Mapping from S-plane to Z-Plane
Digital control

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MAPPING THE S-PLANE INTO THE Z-PLANE

- The pole locations of a closed-loop continuous-time system in the s-plane determine:
- The behaviour and
- The stability of the system,
- We can shape the response of a system by positioning its poles in the s-plane.
- It is desirable to do the same for the sampled data systems.

MAPPING THE S-PLANE INTO THE Z-PLANE

First of all, consider the mapping of the left-hand side of the s-plane into the z-plane. Let $s = \sigma + j\omega$ describe a point in the s-plane. Then, along the $j\omega$ axis,

$$z = e^{sT} = e^{\sigma T} e^{j\omega T}.$$

But $\sigma = 0$ so we have

$$z = e^{j\omega T} = \cos\omega T + j\sin\omega T = 1\angle\omega T.$$

Hence, the pole locations on the imaginary axis in the s-plane are mapped onto the unit circle in the z-plane. As ω changes along the imaginary axis in the s-plane, the angle of the poles on the unit circle in the z-plane changes.

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If ω is kept constant and σ is increased in the left-hand s-plane, the pole locations in the z-plane move towards the origin, away from the unit circle. Similarly, if σ is decreased in the left-hand s-plane, the pole locations in the z-plane move away from the origin in the z-plane. Hence, the entire left-hand s-plane is mapped into the interior of the unit circle in the z-plane. Similarly, the right-hand s-plane is mapped into the exterior of the unit circle in the z-plane. As far as the system stability is concerned, a sampled data system will be stable if the closed-loop poles (or the zeros of the characteristic equation) lie within the unit circle.

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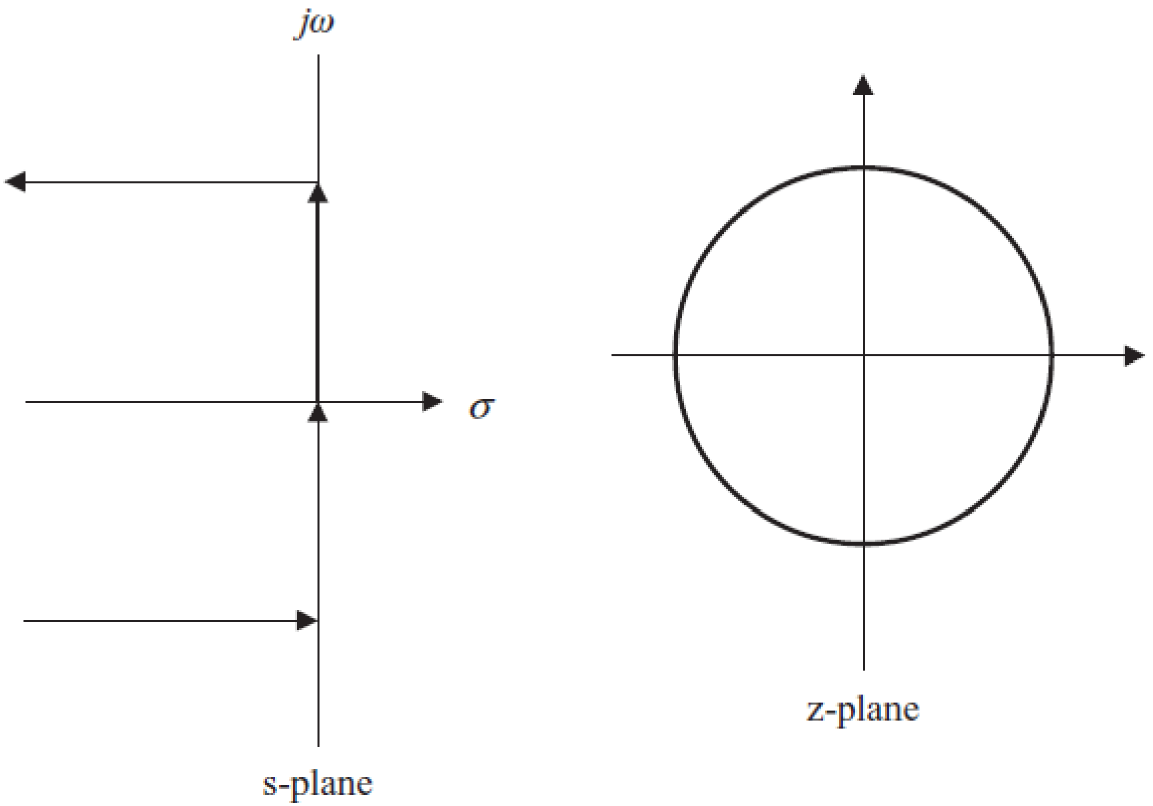

