



PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

NUMBER THEORY

4-2-2006

Part 1: Multiple Choice, each problem is 2 points.

- Which number is relatively prime to 140?
(a) 75 (b) 66 (c) 49 (d) 81
- Which congruence has no solution?
(a) $5x \equiv 1 \pmod{32}$ (b) $5x \equiv 16 \pmod{32}$
(c) $75x \equiv 5 \pmod{30}$ (d) $16x \equiv 8 \pmod{30}$
- Compute $104! \pmod{107}$.
(a) 53 (b) 54 (c) 55 (d) 1
- Suppose $|a|_n = 12$. Find $|a^9|_n$.
(a) 4 (b) 6 (c) 8 (d) 12
- Which number has no primitive root?
(a) 46 (b) 98 (c) 25 (d) 28
- Which number is a quadratic residue modulo 13?
(a) 5 (b) 10 (c) 7 (d) 2
- Choose a prime p such that $\left(\frac{-1}{p}\right) = -1$.
(a) 5 (b) 239 (c) 109 (d) 13

Part 2: Short Answer, each problem is 3 points.

- Find a reduced residue system modulo 9 with only even numbers.
- Evaluate $\varphi(360)$.
- Find all solutions of the congruence $8^x \equiv 5 \pmod{13}$ using this table.

k	1	2	3	4	5	6	7	8	9	10	11	12
$2^k \pmod{13}$	2	4	8	3	6	12	11	9	5	10	7	1



11. Is 2 a primitive root modulo 9? Justify your answer.

12. Evaluate the Legendre symbol $\left(\frac{38}{59}\right)$.

Part 3: Long Answer, each problem is 5 points.

13. Find the smallest positive number x such that $x \equiv 11 \pmod{19}$ and $x \equiv 27 \pmod{53}$.

14. Find the last three digits of 3^{31719} .

15. Find all the solutions of $x^2 \equiv 58 \pmod{77}$.

Part 4: Short Proof, each problem is 2 points.

16. Suppose $m \mid n$. Prove that $\varphi(m) \mid \varphi(n)$.

17. Suppose $a^k \equiv 1 \pmod{n}$ for some positive integer k . Prove that $\gcd(a, n) = 1$.

18. Let a be a primitive root of a prime p . Prove that $a^1 a^2 a^3 \dots a^{p-1} \equiv \square \pmod{p}$.

Table of Prime Numbers

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601
607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733
739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863
877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997	1009	1013