

Matrix Algorithms

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There are only three **elementary row operations**:

1. Interchange two rows.
2. Multiply a row by a non-zero number.
3. Add a multiple of one row to another row.

The **reduced row echelon form** of a matrix satisfies four conditions:

1. In any row, the first non-zero number must equal 1. These are the **leading ones**.
2. In any column that contains a leading one, the other numbers must equal 0.
3. Leading ones must be arranged from left to right down the rows.
4. Any row of all zeros must be at the bottom.

Finding the reduced row echelon form of a matrix using **Gauss-Jordan algorithm**:

1. Find a non-zero element c in the leftmost column then multiply this row by $1/c$ to obtain a leading one. If necessary interchange this row with the first row.
2. Make all the numbers below this leading one equal 0 by adding a suitable multiple of the first row to the appropriate row.
3. Repeat steps 1 and 2 with the sub-matrix below and to the right of this leading one, until there is no more row left.
4. From right to left make all the numbers above a leading one equal 0 by adding a suitable multiple of the row with the leading one to the appropriate row.

Solving a **system of linear equations** $AX = B$ using Gauss-Jordan:

1. Write the **augmented matrix** $[A|B]$ consisting of the coefficients matrix A and the right-hand matrix B side by side.
2. Apply Gauss-Jordan algorithm to convert this matrix to its reduced row echelon form.
3. Translate the reduced matrix back into a system of linear equations.
4. Every column of A that has no leading one gives a **free variable**. Then solve for the variables which are not free.

Computing the **inverse** of a square matrix A using Gauss-Jordan:

1. Write the matrix $[A|I]$ consisting of the matrix A and the identity matrix I side by side.
2. Apply Gauss-Jordan algorithm to convert this matrix to its reduced row echelon form.
3. If the left half of this reduced matrix equals I then the right half is the inverse of A .
4. If not then A has no inverse.

The effects of elementary row/column operations on **determinants**:

1. If a row of A is multiplied by c then the new determinant is $c \det(A)$.
2. If two rows of A are interchanged then the new determinant is $-\det(A)$.
3. If a multiple of one row is added to another row then $\det(A)$ is unchanged.

Finding the **eigenvalues/eigenvectors** of a square matrix A :

1. Solve for λ from the equation $\det(\lambda I - A) = 0$. These are the eigenvalues of A .
2. For each λ , solve the system $(\lambda I - A)X = 0$. These are the eigenvectors of A .

Computing A^k from a **diagonalizable** $n \times n$ matrix A :

1. Let P be a matrix whose columns are the n independent eigenvectors of A .
2. The diagonalized form of A is $D = P^{-1} A P$.
3. Finally compute $A^k = P D^k P^{-1}$.