Mechatronic systems identification using an impulse response recursive algorithm

Tarek Tutunji a,*, Mohammad Molhim b, Eyad Turki b

a Mechatronics Engineering Department, Philadelphia University, Jordan
b Mechanical Engineering Department, JUST, Jordan

Received 8 May 2006; received in revised form 25 April 2007; accepted 9 May 2007
Available online 2 June 2007

Abstract

A recursive identification algorithm is used to identify mechatronic systems using impulse response data. The algorithm is based on an auto regressive moving average (ARMA) model with a steepest descent method to minimize the least square error between the original and predicted outputs. Two mechatronic systems are tested: DC motor and gyroscope. Impulse voltage input is used to excite the system and the angular speed output is measured. In both systems, the torque and angular velocity outputs are dependent on the voltage and current inputs. This relationship is governed by characteristics such as inductance, resistance, moment of inertia, friction, load, and system constants. Once the ARMA model is constructed, the transfer function is realized. Then the input voltage is varied and the identified model results are compared with the original system. Simulation results using Simulink and experimental results using Labview with data acquisition card (DAQ) are presented. Results show that the recursive identification algorithm is able to identify the two systems with minimal error.

© 2007 Elsevier B.V. All rights reserved.

Keywords: Mechatronic systems; System identification; RLS; ARMA; DC motor and gyroscope modeling

1. Introduction

System identification is the field of modeling dynamic systems from experimental data (i.e. input/output patterns). The goal is to approximate the 'unknown' black box with a mathematical model based on differential (or difference) equations that uses the available input/output data.

Soderstrom and Stoica [1] published a comprehensive book detailing many system identification algorithms, nonrecursive and recursive, such as least squares, recursive least squares, instrumentation methods, and recursive prediction error methods. Their work is considered as a cornerstone reference guide. Recently, researchers have developed the system identification methods and applied them to many engineering systems. Kara and Eker [2] and later Ekar [3] presented experimental on-line identification of a three-mass electromechanical system, namely DC motor. They used recursive least squares to estimate the unknown parameters of...
the system by matching input–output behavior of the physical system. In other work [4], impulse response data was used as patterns in a recursive gradient algorithm to identify the transfer function of a DC motor.

Angerer et al. [5] used a structured recurrent neural network to identify physical relevant parameters and nonlinear characteristics of a nonlinear two-mass system with friction and backlash. Koubaa [6] used a least square derived algorithm to estimate the rotor resistance, self-inductance of the rotor winding, as well as the stator leakage inductance of a three phase induction machine.

Yan et al. [7] used a recursive prediction error method based on the ARMA model to identify the transfer function of a CNC milling machine in order to apply a combined self-tuning adaptive control and cross-coupling control to retrofit the machine with DC motors instead of stepper motors. Robotics is another application area for system identification. Ostring et al. [8] identified the behavior of an industrial robot in order to model its mechanical flexibilities while Johansson [9] used a state-space model to identify the a robot manipulator dynamics.

In this paper, a recursive least square (RLS) algorithm is applied to two systems: DC motor and gyroscope. An input impulse voltage is used to excite the system and the angular velocity is measured. This data pattern, input and output, is used in the RLS algorithm in order to identify the ARMA parameters and ultimately the transfer function.

The basic DC motor modeling is well known in the industry where output speed and position are related to the inputs voltage and current through differential equations that depend on several parameters such as coil resistance, inductance, moment of inertia and friction.

In many cases, such parameters are supplied by the manufacturer and therefore, the DC motor behavior can be predicted. However, in some cases not all the parameters are supplied and therefore the transfer function is not known. This is the case for some used motors that do not have parameter plate or data sheets. In order to predict their behavior, experimental data can be used to identify their transfer functions.

The gyroscope system is highly nonlinear. The gyroscope is used in many navigation systems such as airplanes and ships. The relationship between the voltage input and angular velocity output is nonlinear and somewhat complicated. However, linear approximation can be applied about a working point of the system for identification purposes.

In this paper, we propose to use an input voltage pulse and velocity impulse response output as patterns into an auto regressive moving average (ARMA) model with a steepest descent algorithm in order to identify the governing relationship between the pulse and its response. Once the algorithm converges, the transfer function is identified to be the Z-transform of the ARMA model. Then, the input voltage can change relative to the motor application and the output would be predicted accurately. More details of this work is presented in [10].

