Robotic Industrial Manipulator

Introduction

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**Definition**

- Robotic manipulator is a set of links connected by joints to form an open-chain (serial) kinematic chain from base to the end-effector.

- Joints are basically:
  - Revolute (R): like a hinge and has one degree of freedom which rotation about a single axis.
  - Prismatic (P): has single degree of freedom which translation along a specific axis.

- Any other joint can be represented by some combination of these two primary joints.

- Spherical joint can be seen as three revolute joints with zero link lengths.
Examples of Joints

- **Revolute**: 1 Degree of Freedom
- **Prismatic**: 1 Degree of Freedom
- **Screw**: 1 Degree of Freedom
- **Cylindrical**: 2 Degrees of Freedom
- **Spherical**: 3 Degrees of Freedom
- **Planar**: 3 Degrees of Freedom

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Workspace of a Manipulator

• Workspace is defined as the total volume swept out by the end-effector as the manipulator executes all possible motions.
  ➢ Can be known analytically or experimentally
• Workspace is determined by the geometric constraints of the manipulator and joints.
• Workspace can be divided into:
  ➢ Reachable workspace: the entire points that are reachable by the end-effector.
  ➢ Dexterous workspace: the points that are reachable with arbitrary orientation of the end-effector.
• Obviously, dexterous workspace is a subset of the reachable workspace.
Workspace using MATLAB

Workspace of Two link RR Manipulator

Link $L_1 = 4$
Link $L_2 = 2$

$(0, 0) \quad \theta_1 : 30^\circ \leq \theta_1 \leq 150^\circ$

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Power Source

• Manipulators can be:

  ➢ Electrically powered
    ✓ DC or AC servo-motors
    ✓ Cheap, clean, and quiet

  ➢ Hydraulically powered
    ✓ Excellent in speed and torque response and capability.
    ✓ Accurate positioning
    ✓ Leakage of oil is the main drawback
    ✓ Noisy and need for a lot of peripheral equipment and maintenance.

  ➢ Pneumatically powered
    ✓ Simple and inexpensive
    ✓ Cannot be controlled precisely due to air compressibility

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Applications of Manipulators

• Assembly
  ➢ Small
  ➢ Electrically driven
  ➢ Revolute or SCARA

• Non-assembly
  ➢ Welding
  ➢ Spray Painting
  ➢ Material Handling
  ➢ Machine Loading and Unloading.
Method of Control

• Non-Servo:
  - The earliest robotic manipulators were open loop.
  - Uses mechanical stops to control the range of motion.
  - Used primarily for material transfer.

• Servo:
  - Closed-loop computer control
  - Multi-functional and reprogrammable.
Servo-robotic Manipulator

• Point-to-point control: can be taught using a teach pendant between discrete points and then playback. No control between any two successive points.

• Continuous path control: The entire path is under control
  
  ✓ Welding on a straight line or along some predefined profile.
  ✓ Control can be on velocity, acceleration, or even force which require advanced computer and software capabilities.
Geometric Types

- Manipulators are geometrically (kinematically) Classified based on the first three joints on the arm:
  1) Articulated (RRR)
  2) Spherical (RRP)
  3) SCARA (RRP)
  4) Cylindrical (RPP)
  5) Cartesian (PPP)
Articulated (RRR) Manipulator

• Three revolute joints; the axes of second and third R joints are parallel, whereas the axis of first R joint is vertical.
• Relatively large movement freedom in a compact space.
Spherical (RRP) Manipulator

• If the third R joint in the articulate manipulator is replaced with a prismatic joint, then the result is the spherical manipulator.
• It has three mutually perpendicular axes.
• The joint variables are the spherical coordinates of the end-effector with respect to the base.
SCARA Manipulator

- SCARA is the short for Selective Compliant Articulated Robot for Assembly.
- It is RRP but the axes are parallel.
- Specially designed for assembly operations.

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Cylindrical (RPP) Manipulator

• The first joint is revolute and produces a rotation about the base, whereas the second and third joints are prismatic.

• The joint variables are the cylindrical coordinates of the end-effector with respect to the base.
Cartesian (PPP) Manipulator

- The first three joints are prismatic.
- The joint variables are the Cartesian coordinates of the end-effector with respect to the base.
- Useful for transfer of materials, and table-top assembly.
Mathematical Model of Manipulator

• Kinematics: Study of the geometric description of motion regardless of the causing force or torque.

• Kinetics: Study of the relation between the acceleration (linear and angular) and the causing force or torque. (Newton’s second law).

• Dynamics and Control.
Planning and Control

• The motion must be planned in advance, so that the controller can be designed and built accordingly in order to achieve the planned motion.

• All of the mathematical models are used in order to build the controller ultimately.

• Questions:
  - How can the end-effector reach the goal position in the workspace?
  - What is the configuration to reach the goal position?
  - What about redundancy issue?
Overview

• Spatial Description:
  - Position and Orientation Description.
  - Transformation between Frames.

• Manipulator Kinematics
  - Link Description.
  - Denavit – Hartenberg Notation.
  - Forward Kinematics
    - Where is the end-effector in space (task space) given the joint variables (joint space)?
Overview

• Inverse Kinematics
  ✓ What are the joint variables (joint space) in order to locate the end-effector at specific position and orientation in space (task or operational space).

• Jacobean
  ✓ How the joint velocities are related to the end-effector velocity and resulting force.

• Trajectory Generation
  ✓ How to move the manipulator between two points (sequence of configurations) such that it moves with smooth velocity and acceleration and other constraints that might be imposed either in joint space or in task space.

