Philadelphia University

Faculty of Engineering

Student Name:
Student Number:
Serial Number:

Second Exam, First Semester: 2019/2020
Dept. of Communication & Electronics Engineering

Course Title:	Probability and Random Variables	**Date:**	23/12/2019
Course No:	650364	**Time Allowed:**	60 minutes
Lecturer:	Dr. Qadri Hamarsheh	**No. Of Pages:**	4

Instructions:

- **ALLOWED**: pens, calculators and drawing tools (**no red color**).
- **NOT ALLOWED**: Papers, literatures and any handouts. Otherwise, it will lead to the non-approval of your examination.
- **Shut down** Telephones, and other communication devices**.**

Please note:

- *This exam paper contains four questions totaling 20 marks.*

Basic notions: *The aims of the questions in this part are to evaluate the required minimal student knowledge and skills. Answers in the pass category represent the minimum understanding of basic concepts: Statistics of Random Variables (Expectation, Moments), Vector random variables, Joint density and distribution functions, Statistical independence, Central limit theorem, multiple random variables*

Question 1 ***Multiple Choice*** ***(5 marks)***

Identify the choice that best completes the statement or answers the question.

1) When **three** coins are tossed, what is the **expected value** of the number of heads?

a) **1** **b)** **2**
c) **1.5** **d)** **2.5**

2) The payoff (**X**) for a lottery game has the following probability distribution

X = payoff	$0	$5
probability	0.8	0.2

What is the *expected value* of **X= payoff**?

a) **$0** **b)** **$1.00**
c) **$0.50** **d)** **$2.50**

3) Given $E(X) = 5$ and $E(Y) = -2$, then $E(X - Y)$ is:

a) **-2** **b)** **3**
c) **5** **d)** **7**

4) The equation of the mean for **uniform** distribution is

a) **mean = 4(b + a)/2b** **b)** **mean = (b - 2a)/4**
c) **mean = (b + a)/2** **d)** **mean = (2a + 2b)/2a**

5) Consider the following functions:

$$f(x) = \begin{cases} \cos x, & x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \\ 0, & \text{otherwise}, \end{cases} \quad g(x) = \begin{cases} \cos x, & x \in \left[-\frac{\pi}{2}, \pi\right], \\ 0, & \text{otherwise}, \end{cases} \quad h(x) = \begin{cases} \cos x, & x \in \left[0, \frac{\pi}{2}\right], \\ 0, & \text{otherwise}. \end{cases}$$

Which of these functions is/are a **probability density**?

a) **Only h** **b)** **Only g**

c) **f and g** **d)** **f and h**

Familiar and Unfamiliar Problems Solving: *The aim of the questions in this part is to evaluate that the student has some basic knowledge of the key aspects of the lecture material and can attempt to solve familiar and unfamiliar problems: Statistics of Random Variables (Expectation, Moments), Vector random variables, Joint density and distribution functions, Statistical independence, Central limit theorem, multiple random variables*

Question 2 ***(4 marks)***

a) List **four** Properties of the **Joint CDF Distribution (write equations)**: ***(2 marks)***

Solution

b) Explain the **first, second** and **third Central Moments** (description and equation) ***(2 marks)***

Solution

Question 3 ***(6 marks)***

a) Given the **joint probability density** ***(4.5 marks)***

$$f(x,y) = \begin{cases} 4xy & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the **marginal densities** of ***X*** and **Y** and the **conditional density** of ***X*** given ***Y* = *y*.**

Solution

b) Given the **independent random variables** $\mathbf{X_1}$**,** $\mathbf{X_2}$**,** and $\mathbf{X_3}$ with the probability densities

$$f_1(x_1) = \begin{cases} e^{-x_1} & \text{for } x_1 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_2(x_2) = \begin{cases} 2e^{-2x_2} & \text{for } x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_3(x_3) = \begin{cases} 3e^{-3x_3} & \text{for } x_3 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find **their joint probability density**. ***(1.5 marks)***

Solution

Question 4 *(5 marks)*

a) Random variable **X** has **PMF** *(3 marks)*

$$P_X(x) = \begin{cases} 1/4 & x = 0, \\ 1/2 & x = 1, \\ 1/4 & x = 2, \\ 0 & \text{otherwise.} \end{cases}$$

i. Calculate $E[X]$

ii. Calculate the expected value of $V = g(X) = 4X + 7$

iii. Calculate the **second moment about the origin**

Solution

b) The **joint PMF** shown in the following table. Find the **marginal PMFs** for the random variables **X** and **Y.** *(2 marks)*

$P_{X,Y}(x,y)$	$y = 0$	$y = 1$	$y = 2$
$x = 0$	0.01	0	0
$x = 1$	0.09	0.09	0
$x = 2$	0	0	0.81

Solution

GOOD LUCK