This paper is divided as follows: Section 2 gives the mathematical background of the recursive algorithm used to identify the mechatronic systems tested. Section 3 gives the DC motor and gyroscope modeling. Section 4 shows the DC motor simulation results while Section 5 presents the experimental results of the DC motor and gyroscope. Finally, the conclusions are summarized in Section 6.

2. Recursive identification methods

The model structure used to identify the system dynamics for a single-input-single-output is given below [1]

\[ Y = \phi^T \theta + \epsilon \]  

(2.1)

where \( Y \) is the output vector with dimension \( K \times 1 \) composed of the elements \( y(1), y(2), \ldots, y(K) \), \( \epsilon \) is the error vector, \( \phi^T \) is a matrix whose elements are the delayed input and output components

\[
\phi^T = \begin{bmatrix}
\phi^T(1) & \cdots & 0 \\
\cdots & \cdots & \cdots \\
0 & \cdots & \phi^T(K)
\end{bmatrix}
\]  

(2.2)

\[
\phi^T(k) = [-y(k-1) \cdots -y(k-n) \; u(k-1) \cdots u(k-m)]
\]

\( y(k) \) and \( u(k) \) are the system output and input at time \( k \), and \( \theta \) is the parameter vector.
\[
\theta = [a_1 \cdots a_n \ b_1 \cdots b_m]
\]

Simple least square estimate of the parameter vector \( \theta \) is

\[
\hat{\theta} = \left( \Phi^T \Phi \right)^{-1} \Phi^T y
\]  

(2.3)

Eq. (2.3) is appropriate if the function has a unique minimum point. However, when the function has several local minima, a recursive algorithm is used to find the optimal parameters.

Recursive Least Square (RLS), in vector format, gives the following equations:

\[
\hat{\theta} = \hat{\theta} + Q e
\]

\[
e = Y - \Phi^T \hat{\theta}
\]

\[
Q = P \Phi
\]

\[
P = \left( P - P \Phi \Phi^T P / (\lambda + \Phi^T P \Phi) \right) / \lambda
\]

(2.4)

where \( P \) is a positive definite matrix initialized to be \( cI \) (\( I \): identity matrix, \( 100 < c < 10,000 \)) and \( \lambda \) is the forgetting factor (\( 0.95 < \lambda < 0.99 \)).

Eqs. (2.4) are used in a recursive algorithm where the parameter vector \( \theta \) is updated at each iteration until convergence.

In an auto regressive moving average (ARMA) model, the system to be identified is assumed to have the following linear model:

\[
\hat{y}(k) = \sum_{j=1}^{n} a_j y(k-j) + \sum_{i=0}^{m} b_i u(k-i)
\]  

(2.5)

where \( u(k), y(k) \) and \( \hat{y}(k) \) are the input, the original system output, and the predicted model output of the \( k \)th sample.

The goal is to find a linear system model that gives output, \( \hat{y} \), equal to real output \( y \). The error between the actual and modeled output is error \( = \hat{y} - y \).

Input–output patterns \((u, y)\) are available. They are used in the above equations to calculate \( \hat{y} \). The parameters \( a_j \) and \( b_i \) are updated to minimize the least square error as shown in Fig. 1.

The least square error is

\[
E = \frac{1}{2} \sum_{k=1}^{K} (\hat{y}(k) - y(k))^2 = \frac{1}{2} \sum_{k=1}^{K} e(k)^2
\]  

(2.6)

This is the error function that is to be minimized. The variables are the parameters \( a_j \) and \( b_i \).

