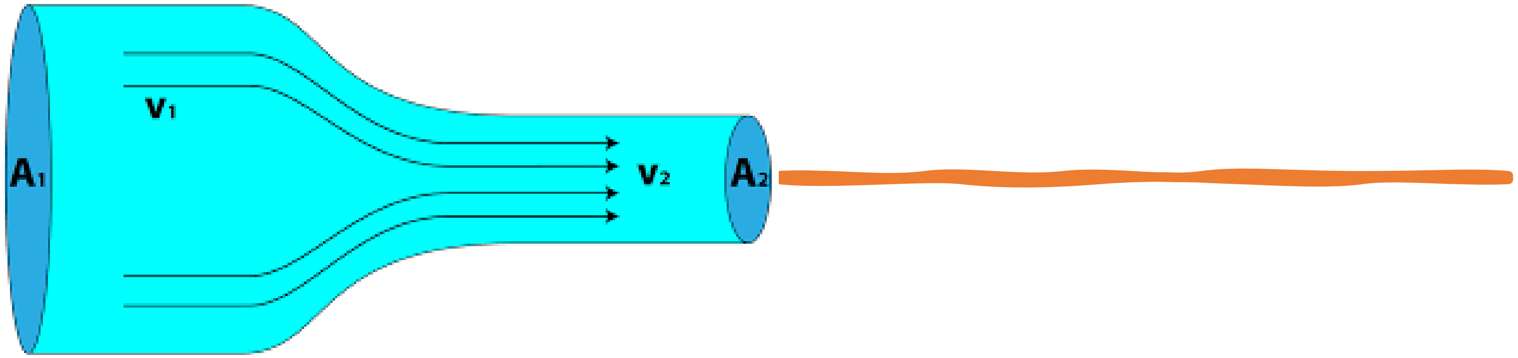
Pneumatics and hydraulics

Energy and Power in Hydraulic Systems part 2



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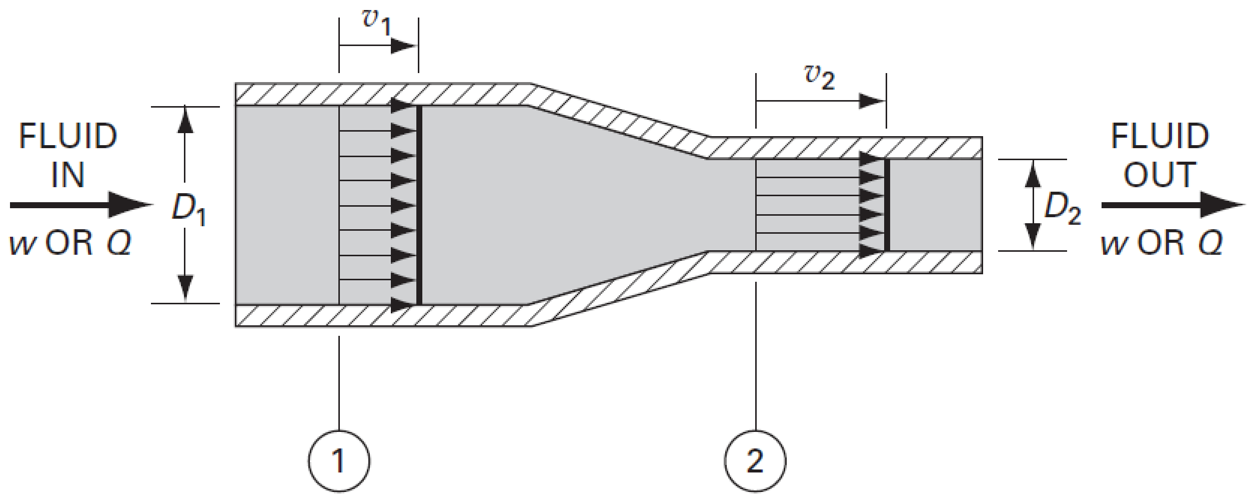
Outline

- THE CONTINUITY EQUATION
- HYDRAULIC POWER
- BERNOULLI’S EQUATION
- ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM
- EXAMPLES USING THE SI METRIC SYSTEM

The Continuity equation

The continuity equation states that for steady flow in a pipeline, the weight flow rate (weight of fluid passing a given station per unit time) is the same for all locations of the pipe.



