Smooth Variable Structure Filter for Pneumatic System Identification

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Abstract—The Smooth Variable Structure Filter (SVSF) is a newly-developed predictor-corrector filter for state and parameter estimation [1]. The SVSF is based on the Sliding Mode Control concept. It defines a hyperplane in terms of the state trajectory and then applies a discontinuous corrective action that forces the system to return back and forth across that hyperplane. The SVSF is suitable for fault detection and identification applications because of its stability and robustness in modeling uncertainties. The SVSF has two indicators of performance; the posteriori output error and the chattering. The latter—as a signal—contains the system’s information which is proven and explored in this paper. The SVSF is applied for the identification of pneumatic systems in order to verify the proposed method. Furthermore, the proposed method is compared with neural network and the results reveal that SVSF is better in identifying nonlinear systems.

Keywords—SVSF, System Identification, Pneumatic Systems.

I. INTRODUCTION

"From earliest time, people have been concerned with interpreting observations and making estimates and predictions.” Kailath said in [2] and continued by “Neugebauer has noted that the Babylonians used a rudimentary form of Fourier series for such purposes”. The beginning of the “theory of estimation” can be traced back to 1632, when Galileo tried to minimize the error of some functions [2]. In 1795, Gauss introduced and used the method of least squares to locate the asteroid Ceres, although Legendre first published it independently in 1805 [2,3]. These were followed by numerous investigations and studies pertaining to the least squares method, leading to the pioneering work of Wiener in the 1940’s [2,4,5]. In 1942, Wiener gave the first explicit solutions for the problem of estimating a stochastic process using the least squares method, referred to as the Wiener Filter [2,5,6]. In [2], Kailath gave a brief history of the estimation problem focusing on the period from 1930s to 1960s. In that article, he summarized the history of optimal filters up to the Kalman Filter (KF) which was one of the earliest filters that was implemented in a predictor-corrector form.

The history of estimation was continued in [4] focusing on the period from 1960s to 2000s. He specifically considered the non-linear Bayesian filtering problems that include the Particle Filter. The Bayesian approach was applied to estimation in stochastic processes in 1964 by Ho and Lee [4,7]. They introduced the iterative Bayesian filtering and explored the concepts of “the sequential state estimation problem” [4]. Later in the 1980’s and the 1990’s, the Bayesian filtering was expanded to include the state space structure. During the twentieth century, the “optimality” derivatives were formulated specifically in the Bayesian framework. The notion of optimality relies on minimizing a measure referred to as the cost function [4,8,9].

Simultaneously with the development of the optimal filters, another type of filter started to rise and take place in estimation utilizing the principles of the Variable Structure System and Control as well as the Sliding Mode Control. These filters and observers are referred to as Sliding Mode Observers (SMO). These observers are used widely in fault detection and signal reconstruction problems due to their robustness and stability against uncertainties. An early example of the SMO was done bySlotine in 1986 and 1987. In [10,11], Slotine et al. modified the Luenberger Observer by adding a discontinuous element that tolerates the nonlinearity in the system. They described their observer using the SMC mold and explored the effectiveness of its gains mathematically. At the same period, Walcott et al. published a landmark paper [12] on the linear SMO algorithm and its design methodologies. Later in 1994, Edwards et al. expanded the design algorithm to a more general form using symbolic manipulation and defined an explicit design algorithm, [13,14]. At the beginning of the 21st century, Tan and Edwards presented their observer as an extended version of the Walcott and Zak observer, with less constraint and a simpler design method [15].

In 2003, a new estimation method referred to as the Variable Structure Filter (VSF) was proposed for its applications to linear systems. This method is a version of the SMO formulated in a recursive predictor-corrector form [16,17,18]. In 2006, the VSF was revised to a new form, referred to as the Extended Variable Structure Filter (EVSF), to accommodate nonlinear systems. The EVSF uses the linearized system and output matrices to obtain its gain [19]. Later in 2007, a revised version of the VSF, referred to as the Smooth Variable Structure Filter (SVSF), was proposed for its applications to linear and nonlinear systems [1]. The benefits of the SVSF over the VSF and the EVSF are its simplicity and the absence of linearizing the system matrix.

The SVSF is discussed in further detail in section (II). This includes the filter’s structure and its concepts. The chattering is developed mathematically in section (III). The new method used to obtain the system information from chattering signal is developed in section (IV). The pneumatic system used for this experiment is developed in section (V), and it is tested by the new algorithm in section (VI). The results are compared to those obtained from the Neural Network.

Table 1. Nomenclature

| J | j | Notations denoting an inverse and matrix transpose, respectively. |
| | | |
| | | |
| | | Absolute value. |
The measurement matrix. If the measurement matrix has partial
consideration be observable, [1].

