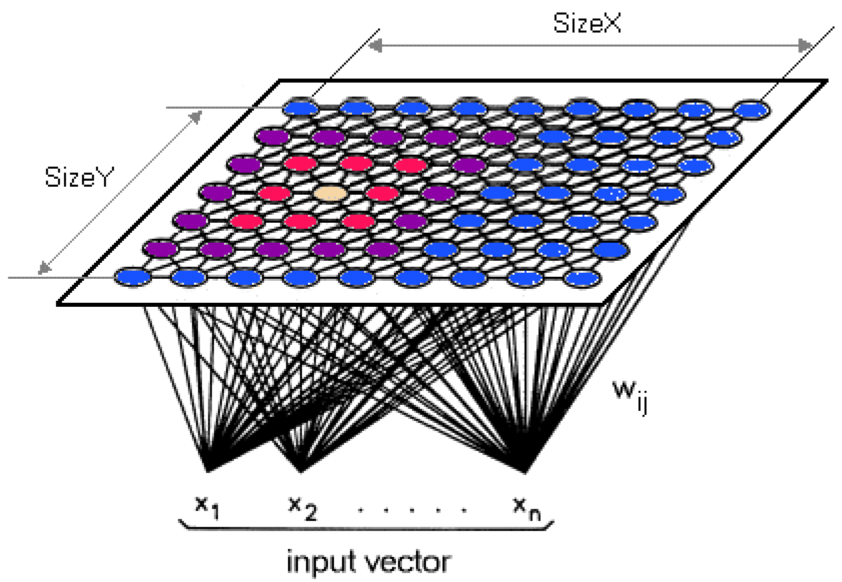
Unsupervised learning and Self Organizing Maps



Machine intelligence

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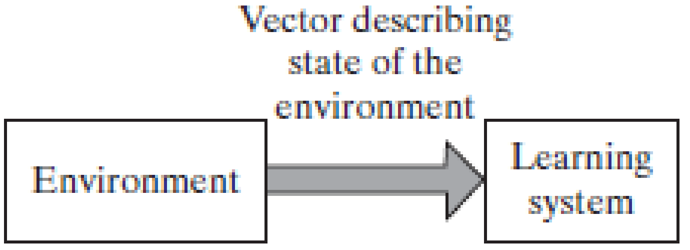
Outline

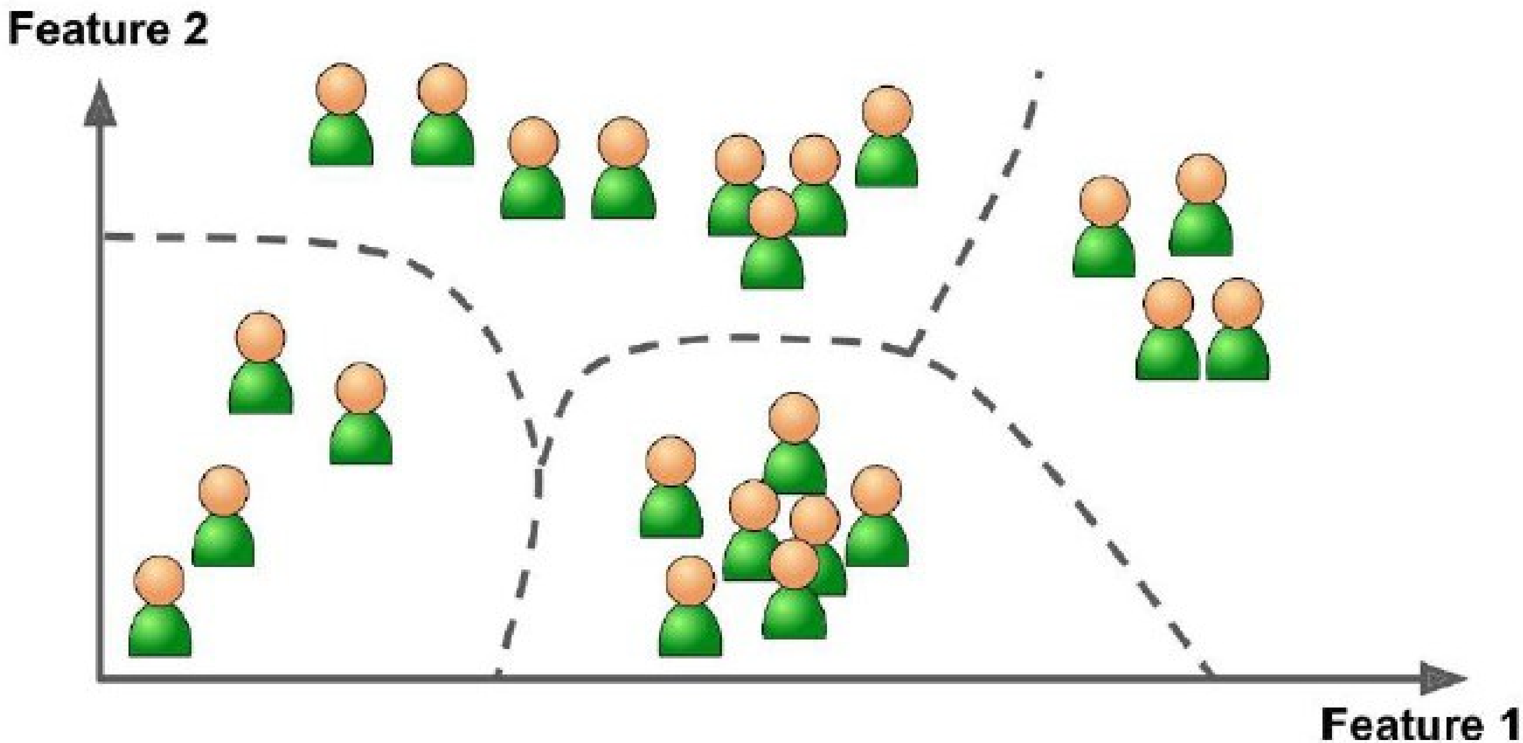
- Unsupervised Learning
- Hebbian learning
- Self-Organized Mapping
- Winner-takes-all SOMS
- Network Example
- Applications

Learning Methods: Unsupervised

- Unsupervised or Self- organized learning does not have a teacher (i.e. desired responses are not available.
- The network is tuned from input data only
- The network weights are updated to find features within the data and therefore does automatic classification.