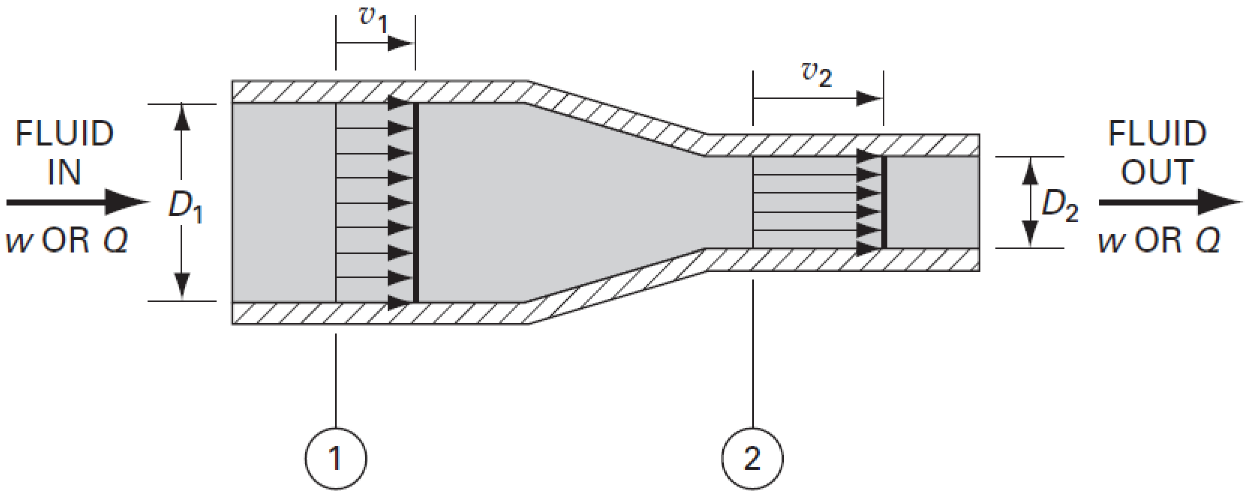
$$\gamma_1 A_1 v_1 = \gamma_2 A_2 v_2$$

where γ = specific weight of fluid (lb/ft³),
A = cross-sectional area of pipe (ft²),
υ = velocity of fluid (ft/s).

Assuming the fluid is incompressible we have $\gamma_1 = \gamma_2$ →

$$Q_1 = A_1 v_1 = A_2 v_2 = Q_2$$

The Continuity equation



The continuity equation for hydraulic systems can be rewritten as follows:

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{(\pi/4)D_2^2}{(\pi/4)D_1^2}$$

where D_1 and D_2 are the pipe diameters at stations 1 and 2, respectively. The final result is

$$\frac{v_1}{v_2} = \left(\frac{D_2}{D_1}\right)^2 \qquad \textbf{(3-19)}$$

Equation (3-19) shows that the smaller the pipe size, the greater the velocity, and vice versa. It should be noted that the pipe diameters and areas are inside values and do not include the pipe wall thickness.

The Continuity equation

EXAMPLE 3-7

For the pipe in Figure 3-23, the following data are given:

$$D_1 = 4 \text{ in} \quad D_2 = 2 \text{ in}$$

$$v_1 = 4 \text{ ft/s}$$

Find:

a. The volume flow rate Q

b. The fluid velocity at station 2

The Continuity equation

Solution

a.

$$Q = Q_1 = A_1 v_1$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}\left(\frac{4}{12}\text{ ft}\right)^2 = 0.0873 \text{ ft}^2$$

$$Q = (0.0873 \text{ ft}^2)(4 \text{ ft/s}) = 0.349 \text{ ft}^3/\text{s}$$

b. Solving Eq. (3-19) for v_2, we have

$$v_2 = v_1\left(\frac{D_1}{D_2}\right)^2 = 4\left(\frac{4}{2}\right)^2 = 16 \text{ ft/s}$$