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proposed, [1]. The SVSF is a predictor corrector filter that is
based on the SMC principles and can be applied to both linear
and nonlinear systems. A requirement of this filter is that the
system is differentiable and hence the word “smooth” is used to
name this filter. The filter also requires that the system under
consideration be observable, [1].

The SVSF’s derivation depends on the rank of the
measurement matrix. If the measurement matrix has partial
rank, the SVSF’s gain is calculated by using Luenberger’s
reduced order technique. This paper considers the case of a
system that has full rank measurement matrices. For these
systems, the SVSF process can be summarized in Figure 1.

\[
\begin{align*}
K_k & = H_\pi^{-1} \left( \gamma \left| e_{z(k)} \right| + \left| e_{z(k-1)} \right| \right) \cdot \text{sgn} \left( e_{z(k-1)} \right) \\
\hat{x}_k & = \hat{x}_{k-1} + K_k \cdot e_{z(k)} = z_k - \hat{H}_k \hat{x}_{k-1}
\end{align*}
\]

Figure 1: The Smooth Variable Structure Filter Steps [1].

The SVSF defines a hyperplane, which is referred to as the sliding surface, and applies a discontinuous force on the
estimate to make it reach the hyperplane in two stages; prediction and update stages. The estimates in each stage
remain within subspace surrounding the sliding hyperplane, referred to as the Existence Subspace as illustrated in Figure 2.
During each stage, an artificial noise referred to as Chattering
is created depending on uncertainties and the SVSF coefficient matrix \( \gamma \). Chattering obtained in prediction stage is referred to as the secondary indicator of performance [1]. Conversely,
chattering in the a posteriori estimate decays with time by the
factor \( \gamma^\alpha \), therefore no information could be obtained [19].

II. THE SMOOTH VARIABLE STRUCTURE FILTER

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K_{SVSF} = \hat{H}_k^{-1} \left( y \left[ e_{z_k-1|k-1} + e_{z_k-1} \right] \right) \text{sat} \left( e_{z_k-1} \right) \psi_k \tag{1}

Ψ_k must be chosen carefully, as large width causes a slower convergence rate and degrades filter performance, and smaller width causes chattering. If the width of the smoothing boundary layer is chosen to be larger than the width of the a priori existence subspace and the difference between them is made to be small, then chattering is removed and the error in the output estimation is limited. The width of the smoothing boundary layer determines the presence and the level of the a priori chattering. If the smoothing boundary layer is overestimated then chattering is removed. However, if due to changes in the system, additional uncertainties are added such that the amplitude of the output’s a priori estimation error grows larger than the width of the smoothing boundary layer, then chattering will be observed. \cite{1,19}. This gives the SVSF the ability to explicitly point out and extract information on modeling uncertainties. Therefore, chattering is considered as the secondary indicators of performance.

In this section, equations are derived to quantify the amplitudes of the a priori and the a posteriori chattering. 

A. The a posteriori chattering

The a posteriori chattering is derived in this section. Subtracting the actual measurement from the a posteriori estimated measurement yields:

\[ e_{z_{k|k-1}} = z_k - \hat{x}_{k|k-1} = \hat{H}_k x_k + v_k - \hat{H}_k \hat{x}_{k|k-1} \tag{2} \]

(2) could be simplified to:

\[ e_{z_{k|k-1}} = -y_k e_{z_k-1} \text{sgn} \left( e_{z_{k|k-1}} \right) \tag{3} \]

For the SVSF with zero-width-smoothing boundary layer, the a posteriori chattering is then defined as:

\[ Ch_{k|k-1} = -y_k e_{z_k-1} \text{sgn} \left( e_{z_{k|k-1}} \right) \tag{4} \]

Knowing that \( Y \) is a diagonal matrix that has elements with magnitudes less than unity, leads the chattering to decay with time.

B. The a priori chattering signal

The a priori estimation error is defined as follows:

\[ e_{z_{k|k-1}} = z_k - \hat{x}_{k|k-1} = \hat{H}_k x_k + v_k - \hat{H}_k \hat{x}_{k|k-1} \tag{5} \]

Substituting system equations of Figure 1 into (5) gives:

\[ e_{z_{k|k-1}} = \left[ A_{z_{k}} - \hat{A}_{z_{k}} - \hat{H}_k - B_{k} - B_{k-1} \right] + v_k \tag{6} \]

Assuming the output matrix is the identity matrix, using the measurement equation, and rearranging (6) give:

\[ e_{z_{k|k-1}} = \left[ A_{z_{k}} - \hat{A}_{z_{k}} - \hat{H}_k - B_{k} - B_{k-1} \right] + v_k \tag{7} \]

The a posteriori estimated measurement is defined as \cite{19}:

\[ \hat{x}_k - \hat{z}_{k|k-1} = z_k + y_k e_{z_{k|k-1}} \tag{8} \]