Optimization algorithms, such as Newton and steepest descent, can be used to minimize the above error. The gradient will be made of the following derivatives:

\[
\frac{\partial e(k)}{\partial a_j} = (\hat{y}(k) - y(k)) y(k-j) = e(k) y(k-j)
\]  

(2.7)

\[
\frac{\partial e(k)}{\partial b_i} = (\hat{y}(k) - y(k)) x(k-i) = e(k) u(k-i)
\]  

(2.8)

Fig. 1. Prediction error model block diagram.
The steepest descent algorithm is used to get the following updates:

\[ a_j = a_j - \alpha y(k - j) e(k) \]  
\[ b_i = b_i - \alpha u(k - i) e(k) \]  

(2.9)  
(2.10)

The above equations are equal to the updates, \( \theta \), in Eqs. (2.4) if matrix \( P \) is set to be the identity matrix throughout the algorithm. Here, \( 0 =< \alpha \) (step length) =< 1.

Once the parameters are identified, the Z-transform of the ARMA(\( m, n \)) model is calculated to yield the estimated transfer function of the model which is given by

\[ \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \]  

(2.11)

The ARMA model can be presented as

\[ AY(z) = BU(z) \]  

(2.12)

where

\[ A = 1 + a_1 Z^{-1} + a_2 Z^{-2} + \cdots + a_n Z^{-n} \]

and

\[ B = b_1 Z^{-1} + b_2 Z^{-2} + \cdots + b_m Z^{-m} \]

It is worth noting here that when the input to a system, \( u(k) \), is a pulse

\[ \delta(k) = \begin{cases} 1 & k = 0 \\ 0 & \text{else} \end{cases} \]

Then the output, \( y(k) \), is equal to the system, \( h(k) \). Therefore, the Z-transform of \( y(k) \) gives \( Y(z) \) which is equal to \( H(z) \) as explained below.

In classical signal processing theory, the output to the system (in the time domain) is equal to the convolution of the input and the system

\[ y(k) = \sum_{q=\infty}^{\infty} x(q) h(k - q) \]  

(2.13)

If the input \( x(k) \) is a pulse, then

\[ y(k) = \sum_{q=\infty}^{\infty} \delta(q) h(k - q) = h(k) \]  

(2.14)

3. Mathematical modeling

In this section we present the mathematical modeling of the considered systems in this research, namely, DC motor and gyroscope.

3.1. DC motor model

There are two main equations governing a DC motor: mechanical and electrical

\[ J \frac{d\omega}{dt} = T - b\omega - T_{\text{Load}} \]  

(3.1)

\[ V_{\text{in}} = Ri + L \frac{di}{dt} + V_{\text{emf}} \]  

(3.2)

where \( T \) is the motor torque, \( T_{\text{Load}} \) is the load torque, \( \omega \) is the motor speed, \( J \) is the motor moment of inertia, \( b \) is the friction coefficient of the motor, \( V_{\text{in}} \) is the input motor voltage, \( V_{\text{emf}} \) is the electromagnetic voltage force, \( i \) is the motor current, \( R \) is the motor winding resistance, and \( L \) is the motor winding inductance.
Furthermore, the torque is related to the current via a mechanical constant and the electromagnetic voltage is related to the angular speed via an electrical constant

\[ T = K_m i \]  
\[ V_{emf} = K_e \omega \]  

(3.3)  
(3.4)

where \( k_m \) is the motor mechanical constant and \( k_e \) is the electrical constant.

Also, the load torque can be further divided to include the load’s moment of inertia

\[ T_{Load} = J_L \frac{d\omega}{dt} + b_L \omega \]  

(3.5)

Substituting Eqs. (3.3)–(3.5) into the Eqs. (3.1) and (3.2) and taking the Laplace transform with voltage as input and angular speed as output yields the model block diagram for a DC motor as shown in Fig. 2. Here, \( S = j\omega \) where \( \omega \) is the radian frequency.

3.2. Gyroscope model

Gyroscope system consists of a DC motor in a solid aluminum frame. The motor is equipped with gearbox. The gearbox output drives external gears. The external gearbox drives gyro module. Fig. 3 shows the gyroscope diagram.

The electrical equations can be derived using Kirchoff’s voltage as follow:

\[ V_m - Ri - L \frac{di}{dt} - V_{emf} = 0 \]  

(3.6)

where \( V_m \) is the armature input voltage, \( i \) is the armature input current, \( R \) is the armature resistance, \( L \) is the armature inductance, and \( V_{emf} \) motor back-emf voltage.