HYDRAULIC POWER: Hydraulic Cylinder Example

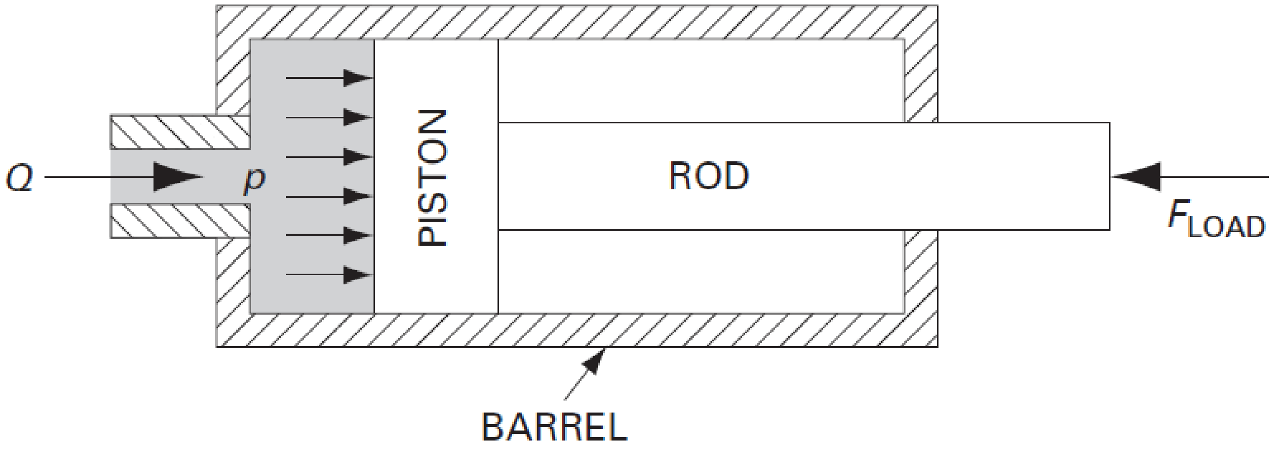


Figure 3-24. Cylinder example for determining hydraulic horsepower.

1. How do we determine how large a piston diameter is required for the cylinder?
2. What is the pump flow rate required to drive the cylinder through its stroke in a specified time?
3. How much hydraulic horsepower does the fluid deliver to the cylinder?

HYDRAULIC POWER: Hydraulic Cylinder Example

Answer to Question 1. A pump receives fluid on its inlet side at about atmospheric pressure (0 psig) and discharges the fluid on the outlet side at some elevated pressure p sufficiently high to overcome the load. This pressure p acts on the area of the piston A to produce the force required to overcome the load:

$$pA = F_{\text{load}}$$

Solving for the piston area A, we obtain

$$A = \frac{F_{\text{load}}}{p} \tag{3-20}$$

The load is known from the application, and the maximum allowable pressure is established based on the pump design. Thus, Eq. (3-20) allows us to calculate the required piston area if the friction between the piston and cylinder bore is negligibly small.

HYDRAULIC POWER: Hydraulic Cylinder Example

Answer to Question 2. The volumetric displacement V_D of the hydraulic cylinder equals the fluid volume swept out by the piston traveling through its stroke S:

$$V_D(\text{ft}^3) = A(\text{ft}^2) \times S(\text{ft})$$

If there is negligibly small leakage between the piston and cylinder bore, the required pump volume flow rate Q equals the volumetric displacement of the cylinder divided by the time t required for the piston to travel through its stroke.

$$Q(\text{ft}^3/\text{s}) = \frac{V_D(\text{ft}^3)}{t(\text{s})}$$

but since $V_D = AS$, we have

$$Q(\text{ft}^3/\text{s}) = \frac{A(\text{ft}^2) \times S(\text{ft})}{t(\text{s})} \qquad \textbf{(3-21)}$$

HYDRAULIC POWER: Hydraulic Cylinder Example

Since stroke S and time t are basically known from the particular application, Eq. (3-21) permits the calculation of the required pump flow.

Recall that for a pipe we determined that $Q = Av$ where v equals the fluid velocity. Shouldn't we obtain the same equation for a hydraulic cylinder since it is essentially a pipe that contains a piston moving at velocity v? The answer is yes, as can be verified by noting that S/t can be replaced by v in Eq. (3-21) to obtain the expected result:

$$Q(\text{ft}^3/\text{s}) = A(\text{ft}^2) \times v(\text{ft/s}) \qquad \textbf{(3-22)}$$

where v = piston velocity.

Note that the larger the piston area and velocity, the greater must be the pump flow rate.