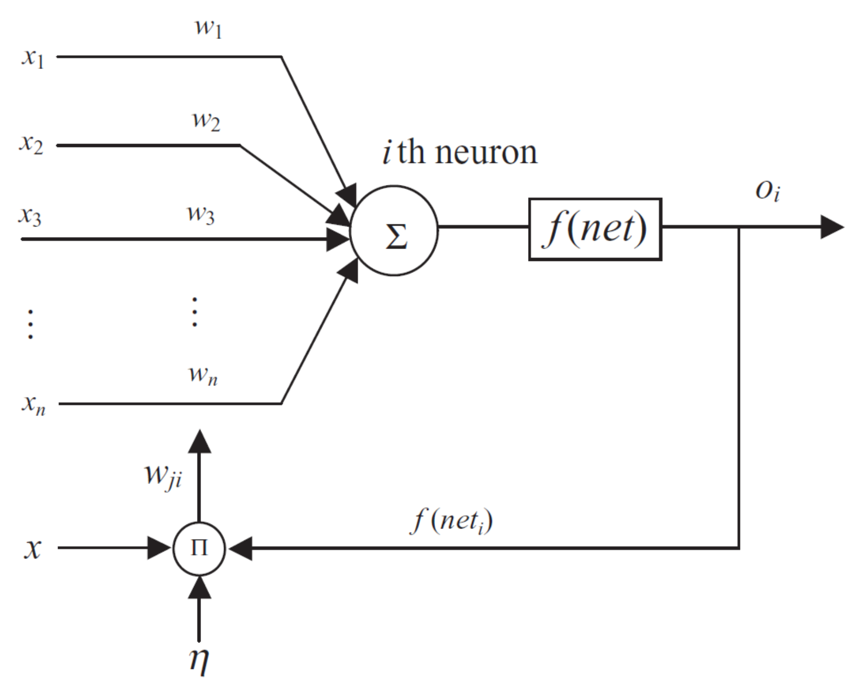
Learning Methods: Unsupervised



Hebbian learning

The basic rule is: ‘If a neuron receives an input from another neuron and if both are highly active (mathematically have the same sign), the weight between the neurons should be strengthened’.

The rule states that if the cross-product of output and input, or correlation term $o_i\, x_j$ is positive, this results in an increase of weight w_{ji} , otherwise the weight decreases.



Fire together, wire together

Hebbian learning

The weight update ΔW in Hebbian learning becomes

$$\Delta w_{ji} = \eta\ o_i x_j = \eta\ f(net_i)\ x_j$$

η is the learning rate

Where

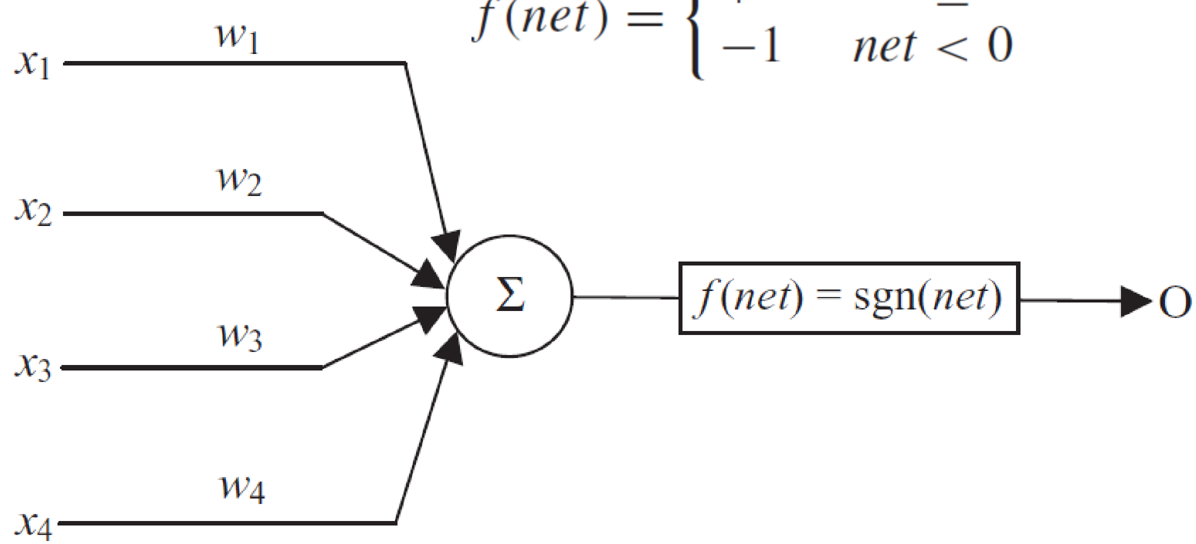
$$o_i = f(net_i)$$

$$net_i = \sum w_{ji} x_j$$

Example of Hebbian learning

$$f(net) = \begin{cases} +1 & net \geq 0 \\ -1 & net < 0 \end{cases}$$

Assume the neural network with a single bipolar binary neuron



having initial weight vector w^1 and three input vectors x^1, x^2 and x^3:

$$w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, x^1 = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}, x^2 = \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} \text{ and } x^3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$

