Case Study: Hard Drives

Dr. Tarek A. Tutunji
Advanced Modeling and Simulation
Mechatronics Engineering Department
Philadelphia University, Jordan
2013
General Structure

- Spindle motor
- Disk
- Magnetic head
- Voice coil motor (VCM)
Hard Disc Drives: HDD

- Hard disc drives are the main storage units in
  - Personal Computers
  - Network Storage Systems
  - Enterprise Workstations
  - DVD Players
  - Game Boxes
Hard Disk Design

- The Hard Disk Design include
  - Electronic Parts
  - Mechanical Parts
- The Mechanical Parts are used to
  - Magnetically Store the data
  - Rotate the disks and move the arm
- The Electronic Parts are used to
  - Control the mechanical movement
  - Transfer Data between the Disks and the Host
- Hard Disks are a good example of Mechatronic Systems
The Mechanical parts are all assembled in a sealed chamber referred to as Head Disk Assembly (HDA).

The HDA includes:
- Platters or Discs
- Spindle Motor
- Actuator Arm
- Voice Coil Motor
- Read/Write Heads
Mechanical: Platters or Discs

- Made of aluminum alloy coated with a magnetic medium
- Stores the data in magnetic patterns
- Each Platter (Disc) is divided into tracks
  - The tracks have circular shapes around the center spindle and are grouped into cylinders
  - The cylinders are divided into sectors of 512 bytes each

Servo Information (stored between data sectors) is used for positioning
The discs are stacked on top of each other through a shaft
The Motor Spindle turns the whole assembly
Magnetic read/write heads are mounted on the end of an Actuator Arm that flies at each side of the platters
The Voice Coil Motor moves the actuator arm
Electronics

• The Electronic parts are assembled on a Printed Circuit Board (PCB)
  ○ Supervises the data transfer
  ○ Encodes / Decodes the stored data pattern
  ○ Converts D/A and A/D
  ○ Digital Filtering of Data
  ○ Controls the Spindle Motor Driver
  ○ Moves the Arm
Controller Tasks

- Head seeking and tracking
- Spindle / spin
- Arm position
- Shock/vibration control
Head Positioning Servo Mechanism
Closed Loop Control
Rotary VCM

- Track
- Sectors
- Pivot
- Head Slider
- Actuator

- Suspension
- Pivot
- Actuator Arm
- Coil

- Pivot
- Part of actuator arm
- Coil

- $B$
- $F$
- $I$
Servo mechanism

Figure 2.5: Movement of suspension arm for rotary VCM.

Figure 2.6: Micro-jog in HDD servomechanism.
Actuator Arm Modeling

The motion of the actuator arm, defined according to the Newton's second law of motion, can be modeled as

$$\ddot{\theta}(t) = \frac{K_t}{J} I(t)$$

where \( J \) is the moment of inertia of the rotating arm, and \( \ddot{\theta} \) is the angular acceleration of the actuator's motion. If the distance between the pivot center and the read head is \( L \) inches, then the linear displacement of the read head corresponding to an angular displacement \( \theta \) is \( x = L\theta \). It is very common in the HDD industry to express the displacement of read head in units of track, i.e., \( y = D_{trk} L\theta \), where \( D_{trk} \) is the track density in units of Tracks per Inch (TPI). Taking all these factors into consideration, the rigid body dynamics of the VCM actuator is given by

$$\ddot{y}(t) = \frac{D_{trk} LK_t}{J} I(t) = KI(t)$$

The corresponding transfer function model is \( G_v(s) = \frac{K}{s^2} \)
Circuit Modeling

\[ V_O(t) = R_v I(t) + L_v \frac{dI(t)}{dt} \]

\[ \frac{I(s)}{U(s)} = \frac{K_{VA}}{L_v s + R_v} \]

\[ G_{v,v} = \frac{Y(s)}{U(s)} = \frac{KK_{VA}}{s^2(L_v s + R_v)} \]

\[ G_{v,c} = \frac{Y(s)}{U(s)} = \frac{KK_{CA}}{s^2} \]
HDD Modeling

- A HDD consists of a voice coil motor (VCM), several magnetic heads, several disks, and a spindle motor.