Most DC motors have negligible \( L \), such that the model (for \( L = 0 \)) is reduced to

\[ i = \frac{(V_m - V_{emf})}{R} \]  

(3.7)

Back emf created by the motor is proportional to the motor shaft velocity \( \omega_m \)

\[ i = \frac{(V_m - K_m \omega_m)}{R} \]  

(3.8)

where \( \omega_m \) motor shaft angular velocity.

![Fig. 2. DC motor block diagram model.](image)

![Fig. 3. Gyroscope diagram.](image)
The mechanical equations are found by applying Newton 2nd law of motion to the motor shaft

\[ J_m \ddot{\omega}_m = T_m - \left( \frac{T_{\text{Load}}}{\eta_g K_g} \right) \]  \hspace{1cm} (3.9)

where \( T_m \) is the torque generated by motor, \( T_{\text{Load}} \) is the torque applied at the load, \( K_g \) is the gear ratio, and \( \eta_g \) is the efficiency of the gearbox.

Applying the 2nd law of motion at the load of the motor gives

\[ J_{\text{total}} \ddot{\omega}_l = \frac{T_{\text{Load}}}{C_0} - B_{\text{equ}} \omega_l \]  \hspace{1cm} (3.10)

\[ J_{\text{total}} = \sqrt{J_f^2 + J_l^2 \cos \theta} \]  \hspace{1cm} (3.11)

\[ \theta = \tan^{-1} \frac{J_f}{J_l} \]  \hspace{1cm} (3.12)

\[ T_{\text{total}} = \sqrt{T_f^2 + T_l^2 \cos \phi} \]  \hspace{1cm} (3.13)

\[ \phi = \tan^{-1} \frac{T_f}{T_l} \]  \hspace{1cm} (3.14)

where \( B_{\text{equ}} \) is the viscous damping coefficient, \( J_{\text{total}} \) is the total moment of inertia, \( J_f \) is the flywheel moment of inertia, \( J_l \) is the load moment of inertia, \( \theta \) is the angle between \( J_{\text{total}} \) and \( J_f \), \( \omega_l \) is the load shaft angular velocity, \( T_f \) is the torque generated by flywheel, \( T_l \) is the torque applied at the load, \( T_{\text{total}} \) is the total torque applied on the system, and \( \phi \) is the angle between \( T_{\text{total}} \) and \( T_f \).

Substituting Eqs. (3.10) in (3.11) gives the following:

\[ J_{\text{total}} \ddot{\omega}_l = \sqrt{\left( \frac{\eta_g K_g T_m}{C_0} \right)^2 + \frac{T_f^2 \cos \phi}{C_0} - \frac{B_{\text{equ}} \omega_l}{C_0}} \]  \hspace{1cm} (3.15)

Using the gear ratio equations

\[ \omega_m = K_g \omega_l \]  \hspace{1cm} (3.16)

\[ T_m = \eta_m K_t \]  \hspace{1cm} (3.17)

where \( \eta_m \) is the motor efficiency and \( K_t \) is the motor torque constant.

Substituting Eqs. (3.8), (3.16) and (3.17) in (3.15) gives the following:

\[ J_{\text{total}} \ddot{\omega}_l = \sqrt{\left( \frac{\eta_g K_g T_m}{C_0} \right)^2 + \frac{T_f^2 \cos \phi}{C_0} - \frac{B_{\text{equ}} \omega_l}{C_0}} \]  \hspace{1cm} (3.18)

Eq. (3.18) is highly nonlinear and therefore the relationship between the angular velocity output, \( \omega_l \), and the motor voltage input, \( V_m \), is nontrivial. However, it is known that the nonlinear system behaves similarly to its linearized approximation around an equilibrium point.

4. Simulation of DC motor

A pulse voltage was used as an input to a DC motor SIMULINK model. The angular speed output was taken to the ARMA algorithm in MATLAB and the model parameters were calculated and therefore the estimated transfer function was identified.

The parameters used for the DC motor where obtained from a DC motor unit TY36A available at the laboratories are shown in Table 1.