Example of Hebbian learning

Compute the weight vector after the second iteration of Hebbian learning

Solution First iteration: net^1 is calculated using the input vector x^1 and initial weight vector w^1

$$net^1 = w^{1^T} x^1 = \begin{bmatrix} 1 & -1 & 0 & .5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = 3.0$$

Since $net > 0$, o^1 will be $o^1 = f(net^1) = +1$

Example of Hebbian learning

$$\Delta w^1 = \eta f\left(net^1\right) x^1 = 1 * (+1) * \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$w^2 = w^1 + \Delta w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix}$$

Example of Hebbian learning

Second iteration: net^2 is calculated using input vector x^2 and weight vector w^2

$$net^2 = w^{2^T} x^2 = \begin{bmatrix} 2 & -3 & 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} = -0.25$$

Since $net < 0$, o^2 will be $o^2 = f(net^2) = -1$

$$\Delta w^2 = \eta f\left(net^2\right) x^2 = 1 * -1 * \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix}$$

$$w^3 = w^2 + \Delta w^2 = \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.5 \\ 3.5 \\ 2.0 \end{bmatrix}$$

Self-Organizing Map (SOM)

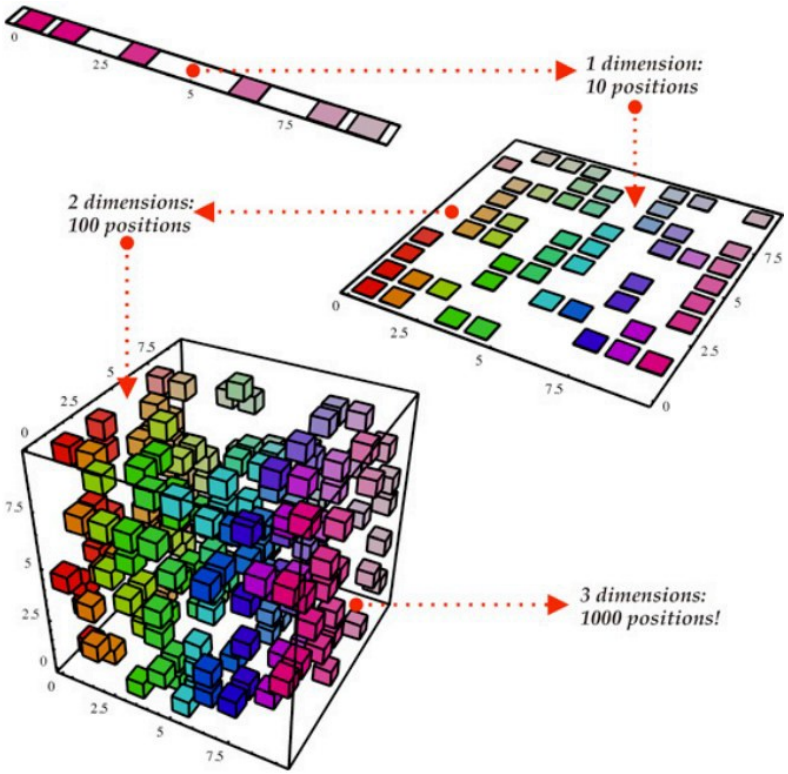
- SOM is to transform an incoming signal pattern of arbitrary dimension into one-or two-dimensional discrete map.

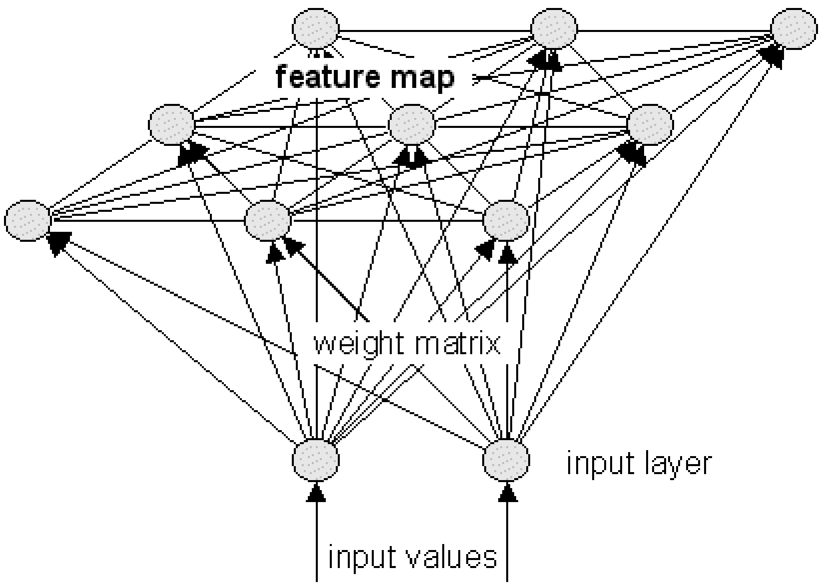
- SOM was originally invented by Kohonen in the mid 1990's
  - sometimes referred to as Kohonen Networks

- A SOM is a multi-dimensional scaling technique which constructs an approximation of the probability density function of some underlying data set and preserves the topological structure of that data set.

- This is done by mapping input vectors, in the data set , to weight vectors, neurons in the feature map.