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Spatial Description

• How to describe the position and orientation of each link?
• Configuration parameter set: how to represent or describe the configuration of the whole manipulator; that is to describe the position and orientation of each link in the kinematic chain from base to end-effector?
• Suggested configuration parameters: three vectors from base to three distinct points on the link; to lock the link at specific location and orientation in space.

✓ 3 vector ➔ 9 parameters for each link ➔ 9n parameters to describe the configuration of n-link manipulator.
Generalized Coordinates

• A set of minimum number of independent configuration parameters.
• Degree of freedom is equal to the number of generalized coordinates.

If the $n$ free-links are connected through $n$ 1-DOF joints $\implies$ the resultant degree of freedom will be $6n-5n=n$

Each free link need $6$ parameters (3 for position and 3 for orientation).

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End-Effector Configuration Parameters

• A set of parameters are needed in order to completely specify the position and orientation of the end-effector with respect to the base frame.

• Parameter set can be:
  - 1 vector to locate the position and 3 angles \( \rightarrow \) 6 parameters.
  - 3 vectors to lock in space \( \rightarrow \) 9 parameters.
  - Direction cosines + position \( \rightarrow \) 12 parameters

• In 3-D, number of these independent parameters (in task or operational space) is 6 at most. 3 for position and 3 for orientation.
Joint Space vs. Task Space

- Joint Space is \( \theta_1, \theta_2, \theta_3 \).
- Task Space is \( x, y, \alpha \).
- If the dimension of joint space is greater than the dimension of task space then the manipulator is said to be redundant.
Position of a Point

• How can we define a point in space?
  ➢ By a vector with respect to a fixed point O.
  ➢ Point P as a geometric entity is different than the vector as its representation.
Representing Position: vectors

- The prefix superscript denotes the reference frame in which the vector should be understood.

\[ \mathbf{b} \mathbf{p} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} \]

\[ a \mathbf{p} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \]

Same point, two different reference frames

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The rotation matrix

\[ A_R B = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \]

\[ A \ p = A_R B \ p \]

\[ A_R B \]: To specify the coordinate vectors for the frame B with respect to frame A

\[ B_R A = A_R B^{-1} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \]

\[ B \ p = B_R A \ p \]

\[ \theta \]: The angle between \( \hat{x}_A \) and \( \hat{x}_B \) in anti clockwise direction

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How to describe the position and orientation of a body in space

- In order to describe the pose of a body, we attach a coordinate system to the body and then give a description of this coordinate system relative to the reference system.
Rotation Matrix

\[
\begin{bmatrix}
A_X^B \\
A_Y^B \\
A_Z^B
\end{bmatrix} = \begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix}.
\]

- Vectors in the rotation matrix represent the axes of the current frame w.r.t. the reference frame. In other words, how the axes of the current frame are seen from the reference frame.

\[
\begin{bmatrix}
\hat{X}_B \\
\hat{Y}_B \\
\hat{Z}_B
\end{bmatrix} = \begin{bmatrix}
\hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \cdot \hat{X}_A & \hat{Z}_B \cdot \hat{X}_A \\
\hat{X}_B \cdot \hat{Y}_A & \hat{Y}_B \cdot \hat{Y}_A & \hat{Z}_B \cdot \hat{Y}_A \\
\hat{X}_B \cdot \hat{Z}_A & \hat{Y}_B \cdot \hat{Z}_A & \hat{Z}_B \cdot \hat{Z}_A
\end{bmatrix}.
\]
Example 1

\[ A \mathbf{p} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \]

Find \( B \mathbf{p} \)

\[ B \mathbf{R}_A = \begin{pmatrix} \cos(-30) & -\sin(-30) \\ \sin(-30) & \cos(-30) \end{pmatrix} \]

\[ B \mathbf{R}_A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \]

\[ B \mathbf{p} = B \mathbf{R}_A A \mathbf{p} \]

\[ B \mathbf{p} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 13.6603 \\ 3.6603 \end{bmatrix} \]

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Basic Rotation Matrices

– Rotation about x-axis with $\theta$

$$R_{x,\theta} = \text{Rot}(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

– Rotation about y-axis with $\theta$

$$R_{y,\theta} = \text{Rot}(y, \theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

– Rotation about z-axis with $\theta$

$$R_{z,\theta} = \text{Rot}(z, \theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Example 2

- A point \( p_{uvw} = (4, 3, 2) \) is attached to a rotating frame, the frame rotates 60 degrees about the OZ axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.

\[
p_{xyz} = \text{Rot}(z, 60) p_{uvw}
\]

\[
\begin{bmatrix}
0.5 & -0.866 & 0 \\
0.866 & 0.5 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
4 \\
3 \\
2
\end{bmatrix}
= \begin{bmatrix}
-0.598 \\
4.964 \\
2
\end{bmatrix}
\]

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Example 3

• A point \( a_{xyz} = (4,3,2) \) is the coordinate w.r.t. the reference coordinate system, find the corresponding point \( a_{uvw} \) w.r.t. the rotated OUVW coordinate system if it has been rotated 60 degree about OZ axis.

\[
p_{uvw} = \text{Rot}(z, 60)^T p_{xyz}
\]

\[OR: \quad p_{uvw} = \text{Rot}(z, 60)^{-1} p_{xyz}\]

\[OR: \quad p_{uvw} = \text{Rot}(z, -60) p_{xyz}\]

\[
\begin{bmatrix}
0.5 & 0.866 & 0 & 4 \\
-0.866 & 0.5 & 0 & 3 \\
0 & 0 & 1 & 2 \\
\end{bmatrix}
= \begin{bmatrix}
4.598 \\
-1.964 \\
2 \\
\end{bmatrix}
\]