Figure 7.5 Mapping the left-hand *s*-plane into the *z*-plane

MAPPING THE S-PLANE INTO THE Z-PLANE

As shown in Figure 7.6, lines of constant σ in the s-plane are mapped into circles in the z-plane with radius $e^{\sigma T}$. If the line is on the left-hand side of the s-plane then the radius of the circle in the z-plane is less than 1. If on the other hand the line is on the right-hand side of the s-plane then the radius of the circle in the z-plane is greater than 1. Figure 7.7 shows the corresponding pole locations between the s-plane and the z-plane.

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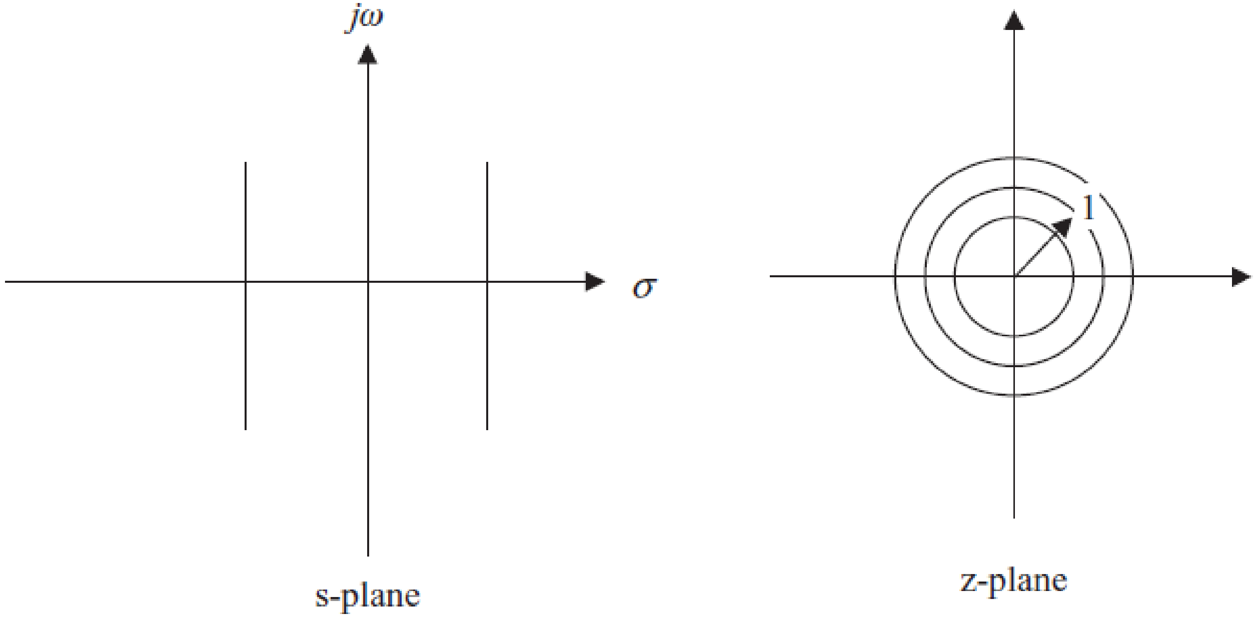