<table>
<thead>
<tr>
<th>DC Motor Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor inertia</td>
<td>5.18E–6 kg m²</td>
</tr>
<tr>
<td>Motor torque constant</td>
<td>0.046 N m/A</td>
</tr>
<tr>
<td>Motor voltage gradient</td>
<td>6 V/100 RPM</td>
</tr>
<tr>
<td>Motor inductance</td>
<td>2.8 mH</td>
</tr>
<tr>
<td>Motor resistance</td>
<td>5.5</td>
</tr>
</tbody>
</table>
Care should be taken when running the ARMA model to avoid singularity and unstable realizations. The step length, $a$, should not be too large (a value between 0.2 and 0.005 was used). Also, the system output data should be large enough to incorporate the system behavior (100 samples were used).

Several ARMA models were used. The model order was varied and different runs for each order were simulated. Three of those models are shown next

$$H_1(z) = \frac{0.0618z^{-1} + 0.0086z^{-2} + 0.0387z^{-3}}{1 - 0.7831z^{-1} + 0.4989z^{-2} - 0.4058z^{-3} - 0.1755z^{-4}}$$

$$H_2(z) = \frac{0.0618z^{-1} + 0.0564z^{-2} + 0.0226z^{-3}}{1 - 0.0097z^{-1} - 0.4749z^{-2} - 0.2548z^{-3} - 0.0816z^{-4}}$$

$$H_3(z) = \frac{0.0618z^{-1}}{1 - 0.5843z^{-1} - 0.3355z^{-2}}$$

The identified transfer function is not unique since the steepest descent converges to a local minimum (i.e. the final solution depends on the initial parameters used). This is noted in $H_1$ and $H_2$ which have the same order, but different identified transfer functions.

All the identified systems gave excellent results with small error. The sample time used for all signals was one unit (1 ms) for both the identification and testing purposes.

---

Fig. 4. Original output signal (solid line) vs. predicted output signal (dots) for simulated DC motor with pulse input. Here the speed was measured in voltages. See Table 3 for V/RPM relationship.

---

Fig. 5. Step response of original (solid) vs. predicted (dots) for simulated DC motor.
An impulse signal was applied as motor input to the simulation of DC motor. The impulse input used was
\[ V_{in}(k) = \begin{cases} 
8 \text{ V} & k = 0 \\
0 & k \neq 0 
\end{cases} \]

Impulse response provides the complete characteristic information of a system. The impulse signal excites all frequencies and the duration of this signal is infinitely small.

Impulse and impulse response were applied as input/output patterns to an ARMA(4,4) model. Recursive Least square (RLS) error was used to minimize the error between desired output (from simulation) and estimated output (from the identified transfer function). The error converge to (0.00005) after (100) iteration.

Fig. 6. Error between original and predicted responses.

Fig. 7. Angular speed (original vs. predicted) with a random input voltage for simulated DC motor.
These results are shown in Fig. 4. Note that the response of the identified system fits perfectly with the original system output.

The following is the transfer function of the identified ARMA model

\[
H(z) = \frac{Y(z)}{U(z)} = \frac{7.1748e - 006 + 0.0003291z^{-1} + 0.00067305z^{-2} + 0.0011488z^{-3}}{1 - 0.19513z^{-1} - 0.089496z^{-2} - 0.66579z^{-3} - 0.042163z^{-4}}
\] (4.4)

Next the input signal was changed from an impulse to a step function and was presented to the original and predicted systems. The response of the two systems are shown in Fig. 5.

The two outputs match almost perfectly. In order to get better insight, Fig. 6 displays the error between the original and predicted responses.

The input voltage was changed to random voltage and both outputs were compared as shown in Fig. 7.

5. Experimental results

The experimental results from the two mechatronic systems, DC motor and gyroscope, are presented in this section. National Instruments NI6063E DAQ Card from with sampling rate up to 200 kHz was used. The PC used was a Pentium 4 operating at 2.19 GHz. All results were obtained using Labview software.

5.1. Servo DC motor

The DC motor experiment duration was set to one second. A sampling rate of 1000 samples per second was used for each channel. A total of three channels from the DAQ were used. The first channel measured the actual motor input signal at the motor rotor (after the drive circuit). The second channel measured the motor speed from the tachometer. The third channel measured the impulse output to the drive circuit. Furthermore, two software filters were used for the motor input and the tachometer output channel. The two filters used were low pass Butterworth first order filters with a cut off frequency of 30 Hz.