HYDRAULIC POWER: Hydraulic Cylinder Example

Answer to Question 3. It has been established that energy equals force times distance:

$$\text{energy} = (F)(S) = (pA)(S)$$

Since power is the rate of doing work, we have

$$\text{power} = \frac{\text{energy}}{\text{time}} = \frac{(pA)(S)}{t} = p(Av)$$

Since $Q = Av$, the final result is

$$\text{hydraulic power (ft} \cdot \text{lb/s)} = p(\text{lb/ft}^2) \times Q(\text{ft}^3/\text{s}) \qquad \textbf{(3-23)}$$

Recalling that 1 hp = 550 ft · lb/s, we obtain

$$\text{hydraulic horsepower} = \text{HHP} = p(\text{lb/ft}^2) \times Q(\text{ft}^3/\text{s}) \times \frac{1\ \text{hp}}{550\ \text{ft} \cdot \text{lb/s}}$$

Hydraulic Horsepower in Terms of psi and gpm Units

Pressure in psi (lb/in²) and flow rate in gallons per minute (gpm) are the most common English units used for hydraulic systems. Thus a hydraulic horsepower equation using these most common English units is developed as follows, noting that hydraulic power equals the product of pressure and volume flow rate:

$$\text{hydraulic power} = p \times Q$$

$$= p\left(\frac{\text{lb}}{\text{in}^2}\right) \times Q\left(\frac{\text{gal}}{\text{min}}\right) \times \frac{231\ \text{in}^3}{1\ \text{gal}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{1\ \text{ft}}{12\ \text{in}} \times \frac{1\ \text{hp}}{550\ \text{ft} \cdot \text{lb/s}}$$

Since all the units cancel out except hp, we have the final result:

$$\text{HHP} = \frac{p(\text{psi}) \times Q(\text{gpm})}{1714} \qquad \textbf{(3-25)}$$

Note that the constants 550 and 1714 in Eqs. (3-24) and (3-25) contain the proper units to make these two equations have units of hp on the right side of the equal sign.

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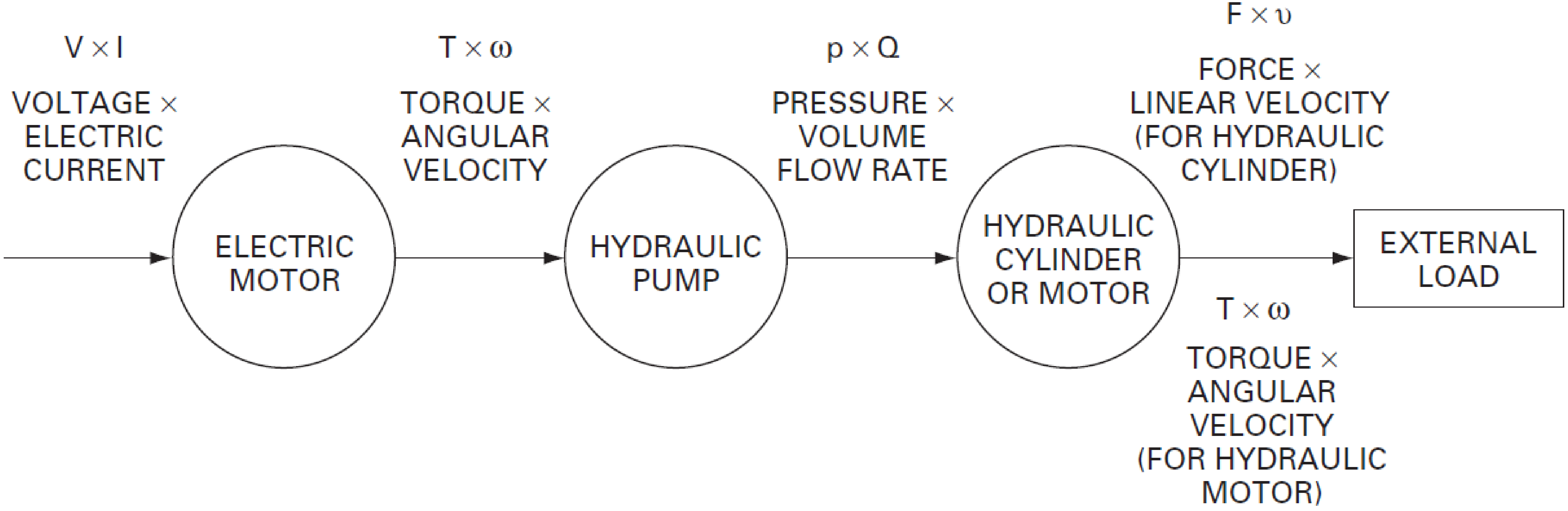
Hydraulic Horsepower in Terms of psi and gpm Units

