(\frac{\pi}{8} n\right) \{u[n] - u[n - 10]\}$

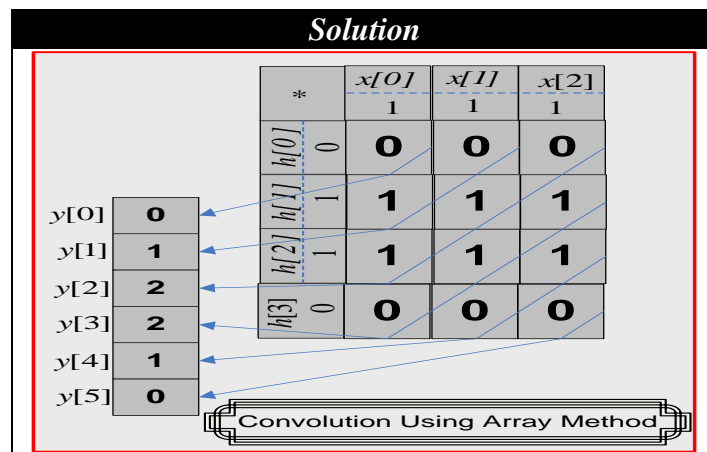
The system has memory, causal and stable

b) $h(t) = e^{-2t} u(t - 1)$

The system has memory, causal and stable

b) (3 marks)

Solution



Question 3: This question is attributed with 8 marks if answered properly,

a)

(2 marks) Bonus

Solution

Given Signal $x(t) = \cos(2\pi t) + 4\sin(6\pi t)$

Period of $\cos(2\pi t)$ is $T_1 = 1$

Period of $\sin(6\pi t)$ is $T_2 = 1/3$

The Fundamental period $x(t)$ is $T = 1\text{sec}$

Expressing $x(t)$ as

$$\begin{aligned} x(t) &= \frac{1}{2} [e^{j(2\pi t)} + e^{-j(2\pi t)}] + \frac{4}{2j} [e^{j6\pi t} - e^{-j6\pi t}] \\ &= \frac{1}{2} e^{j2\pi t} + \frac{1}{2} e^{-j2\pi t} + \frac{2}{j} e^{j2\pi(3)t} - \frac{2}{j} e^{-j2\pi(3)t} \end{aligned}$$

Fourier Series representation is

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

Comparing with above equation

$$X(k) = \begin{cases} -\frac{2}{j} & k = -3 \\ \frac{1}{2} & k = -1 \\ \frac{1}{2} & k = 1 \\ \frac{2}{j} & k = +3 \\ 0 & \text{otherwise} \end{cases}$$

b)

(6 marks)

Solution

Solution: Let $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

Then Euler's constants a_0 , a_n and b_n are given by

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) dx = \frac{1}{\pi} \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_{-\pi}^{\pi} = -\frac{2\pi^2}{3}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \cos nx dx = \frac{1}{\pi} \left[(x - x^2) \left(\frac{\sin nx}{n} \right) - (1 - 2x) \left(-\frac{\cos nx}{n^2} \right) + (-2) \left(-\frac{\sin nx}{n^3} \right) \right]_{-\pi}^{\pi} \\ &= \frac{1}{\pi} \left[(1 - 2\pi) \frac{\cos n\pi}{n^2} - (1 + 2\pi) \frac{\cos n\pi}{n^2} \right] = \frac{-4(-1)^n}{n^2} \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \sin nx dx = \frac{1}{\pi} \left[(x - x^2) \left(-\frac{\cos nx}{n} \right) - (1 - 2x) \left(-\frac{\sin nx}{n^2} \right) + (-2) \left(\frac{\cos nx}{n^3} \right) \right]_{-\pi}^{\pi} \\ &= \frac{1}{\pi} \left[-(\pi - \pi^2) \frac{\cos n\pi}{n} - 2 \frac{\cos n\pi}{n^3} + (-\pi - \pi^2) \frac{\cos n\pi}{n} + 2 \frac{\cos n\pi}{n^3} \right] = \frac{-2(-1)^n}{n} \end{aligned}$$

Therefore the Fourier series for $f(x) = x - x^2$ is

$$f(x) = x - x^2 = -\frac{\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx$$

$$\therefore x - x^2 = -\frac{\pi^2}{3} + 4 \left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \dots \dots \right] + 2 \left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \dots \dots \right]$$

Question 4: This question is attributed with 3 marks if answered properly,

Solution

```
kx = [-1 0 1]; % time indices where x is nonzero  
x = [-1 1 2]; % Sample values for DT sequence x  
kh = [-1 0 1 2 3]; % time indices where y is nonzero  
h = [3 1 -2 3 -2]; % Sample values for DT sequence y  
y = conv(x, h); % Convolve x with h  
ky = kx(1)+kh(1):kx(length(kx))+kh(length(kh));  
% ky= time indices for y
```