- The mathematical model of the mechanical system $P_m(s)$ is given by the following equation, where $I$ is the number of modes under consideration.

$$P_m(s) = K_p \sum_{i=1}^{I} \frac{\alpha_m(i)}{s^2 + 2\zeta_m(i)\omega_m(i)s + \omega_m(i)^2}.$$
HDD Modeling

The transfer function of the HDD servomechanism plant can be described by the model [113]:

\[
G_p(s) = k P_d [P_0 + P_m],
= k \frac{e^{-T_d s}}{T_{amp} s + 1} \left[ \frac{r_0}{(s^2 + 2\zeta_{m0}\omega_{m0}s + \omega_{m0}^2)} + \sum_{i=1}^{N_a} \frac{r_{mi}}{(s^2 + 2\zeta_{mi}\omega_{mi}s + \omega_{mi}^2)} \right],
\]

(3.5)

where the loop gain \( k \) includes gains of various stages of the servo plant e.g. the DAC (Digital-to-Analog Converter) gain*, amplifier gain, torque gain, mass and position gain. The transfer function \( P_d(s) = \frac{1}{T_{amp} s + 1} e^{-T_d s} \) represents both the dynamics of power amplifier with time constant \( T_{amp} \) and the computational delay \( T_d \). The rigid body model of the actuator coupled with linearized pivot friction is modeled as \( P_0(s) \), where as \( P_m(s) = \sum_{i=1}^{N_a} \frac{r_{mi}}{s^2 + 2\zeta_{mi}\omega_{mi}s + \omega_{mi}^2} \) represents \( N_a \) modes of mechanical resonances.
Frequency Response
These methods find the coefficients of the transfer function $G(s) = \frac{B(s)}{A(s)}$ such that the frequency response of the identified transfer function matches as close as possible to the frequency response obtained experimentally. The frequency response data include two vectors:

1. the vector $[\omega_k]$ for $k = 1, \cdots, N$ contains all frequencies for which magnitudes and phases are measured and

2. the vector $[G_{fr}(\omega_k)]$ contains the frequency response measured at each of the $N$ frequencies.
Identification

The transfer function model $G(s)$ is the ratio of two polynomials of Laplace Transform parameter $s$,

$$G(s) = \frac{B(s)}{A(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n},$$

the response at any frequency $\omega_k$ is

$$\frac{b_0 (j\omega_k)^m + b_1 (j\omega_k)^{m-1} + \cdots + b_m}{(j\omega_k)^n + a_1 (j\omega_k)^{n-1} + \cdots + a_n} = G_{fr}(\omega_k).$$

The parameters of the transfer function $G$ can be obtained by solving the least squares estimation problem that minimizes the error criterion

$$\sum_{k=1}^{N} \left| \frac{B(j\omega_k)}{A(j\omega_k)} - G_{fr}(\omega_k) \right|^2.$$
Identification

\[
\frac{b_0(j\omega_k)^m + b_1(j\omega_k)^{m-1} + \cdots + b_m}{(j\omega_k)^n + a_1(j\omega_k)^{n-1} + \cdots + a_n} = G_{fr}(\omega_k).
\]

\[
b_0(j\omega_k)^m + b_1(j\omega_k)^{m-1} + \cdots + b_m = G_{fr}(\omega_k) \left( (j\omega_k)^n + a_1(j\omega_k)^{n-1} + \cdots + a_n \right)
\]

\[
b_0(j\omega_k)^m + \cdots + b_m - G_{fr}(\omega_k) \left( a_1(j\omega_k)^{n-1} + \cdots + a_n \right) = (j\omega_k)^n G_{fr}(\omega_k)
\]

\[
\phi^T(j\omega_k) \begin{bmatrix} b_0 & b_1 & \cdots & b_m & a_1 & a_2 & \cdots & a_n \end{bmatrix} = x(\omega_k) + jy(\omega_k)
\]