Substituting (8) and (3) into (7), rearranging and assuming time invariant system yield:

\[ e_{z_{k|k-1}} = A_{z_{k}} - \hat{A}_{z_{k}} - \hat{H}_k - B_{k} - B_{k-1} \right] + v_k \tag{9} \]

From (9), the terms \( A_{z_{k}} \) and \( B_{k} \) capture the influence of the modeling errors. The terms \( \left[ B_{k} - B_{k-1} \right] \) quantifies the impact of the system and measurement noise. The last term in (9), describes the effects of the uncertainty in initial conditions. According to the latter term, the effect of the error in initial conditions decays in time at a rate of \( y^{k-1} \), and becomes negligible as \( k \to \infty \).

The a priori existence subspace represents the error in the a priori estimate. In other words, it describes the chattering of the a priori estimate around the true trajectory. In this paper, the magnitude of the resultant chattering is referred to as the a priori chattering. Because of the predictor-corrector nature of the SVSF and its gain, the a priori chattering is different from the chattering observed in other SMOs \cite{19}. The a priori chattering is then obtained by eliminating the effect of initial conditions as:

\[ Ch_{k|k-1} = \Delta A z_{k-1} + \Delta B u_{k-1} + \omega_{k-1} + v_k - \hat{A}_{v_k-1} \tag{12} \]

III. THE CHATTERING AMPLITUDES AND ITS INFORMATION CONTENT

The a priori chattering can be used to point out the source and amplitude of modeling errors. Chattering gives an indication that the current filter’s model is uncertain and it needs to be re-estimated or tuned. Chattering thus provides an opportunity for combining the SVSF with adaptive techniques for model refinement.

According to (12), the SVSF has \( n \) a priori chattering signals, one associated with each estimate (assuming that there is \( n \)-states). Thus, each chattering signal points out the modeling errors in the corresponding row of the system and input matrices. As such, the a priori chattering contents provide means for extracting the modeling error explicitly. In this paper, an approach is presented to obtain these uncertainties assuming the input is non-stationary. In order for this method to work, the measurement and system noise signals must be stationary. Since the expectation vector is not directly available, a segment of the a priori chattering is taken and its statistics are calculated; i.e. mean. Using the law of large numbers, the means vectors of the measurement and system noise in the a priori chattering segment approach their expectations and hence they are known.

Lemma - The Law of Large Numbers \cite{6}

The Law of Large Numbers states that: if \( d \) stationary uncorrelated random variables, such as \( u_l \), \( l = 1 \ldots d \), share the same expectation value, then their average tends towards its expected value as \( d \to \infty \) as follows, \cite{6}:

\[ E[u] = E[v] = \ldots = E[u_d] = \bar{u} \quad \text{then} \lim_{d \to \infty} \left( \frac{1}{d} \sum_{l=1}^{d} u_l \right) = \bar{u} \]

The method involves obtaining the modeling errors of a time invariant system from the a priori chattering signals by using an averaging technique. This can be done by taking a segment of length \( d \) (starting from the time step \( l \), when the term \( Y \) becomes negligible, to time step \( l + d - 1 \)) from the a priori chattering, the input and the measurement signals, and then calculating their averages. From (12), the average of the chattering segment is obtained as:

\[ \frac{1}{d} \sum_{l=1}^{d-1} Ch_{l|l-1} = \frac{1}{d} \left[ \Delta A z_{l|l-1} + \Delta B u_{l|l-1} + \omega_{l|l-1} + v_l - \hat{A}_{v_l-1} \right] \tag{13} \]

If \( d \) is chosen to be large enough to satisfy the lemma, then the averages of the measurement and system noise converge to their expectations which are zero (zero-mean assumption) as:

\[ \lim_{d \to \infty} \left( \frac{1}{d} \sum_{l=1}^{d-1} \omega_{l|l-1} + v_l - \hat{A}_{v_l-1} \right) = 0_{n \times 1} \tag{14} \]

If the measurement matrix is the identity matrix, system and measurement noise are white, and the smoothing boundary layer is set to have zero width, then (13) is simplified to:
There are two unknowns \((P_a, P_b)\) in these equations. Rearrange for \(P_a\) and \(P_b\):

\[ P_a = \frac{\int (m_a + m_\lambda)RT_a - \tau A_aP_aY}{(A_aY)} \]

\[ P_b = \frac{\int (m_b + m_\lambda)RT_b - \tau A_bP_bY}{(A_b(L - Y))} \]

**B. Load Model**

The dynamic behavior of the load is derived as:

\[ (A_aP_a - A_bP_b) - (A_{\text{sys}} - A_2)P_{\text{out}} - F_{\text{t}} = F_{\text{e}} \text{sign}(Y) + M\ddot{Y} + c_p\dot{Y} \]

