PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam A **DISCRETE STRUCTURES** **04–02–2008**

PART (I) Each problem is worth 3 points. Circle one answer.

1) Which proposition is equivalent to $p \rightarrow q$?

a) $\neg p \rightarrow \neg q$ b) $\neg p \vee q$
c) $q \rightarrow p$ d) $\neg q \vee p$

2) The number 2008 is decimal. Convert it to hexadecimal.

a) 7D8 b) 820 c) 728 d) 8D0

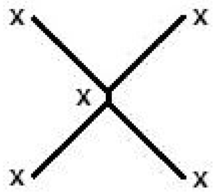
3) How many integer solutions of the equation $x + y + z = 20$ such that $x \geq 3$ and $y \geq 5$ and $z \geq 1$?

a) 78 b) 66 c) 55 d) 45

4) Let A = {0, 1, 4, 6, 9} and R is an equivalence relation on A given by R = {(a,b) | a mod 4 = b mod 4}. Find the equivalence classes.

a) {0, 4, 6}, {1, 9} b) {0}, {1, 4}, {6, 9}
c) {0}, {1, 6}, {4, 9} d) {0, 4}, {1, 9}, {6}

5) A partial order relation is given by this Hasse diagram. Find the zero-one matrix.



a $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ b $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ c $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ d $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

6) A complete graph Kn has 66 edges. How many points does it have?

a) 14 b) 24 c) 12 d) 28

7) Which graph is an Euler path but not Euler circuit?

a) K 11 b) K 2,10 c) K 2,11 d) K 10,11

8) Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

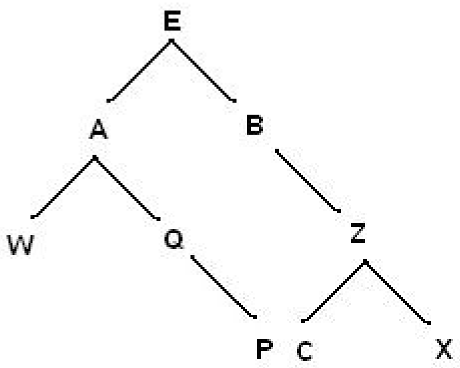
a) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

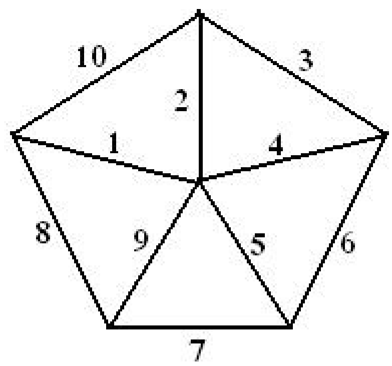
9) Find the output using the in-order algorithm. (The tree on the left)

a) W-P-Q-A-B-C-X-Z-E b) W-P-Q-A-C-X-Z-B-E
c) W-A-Q-P-E-C-B-Z-X d) W-A-Q-P-E-B-C-Z-X

10) Find the minimum spanning tree. (The graph on the right) The value is

a) 17 b) 18 c) 19 d) 20





PART (II) Each problem is worth 4 points. Write complete solutions.

11) Convert the proposition $(p \leftrightarrow r) \rightarrow q$ to a CNF.

12) Prove: If $x^2 - 8x + 5$ is odd then x is even.

13) Find an explicit formula for the recurrence relation given by

$f(0) = 1$
$f(1) = 2$
$f(n) = -f(n-1) + 12f(n-2)$

14) How many positive integers ≤ 300 which are multiples of 6 or 8 or 9 ?

15) Let $A = \{1, 2, 3, 4\}$. Give an example of a relation on A which is

a) reflexive, symmetric, not transitive
b) symmetric, transitive, not reflexive

-Amin Witno