Philadelphia University
Faculty of Engineering

Marking Scheme

Examination Paper

Department of Communication & Electronics Engineering

Probability and Random Variables

(650364)

Second Exam First semester Date: 23/12/2019

Section 1

Weighting 20% of the module total

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Marking Scheme

Probability and Random Variables *(*650364*)*

The presented exam questions are organized to overcome course material through 4 questions. The *all questions* are compulsory requested to be answered.

Marking Assignments

Question 1: This question is attributed with 5 marks if answered properly, the answer is the following:

1) When **three** coins are tossed, what is the **expected value** of the number of heads?

a)	**1**	**b)**	**2**
c)	**1.5**	**d)**	**2.5**

2) The payoff (X) for a lottery game has the following probability distribution

X = payoff	\$0	\$5
probability	0.8	0.2

What is the *expected value* of **X= payoff**?

a)	**\$0**	**b)**	**\$1.00**
c)	**\$0.50**	**d)**	**\$2.50**

3) Given $E(X) = 5$ and $E(Y) = -2$, then $E(X - Y)$ is:

a)	**-2**	**b)**	**3**
c)	**5**	**d)**	**7**

4) The equation of the mean for **uniform** distribution is

a)	**mean = 4(b + a)/2b**	**b)**	**mean = (b - 2a)/4**
c)	**mean = (b + a)/2**	**d)**	**mean = (2a + 2b)/2a**

5) Consider the following functions:

$$f(x) = \begin{cases} \cos x, & x \in [-\frac{\pi}{2}, \frac{\pi}{2}], \\ 0, & \text{otherwise}, \end{cases} \quad g(x) = \begin{cases} \cos x, & x \in [-\frac{\pi}{2}, \pi], \\ 0, & \text{otherwise}, \end{cases} \quad h(x) = \begin{cases} \cos x, & x \in [0, \frac{\pi}{2}], \\ 0, & \text{otherwise}. \end{cases}$$

Which of these functions is/are a **probability density**?

a)	**Only h**	**b)**	**Only g**
c)	**f and g**	**d)**	**f and h**

Question 2: This question is attributed with 4 marks if answered properly, the answer is the following:

a) ***(2 marks)***

Solution

1. $F_{X,Y}(-\infty,-\infty) = 0, \quad F_{X,Y}(-\infty, y) = 0, \quad F_{X,Y}(x,-\infty) = 0$
2. $F_{X,Y}(\infty,\infty) = 1$
3. $0 \le F_{X,Y}(x, y) \le 1$
4. $F_{X,Y}(x,y)$ is a nondecreasing function
5. $P\{x_1 < X \le x_2, y_1 < Y \le y_2\} =$
$$F_{X,Y}(x_2, y_2) + F_{X,Y}(x_1, y_1) - F_{X,Y}(x_2, y_1) - F_{X,Y}(x_1, y_2)$$
6. $F_{X,Y}(x,\infty) = F_X(x)$ and $F_{X,Y}(\infty, y) = F_Y(y)$

b) *(2 marks)*

Solution

- ✓ The n_{th} central moment of the random variable X

$$\mu_n = E[(X-\overline{X})^n] = \int_{-\infty}^{+\infty}(x-\overline{X})^n f_X(x)dx$$

Clearly:

$$\mu_0 = E[(X-\overline{X})^0] = 1$$
$$\mu_1 = E[(X-\overline{X})^1] = 0$$
$$\mu_2 = E[(X-\overline{X})^2] = \sigma_X^2 \quad , \text{ the variance of } X$$

σ_X , is called the standard deviation of X

- ✓ The second central moment called the variance of X

$$\sigma_X^2 = \mu_2 = E[(X-\overline{X})^2] = E[X^2 - 2\overline{X}X + \overline{X}^2]$$
$$= E[X^2] - 2\overline{X}E[X] + \overline{X}^2$$
$$= E[X^2] - \overline{X}^2 = m_2 - m_1^2$$

- ✓ The third central moment is a measure of the asymmetry of the density function about the mean value m0 and called the skew of the density function

$$\mu_3 = E[(X-\overline{X})^3]$$

- ✓ The measure will be positive or negative according to whether the distribution is skewed to the right or left, respectively

<u>Question 3:</u> This question is attributed with 6 marks if answered properly, the answer is the following:

a) *(4.5 marks)*

Solution

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 4xy\,dy$$
$$= 2xy^2\Big|_{y=0}^{y=1} = 2x$$

for $\mathbf{0 < x < 1}$, and $\boldsymbol{g(x) = 0}$ elsewhere

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 4xy\,dx$$
$$= 2x^2y\Big|_{x=0}^{x=1} = 2y$$

for $\mathbf{0 < y < 1}$, and $\boldsymbol{h(y) = 0}$ elsewhere.

$$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{4xy}{2y} = 2x$$

for $\mathbf{0 < x < 1}$, and $\boldsymbol{f(x|y) = 0}$ elsewhere.

b) *(1.5 marks)*

Solution

$$f(x_1,x_2,x_3) = f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3)$$
$$= e^{-x_1}\cdot 2e^{-2x_2}\cdot 3e^{-3x_3}$$
$$= 6e^{-x_1-2x_2-3x_3}$$

for $\boldsymbol{x1 > 0, x2 > 0, x3 > 0}$, and $\boldsymbol{f(x1,x2,x3) = 0}$ elsewhere.

Question 4: This question is attributed with 5 marks if answered properly, the answer is the following:

a) ***(3 marks)***

Solution

a)

$$E[X] = \mu_X = 0 \cdot P_X(0) + 1 \cdot P_X(1) + 2 \cdot P_X(2) = 0(1/4) + 1(1/2) + 2(1/4) = 1.$$

b)

$$E[V] = E[g(X)] = E[4X+7] = 4E[X] + 7 = 4(1) + 7 = 11.$$

c) $W = E(X^2)$

$$E[W] = \sum h(x) P_X(x) = (1/4)0^2 + (1/2)1^2 + (1/4)2^2 = 1.5.$$

b) ***(2 marks)***

Solution

$P_{X,Y}(x,y)$	$y=0$	$y=1$	$y=2$	$P_X(x)$
$x=0$	0.01	0	0	0.01
$x=1$	0.09	0.09	0	0.18
$x=2$	0	0	0.81	0.81
$P_Y(y)$	0.10	0.09	0.81	