For the pneumatic cylinder considered in this paper, the friction model is simplified to obtain the more commonly used threshold model. (25) describes the adopted model.

\[ F_{\text{t}} = \begin{cases} F_e & |Y| > \delta v \\ 0 & |Y| \leq \delta v \end{cases} \]

**C. Valve Model**

According to the standard orifice theory, the mass flow rates across the two control ports of the control valve can be regarded as a function of the valve spool displacement and the chamber pressures, which can be expressed as

\[ m_a = C_dC_owXf_{pa}(P_a) \]

\[ m_b = C_dC_owXf_{pb}(P_b) \]

For air \(C_o = 0.8\), \(C_d\) can be expressed as following:

\[ C_o = \sqrt{r/(R(T_{sat})^2)} \]

where \(r = 1.4, R = 280\), then \(C_o = 0.04\). The mass flow function for chamber A and chamber B can be expressed as

\[ f_{pa}(P_a) = P_a/\sqrt{T_s}f(P_a/P_b) \quad \text{chamber A is a drive} \]

\[ f_{pb}(P_b) = P_b/\sqrt{T_s}f(P_b/P_a) \quad \text{chamber B is a drive} \]

The above component-based model was created and simulated using Matlab/Simulink as shown in Figure 3.

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**IV. PNEUMATIC SYSTEM MATHEMATICAL MODEL**

This section presents the mathematical model of a pneumatic system that consists of double acting asymmetric cylinder, Five-port proportional valve, load, and tubes.

**A. Cylinder model**

Assuming that chamber A is the driving chamber \((P_a > P_b)\), then the control volumes in the cylinder is:

\[ m_aC_p\dot{T}_{pa} = \dot{m}_aC_pT_a - P_a\dot{V}_a/dt = d(C_p\rho_aV_a\dot{T}_a)/dt \]

Similarly, if chamber B is the driving chamber \((P_b > P_a)\), then:

\[ m_bC_p\dot{T}_{pb} = \dot{m}_bC_pT_b - P_b\dot{V}_b/dt = d(C_p\rho_bV_b\dot{T}_b)/dt \]

Assuming an ideal gas \((P = RT\) and \(P/R = \rho T\)), then the right hand side can be simplified to:

\[ C_o\rho_aV_a\dot{T}_a = C_oP_aV_a/R \]

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**V. THE APPLICATION OF THE SVSF INTO THE PNEUMATIC SYSTEM**

The SVSF’s chattering signal can be used to tune the filter model in order to reduce the modeling errors. This was tested and verified by applying the SVSF to the pneumatic system described in the previous section. The basic idea of this experiment was to create a linearized system that gives the best approximation of the non-linear parts in the actual system. Using the chattering equation, the approximated linear system was tested every time step to ensure its efficiency. The results obtained from the SVSF are compared to the results obtained from a Neural Network with three hidden layers.
A. Simulation setup

The SVSF is designed to have $\Psi = 0.2 \times I_{n \times n}$ and $\Psi = 0.0 \times I_{n \times 1}$. The order of the filter model is unknown as the filter model supposed to approximate the non-linear system model. Therefore, several iterations have been used with different order values to obtain the best value that reduces the root mean squared error. The uncertainties caused from the linearization process should have magnitudes less than 10% of the corresponding states’ magnitudes. The filter uses a sample consisting of 200 points from the chattering, the measurements, and the input signals to extract the modeling errors (system parameters) as discussed in section (IV). The NN used is a feedforward network architecture with four layers: one input layer with six nodes, two hidden layers with 15 nodes each, and an output layer with three nodes. The six inputs used were the two random inputs (at time steps k and k-1), two position signals (at time steps k and k-1), one velocity signal (at time step k-1) and one acceleration signal (at time step k-1). The three outputs were the position, velocity, and acceleration (at time step k). The training algorithm used was the Levenberg-Marquardt Backpropagation.

The experiment consisted of two parts: training (2297 patterns) and testing (10,000 patterns). For the former part, a random input signal that has maximum absolute value of 10 volts was used to excite the system. The outputs and the input were then used to obtain the system model or its equivalent by training the NN and the SVSF. The outputs are shown in Figures 4 to 6. The model/neurons were then used in the latter part with different input profile to obtain the estimated states which are the position, velocity and acceleration. The tested input and outputs are shown in Figures 7 to 10.

![Figure 4: The training position for NN and SVSF](image)

![Figure 5: The training velocity for NN and SVSF](image)

![Figure 6: The training acceleration for NN and SVSF](image)

![Figure 7: The input used in testing NN and SVSF](image)

![Figure 8: The position used in testing NN and SVSF](image)

![Figure 9: The velocity used in testing NN and SVSF](image)

![Figure 10: The acceleration used in testing NN and SVSF](image)
**REFERENCES**