Figure 7.6 Mapping the lines of constant σ

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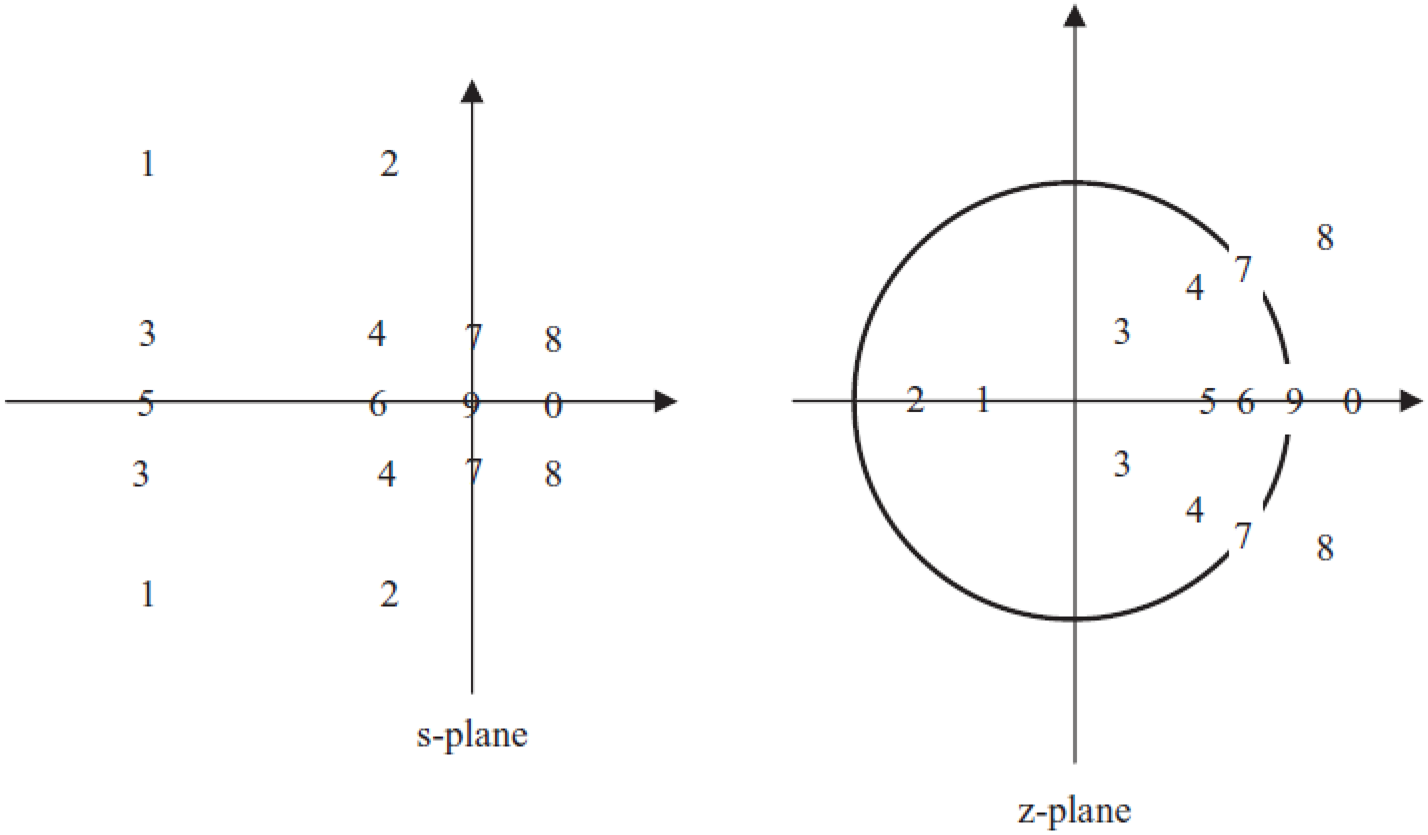

Mapping S-plane to Z-plane

- The time responses of a sampled data system based on its pole positions in the z-plane are shown in Figure 7.8.
- It is clear from this figure that the system is stable if all the closed-loop poles are within the unit circle.