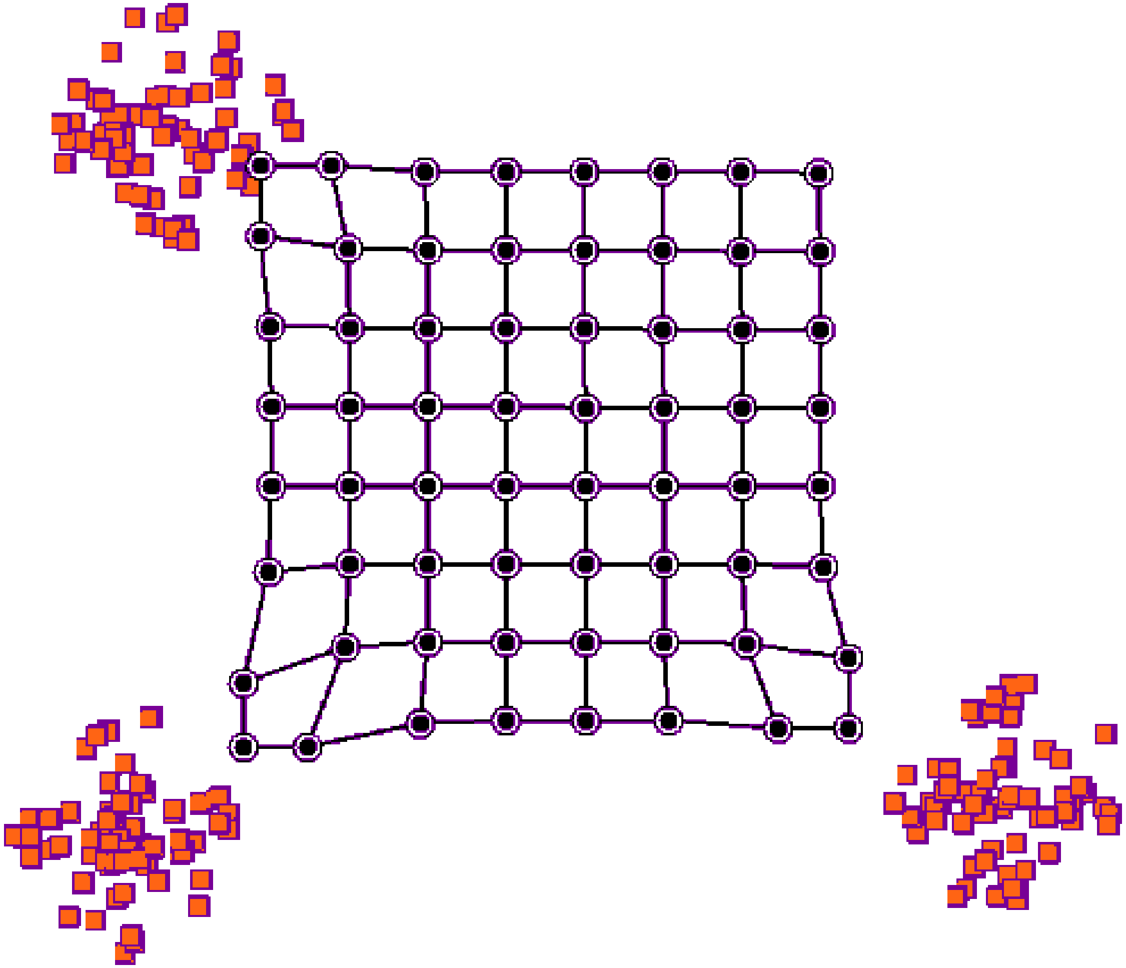
Self-Organized Map (SOM)



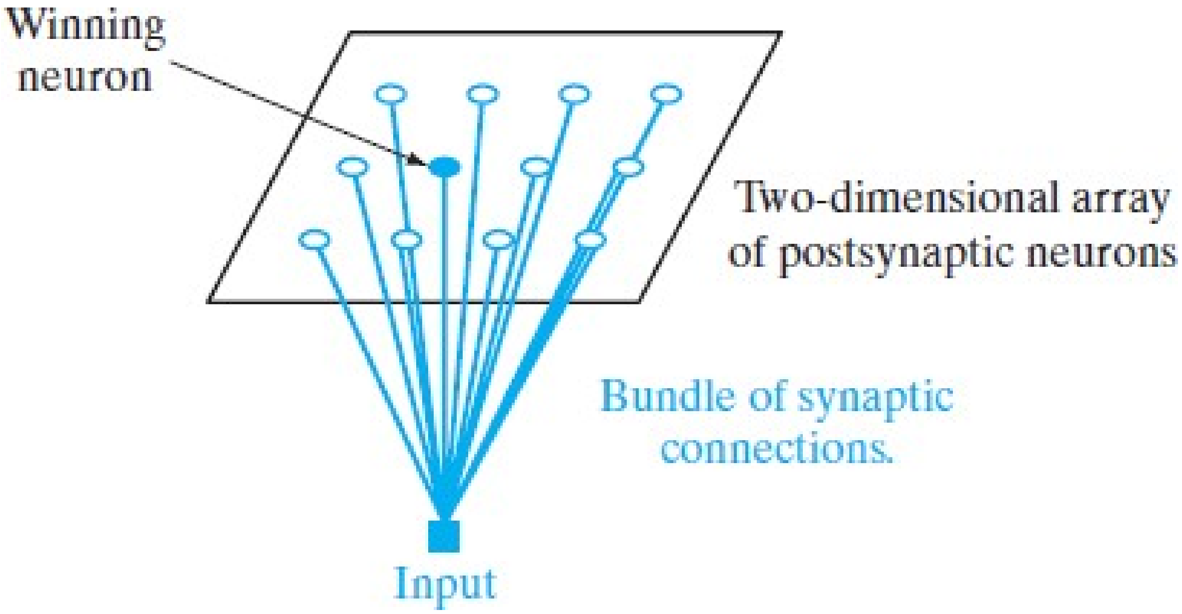


Source: Turingfinance.com

Self-Organized Map (SOM)