Observe the following power analogy among mechanical, electrical, and hydraulic systems:

$$\text{mechanical power} = \text{force} \times \text{linear velocity}$$
$$= \text{torque} \times \text{angular velocity}$$
$$\text{electrical power} = \text{voltage} \times \text{electric current}$$
$$\text{hydraulic power} = \text{pressure} \times \text{volume flow rate}$$



Hydraulic Horsepower in Terms of psi and gpm Units

EXAMPLE 3-8

A hydraulic cylinder is to compress a car body down to bale size in 10 s. The operation requires a 10-ft stroke and a 8000-lb force. If a 1000-psi pump has been selected, and assuming the cylinder is 100% efficient, find

a. The required piston area
b. The necessary pump flow rate
c. The hydraulic horsepower delivered to the cylinder
d. The output horsepower delivered by the cylinder to the load

Solution

a.

$$A = \frac{F_{\text{load}}}{p} = \frac{8000 \text{ lb}}{1000 \text{ lb/in}^2} = 8 \text{ in}^2$$

b.

$$Q(\text{ft}^3/\text{s}) = \frac{A(\text{ft}^2) \times S(\text{ft})}{t(\text{s})} = \frac{\left(\frac{8}{144}\right)(10)}{10} = 0.0556 \text{ ft}^3/\text{s}$$

Hydraulic Horsepower in Terms of psi and gpm Units

Per Appendix E, 1 ft^3/s = 449 gpm. Thus,

$$Q(\text{gpm}) = 449Q(\text{ft}^3/\text{s}) = (449)(0.0556) = 24.9 \text{ gpm}$$

c. $$\text{HHP} = \frac{(1000)(24.9)}{1714} = 14.5 \text{ hp}$$

d. $$\text{OHP} = \text{HHP} \times \eta = 14.5 \times 1.0 = 14.5 \text{ hp}$$

Thus, assuming a 100% efficient cylinder (losses are negligibly small), the hydraulic horsepower equals the output horsepower.

Hydraulic Horsepower in Terms of psi and gpm Units

Example 3-9

Solve the problem of Example 3-8 assuming a frictional force of 100 lb and a leakage of 0.2 gpm.

Hydraulic Horsepower in Terms of psi and gpm Units

Solution

a. $$A = \frac{F_{\text{load}} + F_{\text{friction}}}{p} = \frac{8000\text{ lb} + 100\text{ lb}}{1000\text{ lb/in}^2} = 8.10\text{ in}^2$$

b. $$Q_{\text{theoretical}} = \frac{A(\text{ft}^2) \times S(\text{ft})}{t(\text{s})} = \frac{\left(\frac{8.10}{144}\right)(10)}{10} = 0.0563\text{ ft}^3/\text{s} = 25.2\text{ gpm}$$

$$Q_{\text{actual}} = Q_{\text{theoretical}} + Q_{\text{leakage}} = 25.2 + 0.2 = 25.4\text{ gpm}$$

c. $$\text{HHP} = \frac{1000 \times 25.4}{1714} = 14.8\text{ hp}$$

d. $$\text{OHP} = \frac{F(\text{lb}) \times v(\text{ft/s})}{550} = \frac{8000 \times 1}{550} = 14.5\text{ hp}$$

Thus, a hydraulic horsepower of 14.8 must be delivered by the fluid to the cylinder to produce an output horsepower at 14.5 for driving the load. The efficiency of the cylinder is

$$\eta = \frac{\text{OHP}}{\text{HHP}} = \frac{14.5}{14.8} = 0.980 = 98.0\%$$

BERNOULLI'S EQUATION

the total energy possessed by the *W* lb of fluid at station 1 equals the total energy possessed by the same *W* lb of fluid at station 2, provided frictional losses are negligibly small:

$$WZ_1 + W\frac{p_1}{\gamma} + \frac{Wv_1^2}{2g} = WZ_2 + W\frac{p_2}{\gamma} + \frac{Wv_2^2}{2g}$$