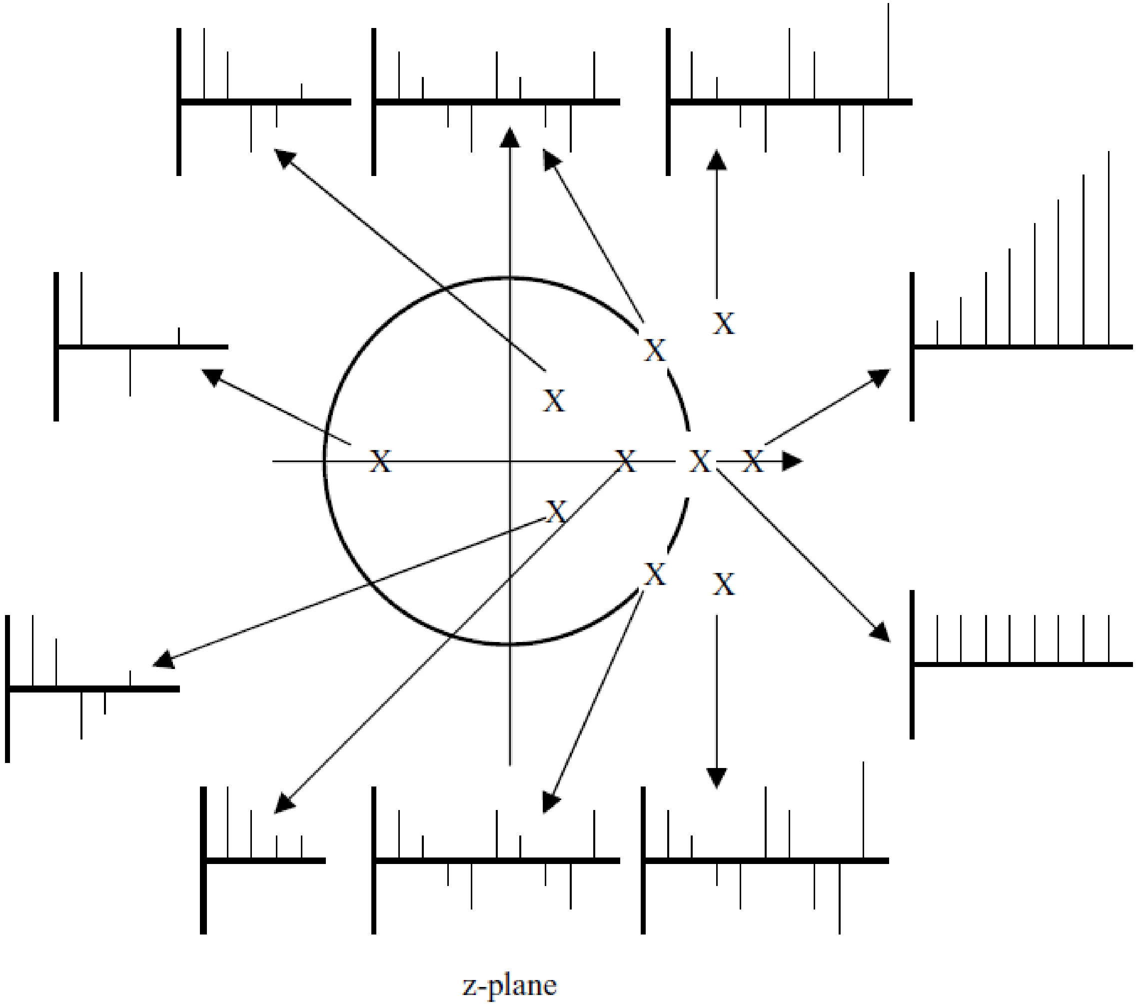

Figure 7.8 Time response of z-plane pole locations