Kohonen Model

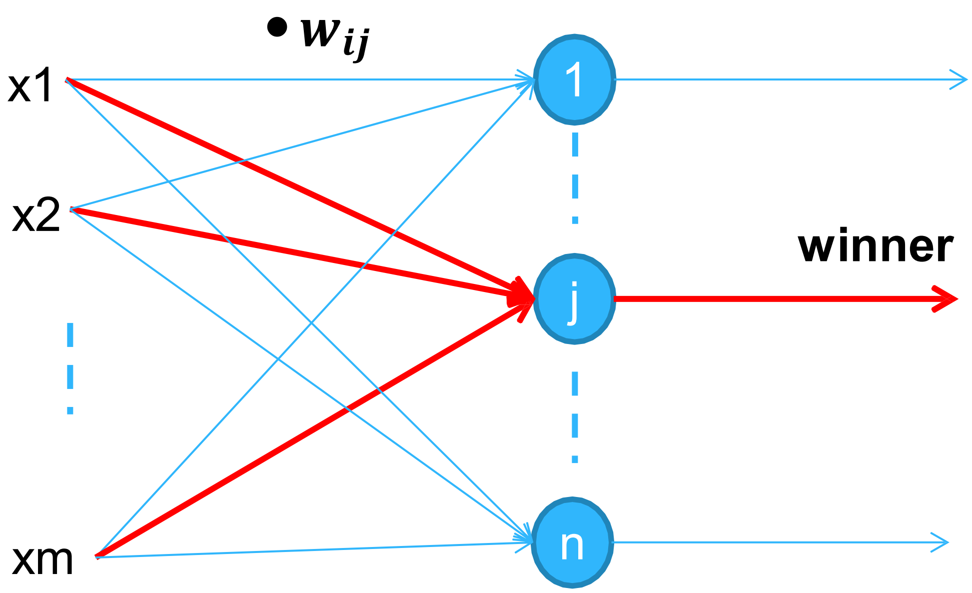


Competitive Learning

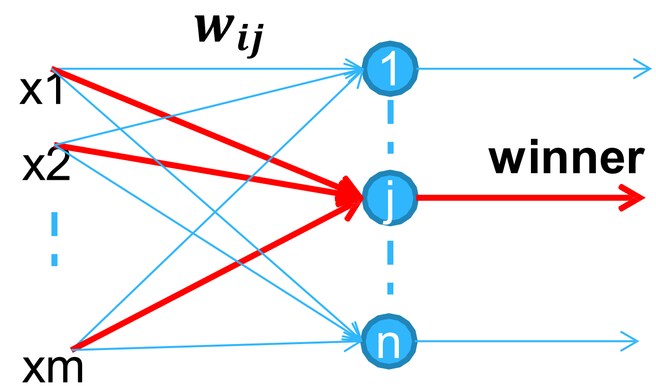
- A set of neurons are fully connected to the inputs
- Neurons compete for the best response to input patterns
- A mechanism for this competition should be decided
  - such as Euclidean distance or Mahalanobis distance
- Typically, a winner-takes-all method is used
  - The idea is one of the neurons will have the maximum response and therefore only its weights are adjusted

Winner-takes-all

- Winner-take-all algorithm works with single node in a layer of nodes that responds most strongly to the input pattern.



SOM Algorithm winner takes all



Initialize weights

Iteration Loop

Pattern Loop

Calculate Euclidean distance

$$D_j = \sum_{i=1}^{m} (x_i - w_{ij})^2 \quad for\ j = 1{:}n$$

Determine the winner neuron

$$\boldsymbol{min\ D_j}$$

Adjust the weights for the winner neuron only

$$\boldsymbol{w_{ij} = w_{ij} + \alpha(x_i - w_{ij})}$$

End Pattern Loop

End Iteration Loop

SOM Algorithm with neighborhood function

Initialize weights

Iteration Loop

Pattern Loop

Calculate Euclidean distance

$$D_j = \sum_{i=1}^{m} (x_i - w_{ij})^2 \quad for\ j = 1{:}n$$

Determine the winner neuron

$\boldsymbol{min\ D_j}$

Adjust the weights for the winner neuron only

$\boldsymbol{w_{ij} = w_{ij} + h_{ij}(x_i - w_{ij})}$

Where h_{ij} is the neighbourhood function

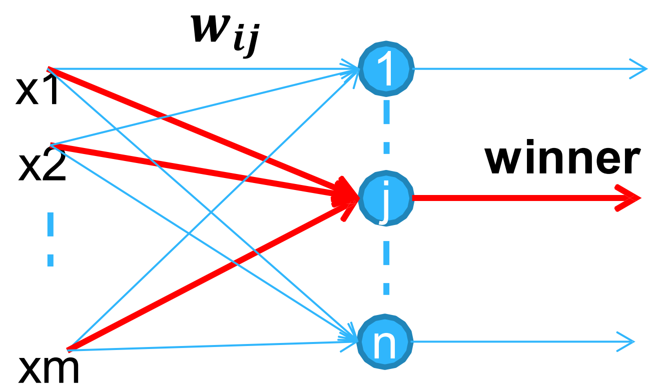
$$h_{ij} = \alpha(t) e^{\left(\frac{-||r_i - c_i||}{2\sigma^2}\right)}$$

Where $0 < \alpha(t) < 1$ *is the learning rate parameter* σ is the width of the kernel

$$\alpha(t) = 0.9\left(1 - \frac{t}{1000}\right)$$

End Pattern Loop

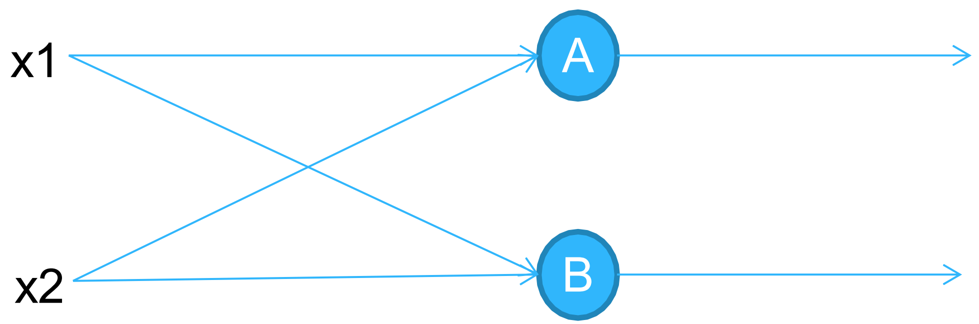
End Iteration Loop



Example

- Consider the 2-dimensional data in table below
- Use two winner-takes-all neurons to classify the data as shown in figure below