\[
x(\omega_k) + jy(\omega_k) = (j\omega_k)^n G_{fr}(\omega_k)
\]

\[
\phi(j\omega_k) = \\
\begin{bmatrix} (j\omega_k)^m & (j\omega_k)^{m-1} & \cdots & 1 & -G_{fr}(\omega_k)d_1(j\omega_k)^{n-1} & \cdots & -G_{fr}(\omega_k)d_n \end{bmatrix}^T
\]
Identification

\[
\begin{bmatrix}
\phi_R^T(\omega_k) \\
\phi_I^T(\omega_k)
\end{bmatrix}
\cdot \theta = \begin{bmatrix}
x(\omega_k) \\
y(\omega_k)
\end{bmatrix}, \quad \theta = [b_0 \ b_1 \ \cdots \ b_m \ a_1 \ a_2 \ \cdots \ a_n]^T
\]

The frequency response is measured for \( N \) different frequencies, and we get \( N \) sets of the above equation. That is,

\[
\Phi_{2N \times np} \Theta_{np \times 1} = Y_{2N \times 1}
\]

where \( np \) is the number of parameters to be identified. This is a linear in the parameters (LIP) model and can be solved using linear least-squares method, i.e., to find the estimate \( \hat{\Theta} \) of the parameter vector by minimizing the cost function,

\[
J_{LS} = (Y - \Phi \hat{\Theta})^T (Y - \Phi \hat{\Theta})
\]

Solution of this least squares problem is,

\[
\hat{\Theta} = (\Phi^T \Phi)^{-1} \Phi^T Y.
\]
State Estimator

Considering the rigid body model \( \left( \frac{a}{s^2} \right) \) of the VCM actuator, corresponding continuous-time state space model is

\[
\frac{dp}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} p + \begin{bmatrix} 0 \\ a \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} p,
\]

state vector \( p = [p_1 \ p_2]^T \) includes the position \( (p_1) \) and velocity \( (p_2) \)

the transformed state equation is,

\[
\frac{dx}{dt} = \begin{bmatrix} 0 & \frac{1}{T_s} \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ aT_s \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x,
\]

Corresponding discrete-time state space model is

\[
x(k+1) = \Phi x(k) + \Gamma u(k),
\]

\[
y(k) = H x(k).
\]
State Estimator

\[
\begin{bmatrix}
    x(k+1) \\
    w(k+1)
\end{bmatrix}
= \begin{bmatrix}
    \Phi & \Gamma \\
    0 & 0
\end{bmatrix}
\begin{bmatrix}
    x(k) \\
    w(k)
\end{bmatrix}
+ \begin{bmatrix}
    \Gamma \\
    0
\end{bmatrix} u(k)
\]

\begin{equation}
y(k) = \begin{bmatrix}
    H \\
    0
\end{bmatrix}
\begin{bmatrix}
    x(k) \\
    w(k)
\end{bmatrix}
\end{equation}

or

\[
z(k+1) = \Phi_a z(k) + \Gamma_a u(k); \quad y(k) = H_a z(k).
\]

Here \( z = [x^T \ w]^T \) is the augmented state vector, \( \Phi_a = \begin{bmatrix}
    \Phi & \Gamma \\
    0 & 0
\end{bmatrix} \), \( \Gamma_a = \begin{bmatrix}
    \Gamma \\
    0
\end{bmatrix} \), and \( H_a = [H \ 0] \). The prediction observer using this model is

\[
\ddot{z}(k+1) = \Phi_a \ddot{z}(k) + \Gamma_a u(k) + L_p[y(k) - H_a \ddot{z}(k)],
\]
Proximate Time Optimal Servomechanism

Figure 2.32: Schematic diagram of PTOS with integral control.

Figure 2.33: Schematic diagram of PTOS with bias estimator.
References

- Hard Disc Drive: Mechatronics and Control (Chapter 1, 2, and 3)
  - Al Mamun, Guo, and Bi