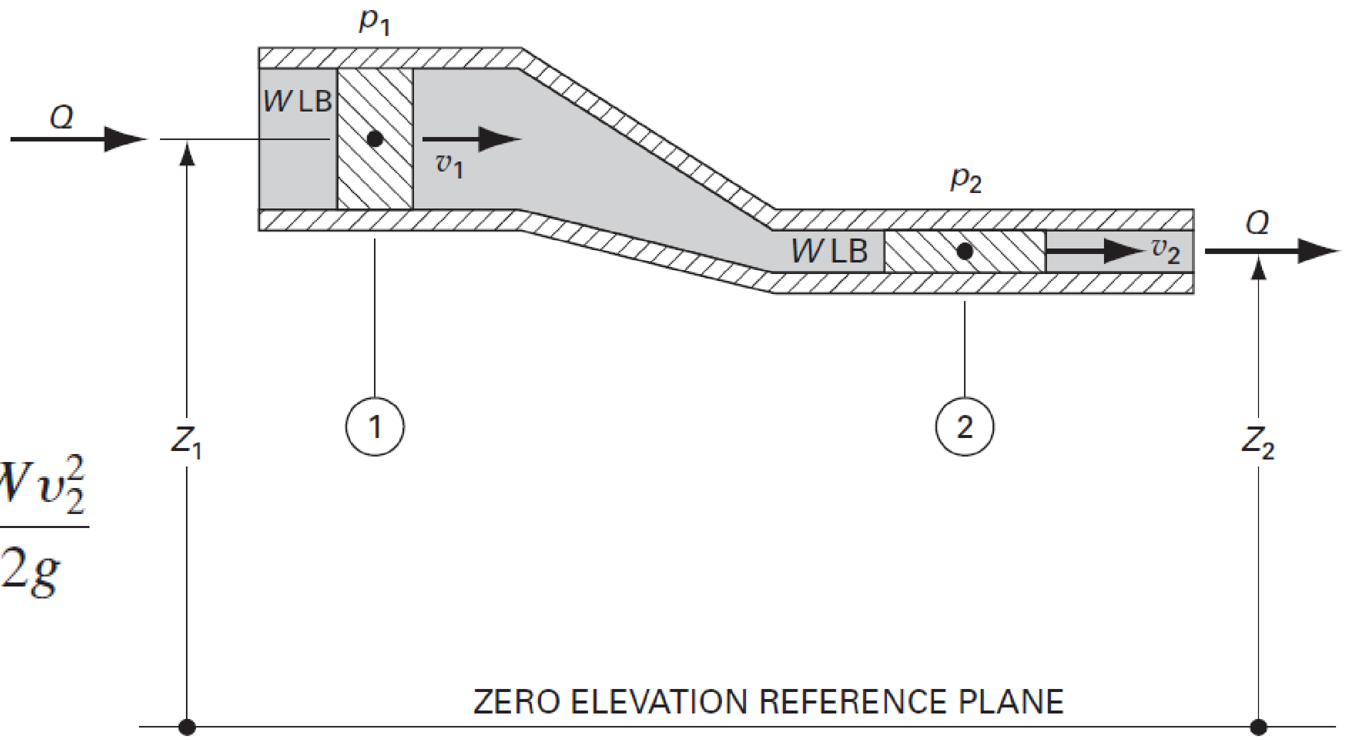


Figure 3-26. Pipeline for deriving Bernoulli's equation.