X1	X2
1.0	1.0
9.4	6.4
2.5	2.1
8.0	7.7
0.5	2.2
7.9	8.4
7.0	7.0
2.8	0.8
1.2	3.0
7.8	6.1



Initialize the neuron weights as **WA** = [5 ; 3] and **WB** = [6 ; 8]

Example

Iteration One .. Pattern One

1. **Competition:**

Calculate the distance between the input pattern and neuron A

$$D_1{}^2 = (x_1 - w_{1A})^2 + (x_2 - w_{2A})^2$$

$$D_1{}^2 = (1-5)^2 + (1-3)^2 = 20$$

Calculate the distance between the input pattern and neuron B

$$D_2{}^2 = (x_1 - w_{1B})^2 + (x_2 - w_{2B})^2$$

$$D_2{}^2 = (1-6)^2 + (1-8)^2 = 74$$

Compare

$$D_1 < D_2 \Longrightarrow \boldsymbol{winner}\ is\ neuron\ \boldsymbol{A}$$

Example

2. Update Weights

Only weights of the winner neuron are updated

$$W_A = W_A + \alpha \ (X - W_A)$$

$$W_A = \begin{bmatrix} 5 \\ 3 \end{bmatrix} + 0.5 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$W_B = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ unchanged

Example

Iteration One .. Pattern Two

1. **Competition:**

Calculate the distance between the input pattern and neuron A