The DAQ output signal is the impulse applied to power drive circuit at 1 Hz Frequency with an amplitude of 5 V. The DC servo motor rated voltage was 10 V that was provided to the motor by power drive circuit.

All the signals were saved to a file. Each file had 20 signals. Two files were used, one for acquiring the motor input signals and another for acquiring tachometer output signals. Each signal had 1000 data samples. The DC motor used is shown in Fig. 8.
Technical characteristics for the servo DC motor and the tachometer are given in Tables 1 and 2 respectively.

The experiment setup is shown in Fig. 9.

The measured impulse signal sent to the motor drive circuit through channel 0 in DAQ card is shown in Fig. 10.
The measured motor rotor input signal and the measured tachometer signal, angular velocity presented as voltage (Table 3 shows voltage vs. speed relationship), through channels 1 and channel 2 are shown in Fig. 11a and b.

Those signals were modified through a soft low pass (LP) filter with a 30 Hz cutoff frequency. The results are shown in Fig. 12.

Table 3
Tacho-generator results (voltage vs. speed)

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.08</td>
<td>5620</td>
</tr>
<tr>
<td>15.22</td>
<td>4919</td>
</tr>
<tr>
<td>12.23</td>
<td>3979</td>
</tr>
<tr>
<td>10.72</td>
<td>3474</td>
</tr>
<tr>
<td>8.98</td>
<td>2906</td>
</tr>
<tr>
<td>7.5</td>
<td>2491</td>
</tr>
<tr>
<td>5.23</td>
<td>1664</td>
</tr>
<tr>
<td>3.41</td>
<td>1097</td>
</tr>
<tr>
<td>1.38</td>
<td>446.7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 11. (a) Motor input signal and (b) tachometer output signal.

Fig. 12. (a) Motor input signal with LP filter and (b) tachometer output signal with low LP filter.
The low pass filter was used to filter out noisy spikes in the acquired data. The time constant for the motor response is about 500 ms (2 Hz which falls inside the LP cutoff). Note that the characteristic of the signal did not change and therefore no restrictions were enforced. This can be shown clearly by comparing Figs. 11 and 12 (signals before and after the applied filter).

The average of 26 input and 26 output signals were used as the input/output patterns to the ARMA model in order to get more accurate results. Figs. 13 and 14 present those averaged signals. Note here that the input signal is the voltage pulse measured at the motor input while the output signal is the angular velocity measured by the tachometer and presented as voltage signal.

We ran several experiments to find the optimal ARMA model order. The order of AR = 4 and MA = 5 gave best results.

Then the identified transfer function of the system was

\[
H(z) = \frac{-0.3215 + 0.3659Z^{-1} + 0.1836Z^{-2} - 0.2071Z^{-3} - 0.021Z^{-4}}{1 - 1.1343Z^{-1} - 0.4964Z^{-2} + 0.3828Z^{-3} + 0.2478Z^{-4}}
\]  

(5.1)

The transfer function was rewritten to identify the zeros and poles

\[
H(z) = \frac{-0.315(Z + 0.0951)(Z - 0.958)(Z - 0.9957)(Z + 0.7205)}{Z(Z - 0.9966)(Z - 1.0094)(Z^2 + 0.8718Z + 0.2464)}
\]  

(5.2)

A plot of these poles and zeros in z-domain is shown in Fig. 15.

---

**Fig. 13.** Average input signal for DC motor experiment.

**Fig. 14.** Average output signal for DC motor experiment.
Fig. 15. Poles–zeros plot of identified DC motor system (zero (○), pole (×)).

Fig. 16. Predicted output (dot line) vs. physical original system output (solid line) for DC motor impulse response.
Table 4
Statistical error results for the DC motor experiment with an impulse input

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$: Mean square error</td>
<td>3.90E−6</td>
</tr>
<tr>
<td>$E_{\text{max}}(k)$: Maximum error</td>
<td>1.09E−1</td>
</tr>
<tr>
<td>$\sigma(E)$: Standard deviation</td>
<td>6.28E−3</td>
</tr>
</tbody>
</table>
Fig. 19. (a) Filtered input to the gyroscope system and (b) filtered output from tachometer.