BERNOULLI'S EQUATION

$$Z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

$$Z = \text{ft}$$

$$\frac{p}{\gamma} = \frac{\text{lb/ft}^2}{\text{lb/ft}^3} = \text{ft}$$

$$\frac{v^2}{2g} = \frac{(\text{ft/s})^2}{\text{ft/s}^2} = \text{ft}$$

Z is called *elevation head.*

p/γ is called *pressure head.*

$v^2/2g$ is called *velocity head.*

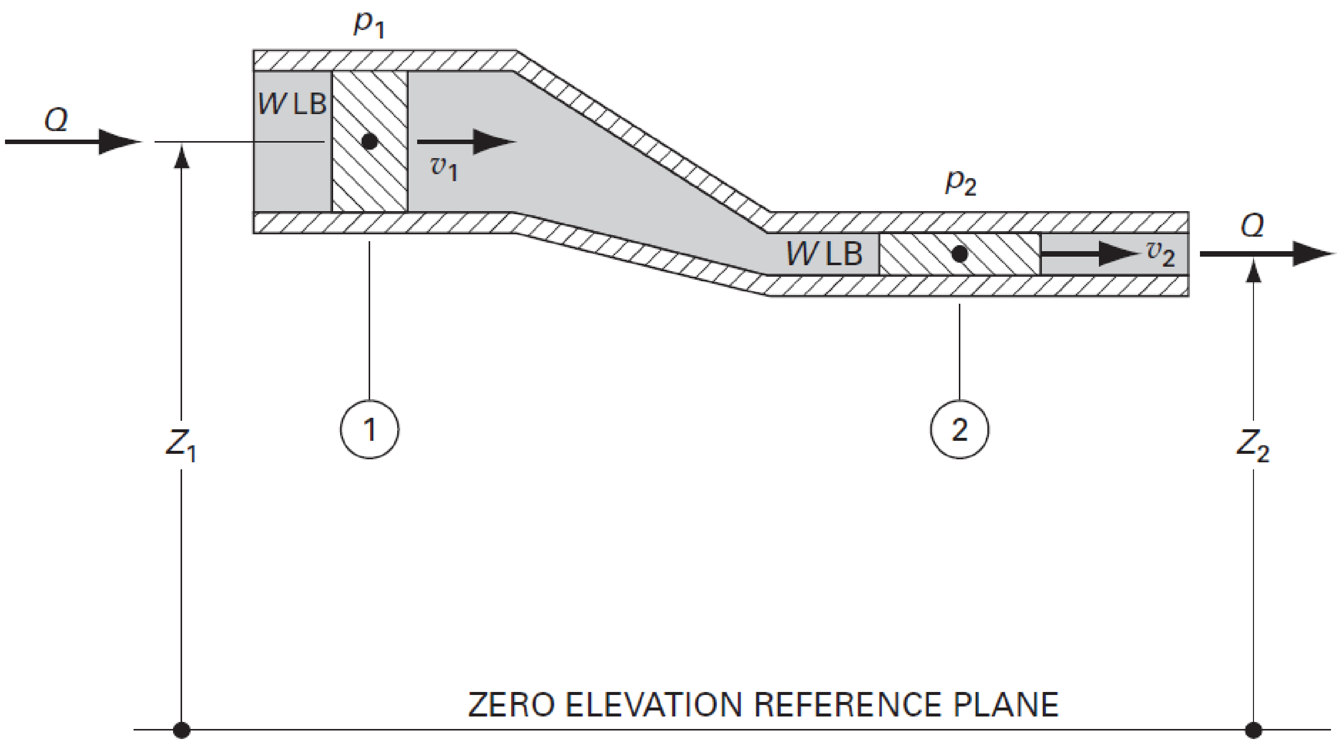


Figure 3-26. Pipeline for deriving Bernoulli's equation.

The Energy Equation

Bernoulli modified his original equation to take into account that frictional losses (H_L) take place between stations 1 and 2. H_L (called head loss) represents the energy per pound of fluid loss due to friction in going from station 1 to station 2. In addition, he took into account that a pump (which adds energy to fluid) or a hydraulic motor (which removes energy from fluid) may exist between stations 1 and 2. H_P (pump head) represents the energy per pound of fluid added by a pump, and H_m (motor head) represents the energy per pound of fluid removed by a hydraulic motor.

The energy equation is given as follows, where each term represents a head and thus has units of length:

$$Z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} \qquad \textbf{(3-28)}$$

The Energy Equation

$$H_p(\text{ft}) = \frac{3950 \times (\text{HHP})}{Q(\text{gpm}) \times \text{SG}} \qquad \textbf{(3-29)}$$

The motor head can also be calculated using Eq. (3-29), where the H_p term is replaced by H_m. The HHP and Q terms then represent the motor hydraulic horsepower and gpm flow rate, respectively.