$$D_1{}^2 = (x_1 - w_{1A})^2 + (x_2 - w_{2A})^2$$

$$D_1{}^2 = (9.4 - 3)^2 + (6.4 - 2)^2 = 112$$

Calculate the distance between the input pattern and neuron B

$$D_2{}^2 = (x_1 - w_{1B})^2 + (x_2 - w_{2B})^2$$

$$D_2{}^2 = (9.4 - 6)^2 + (6.4 - 8)^2 = 14$$

Compare

$$D_2 < D_1 \Longrightarrow \boldsymbol{winner} \text{ is neuron } B$$

Example

2. Update Weights

Only weights of the winner neuron are updated

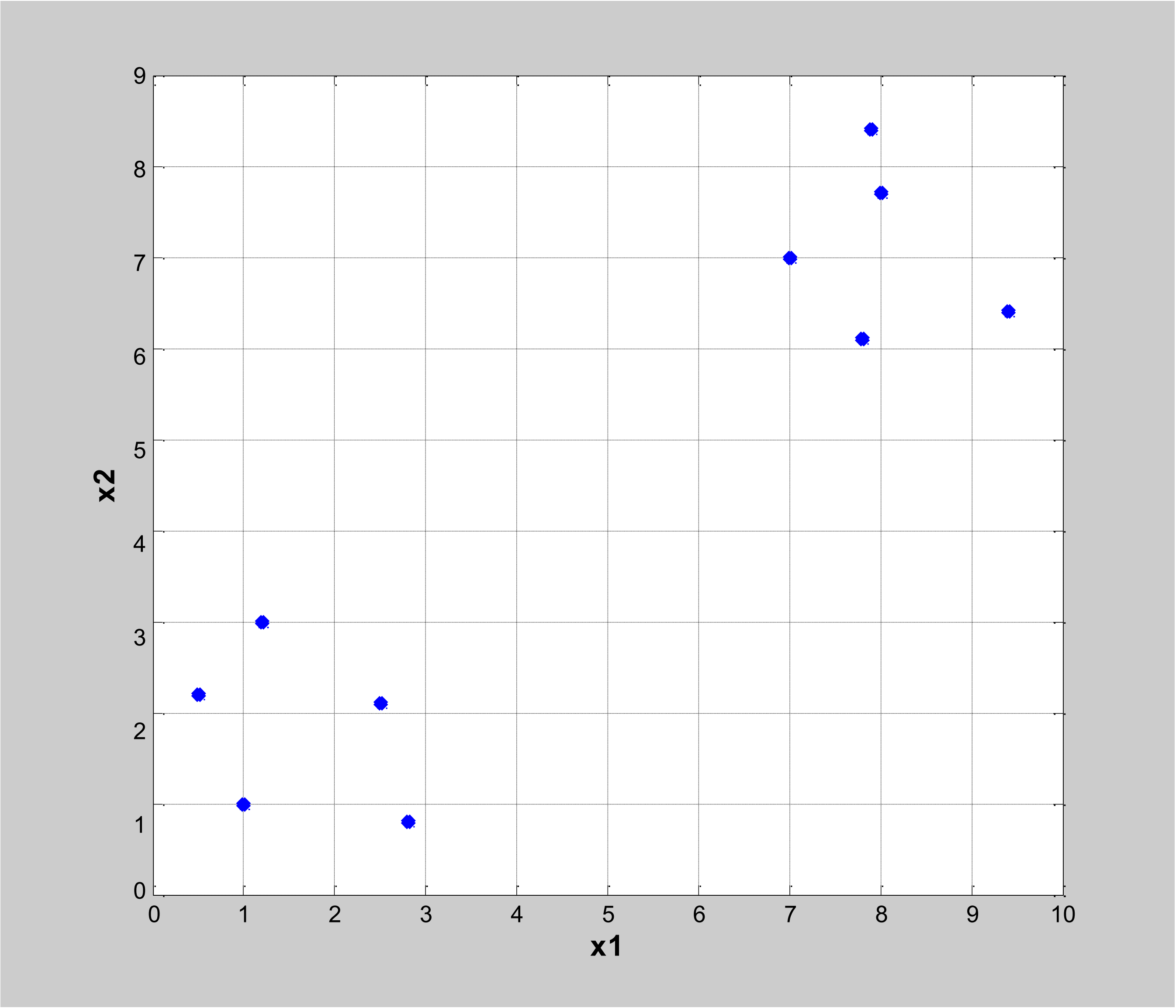
$$W_B = W_B + \alpha \ (X - W_B)$$

$$W_B = \begin{bmatrix} 6 \\ 8 \end{bmatrix} + 0.5 \left(\begin{bmatrix} 9.4 \\ 6.4 \end{bmatrix} - \begin{bmatrix} 6 \\ 8 \end{bmatrix} \right) = \begin{bmatrix} 7.7 \\ 7.2 \end{bmatrix}$$

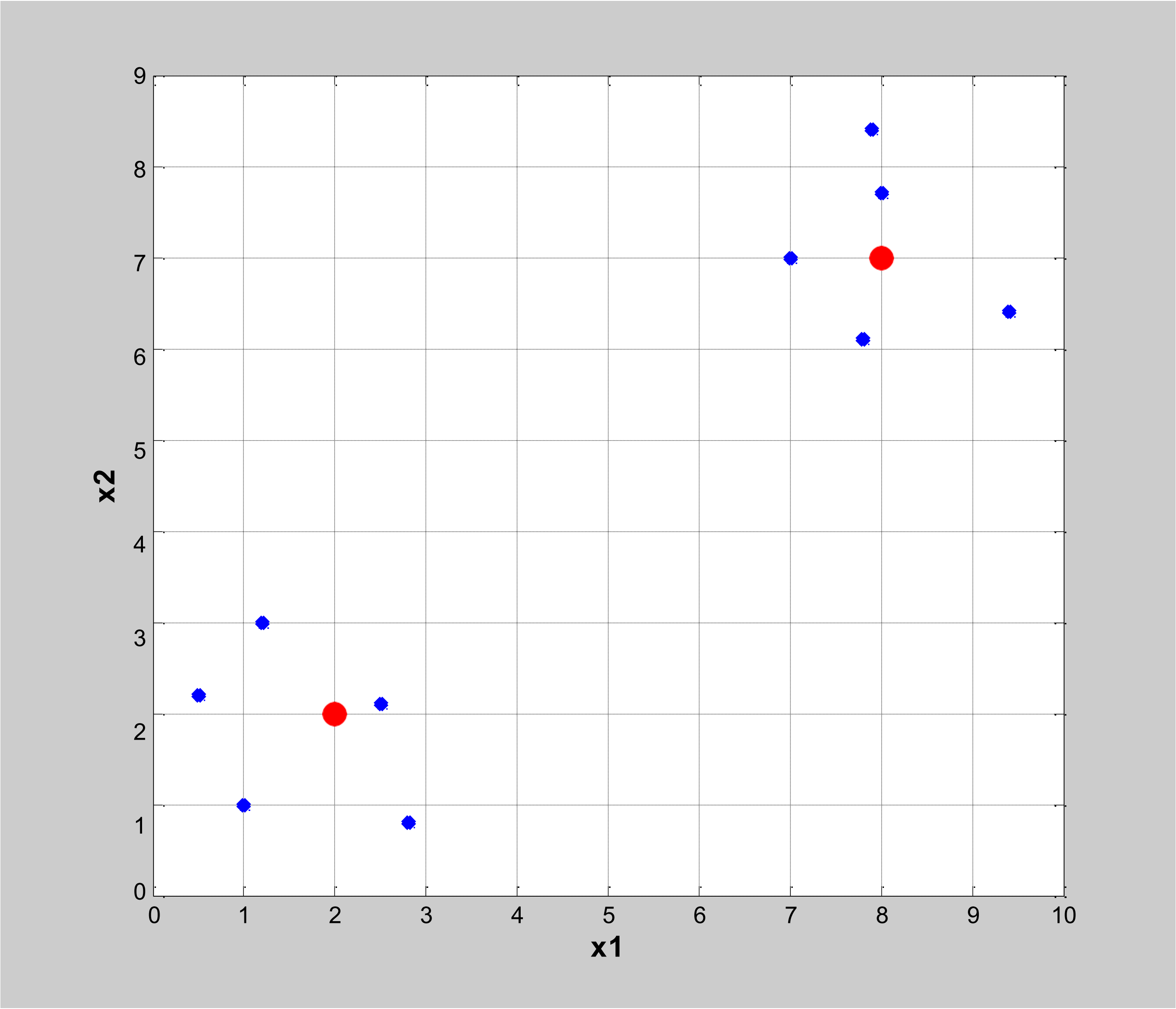
$W_A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ unchanged

Example

- Iteration One
  - Pattern 3; Competition; Update
  - Pattern 4; Competition; Update
  - …
  - Pattern 10; Competition; Update
- Iteration Two
  - Pattern 1; Competition; Update
  - …
  - Pattern 10; Competition; Update
- Iteration Three
  - …