Fig. 20. Predicted output (dot line) vs. physical original output (solid line) for gyroscope impulse response.
Next, another impulse input was given to the servo DC motor and the error between the physical system and identified transfer function responses was calculated. Fig. 16 displays both responses.

Fig. 16 shows that both signals fitting almost perfectly. In order to get better insight, Fig. 17 displays the error between the physical and identified responses.

Table 4 shows the statistical error results for the identified DC motor.

5.2. Gyroscope system

Gyroscope system is a nonlinear system. We tested the recursive algorithm on the gyroscope to further validate the system identification method used. The objective was to identify the motor behavior in the gyroscopic load.

In the gyroscope system experiment, the total duration time was set to 2.5 s and the sampling rate was set to 1000 samples per second for each channel. A total of three DAQ channels were used. The first channel was the input measured at the actual motor input signal measured at the motor rotor (after the drive circuit). The second channel was the measured motor speed input from the tachometer. The third channel was the impulse output to the drive circuit. As in the DC motor experiment, two software filters were used for the motor input and for tachometer output channel. Fig. 18 shows the gyroscope used in the experiment.

The impulse and impulse response signals are shown in Fig. 19.

The transfer function was found to be

\[
H(z) = \frac{0.099 - 0.0883Z^{-1} - 0.1391Z^{-2} + 0.2122Z^{-3} - 0.0838Z^{-4}}{1 - 2.2005Z^{-1} + 1.5504Z^{-2} - 0.456Z^{-3} + 0.1065Z^{-4}}
\] (5.3)

Note here that the motor behavior in a nonlinear load inertia was approximated using a linear model. Another impulse was given the physical system. The response of the identified transfer function was compared to the original physical system. These results are shown in Fig. 20.

![Fig. 21. Error between the identified and the physical system.](image)

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical error results for the gyroscope experiment with an impulse input</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$E$: Mean square error</td>
</tr>
<tr>
<td>$E_{\text{max}}(k)$: Maximum error</td>
</tr>
<tr>
<td>$\sigma(E)$: Standard deviation</td>
</tr>
</tbody>
</table>
Fig. 22. Output signal (solid line), input signal (dot line) with noise as applied to the gyroscope system.

Fig. 23. Predicted output (dot line) vs. physical original output (solid line) for gyroscope with noisy impulse response.
In order to get better insight, Fig. 21 displays the error between the identified and the physical system. Table 5 shows the statistical error results for the gyroscope experiment.

Next noise was added impulse input and applied to the system. The results are shown in Figs. 22 and 23. Table 6 shows the statistical error results for the gyroscope experiment when noise is added.

Finally, a step input was applied to the gyroscope and the physical system response was compared to the predicted response as shown in Fig. 24 and Table 7.

**Table 6**
Statistical error results for gyroscope experiment with an impulse input and noise added

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$: Mean square error</td>
<td>1.00E−3</td>
</tr>
<tr>
<td>$E_{\text{max}}(k)$: Maximum error</td>
<td>1.30E−1</td>
</tr>
<tr>
<td>$\sigma(E)$: Standard deviation</td>
<td>3.16E−2</td>
</tr>
</tbody>
</table>

**Table 7**
Statistical error results for the gyroscope experiment with step input

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$: Mean square error</td>
<td>0.0050</td>
</tr>
<tr>
<td>$E_{\text{max}}(k)$: Maximum error</td>
<td>0.8678</td>
</tr>
<tr>
<td>$\sigma(E)$: Standard deviation</td>
<td>0.0757</td>
</tr>
</tbody>
</table>

![Fig. 24. Predicted output (dot line) vs. physical original output (solid line) for gyroscope step response.](image)
6. Conclusions

Recursive identification algorithm was used to identify two mechatronic systems: DC motor and gyroscope system. The algorithm used a recursive least squares to identify the ARMA parameters of the transfer function. The input–output patterns used were the impulse (excitation voltage) and impulse response (angular speed) to the motors. Once the transfer function was identified, the algorithm was able to predict the angular speed of the systems for several input signals with excellent accuracy. Simulation results using Simulink and experimental results using NI DAQ with Labview are presented.

References