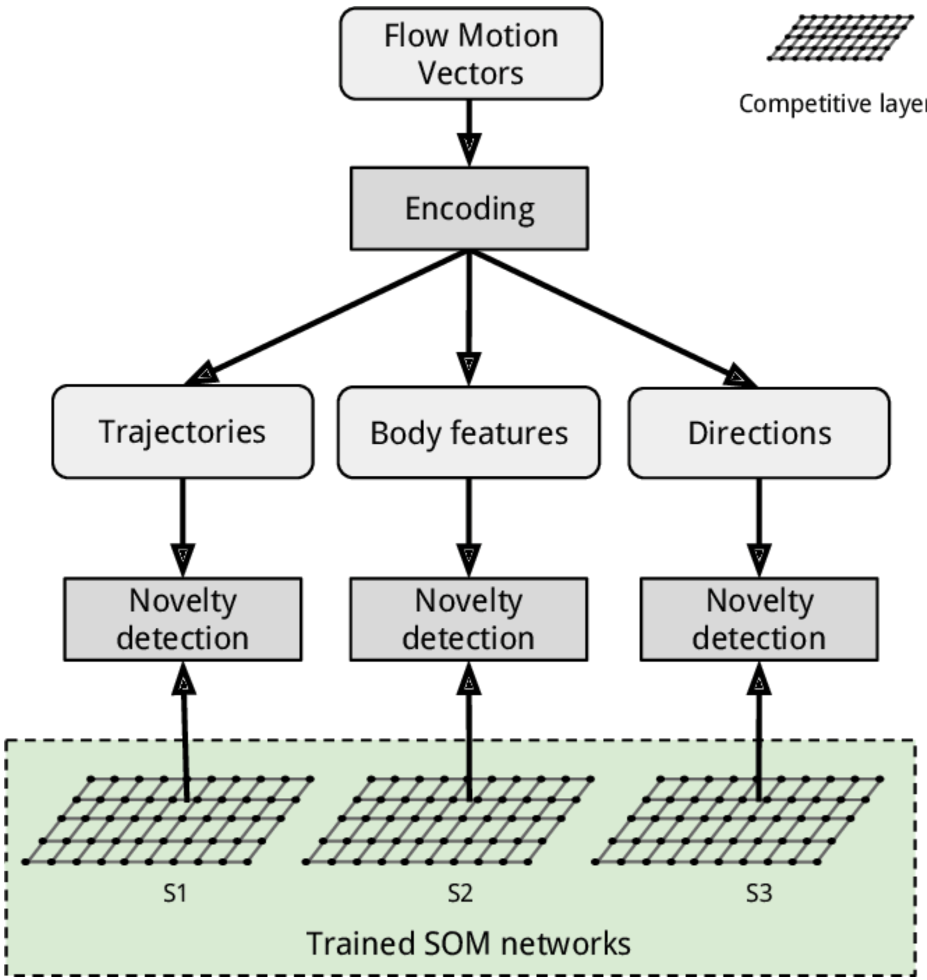


x2
x1
0
1
2
3
4
5
6
7
8
9
10

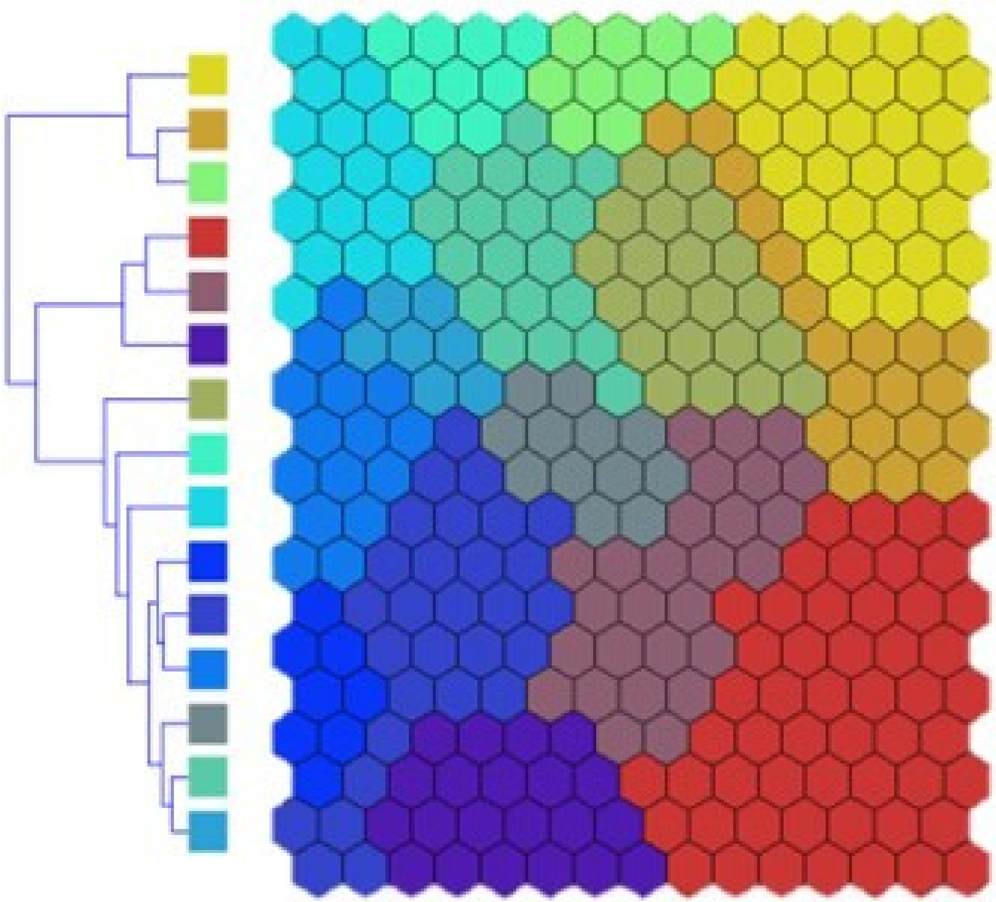


x2
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
x1

Application Example



Application Example



References

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- Neural Networks and Learning Machine (Chapter 3) by Simon Haykin 3rd Edition. Pearson 2009