BERNOULLI’S EQUATION: example

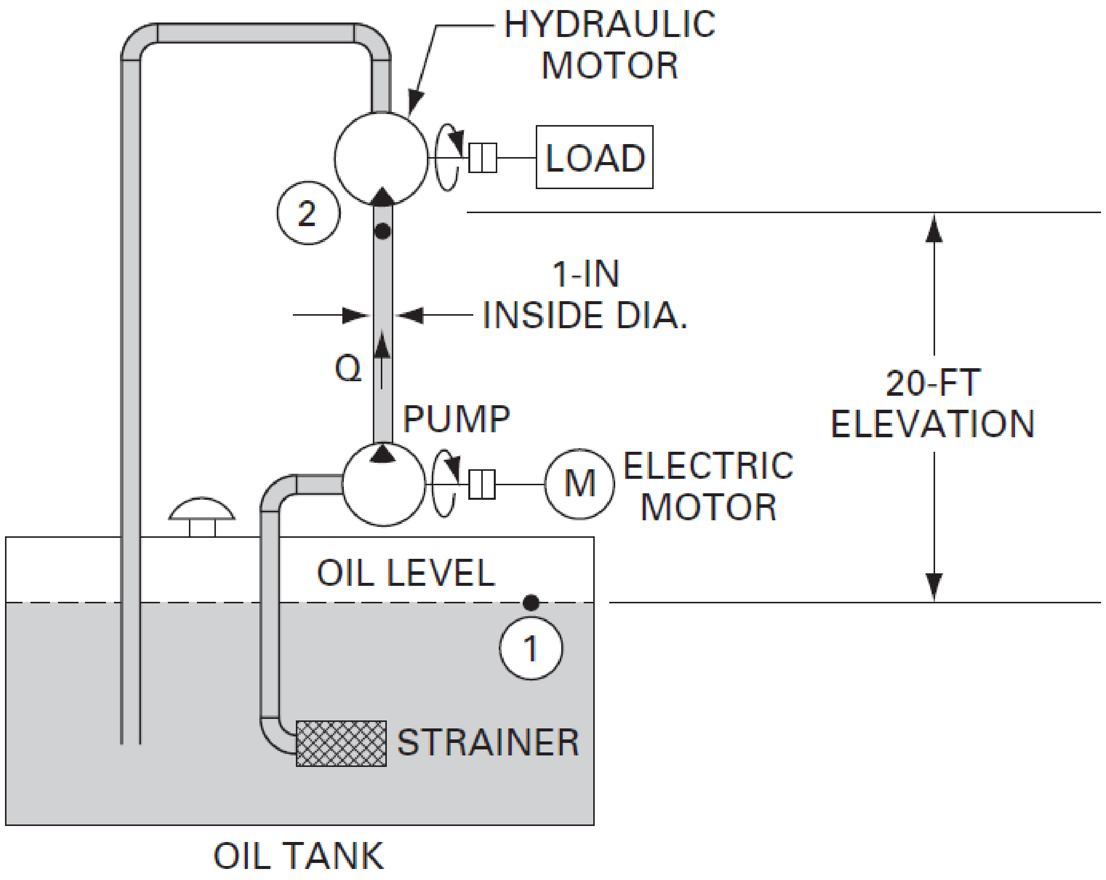


Figure 3-29. Hydraulic system for Example 3-10.

BERNOULLI'S EQUATION: example

EXAMPLE 3-10

For the hydraulic system of Figure 3-29, the following data are given:

a. The pump is adding 5 hp to the fluid (pump hydraulic horsepower = 5).
b. Pump flow is 30 gpm.
c. The pipe has a 1-in. inside diameter.
d. The specific gravity of the oil is 0.9.

Find the pressure available at the inlet to the hydraulic motor (station 2). The pressure at station 1 in the hydraulic tank is atmospheric (0 psig). The head loss H_L due to friction between stations 1 and 2 is 30 ft of oil.

Solution Writing the energy equation between stations 1 and 2, we have

$$Z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Since there is no hydraulic motor between stations 1 and 2, $H_m = 0$. Also, $v_1 = 0$ because the cross section of an oil tank is large. Thus, the velocity of the

BERNOULLI'S EQUATION: example

oil surface is negligible, and the value of υ_1 approaches zero. Per Figure 3-29, $Z_2 - Z_1 = 20$ ft. Also, $H_L = 30$ ft, and $p_1 = 0$ gage pressure per the given input data.

Substituting known values, we have

$$Z_1 + 0 + 0 + H_p - 0 - 30 = Z_2 + \frac{p_2}{\gamma} + \frac{\upsilon_2^2}{2g}$$

Solving for p_2/γ, we have

$$\frac{p_2}{\gamma} = (Z_1 - Z_2) + H_p - \frac{\upsilon_2^2}{2g} - 30$$

Since $Z_2 - Z_1 = 20$ ft, we have

$$\frac{p_2}{\gamma} = H_p - \frac{\upsilon_2^2}{2g} - 50$$

Using Eq. (3-29) yields

$$H_p = \frac{(3950)(5)}{(30)(0.9)} = 732 \text{ ft}$$

BERNOULLI'S EQUATION: example

Then, solve for $v^2_2/2g$ as follows

$$Q(\text{ft}^3/\text{s}) = \frac{Q(\text{gpm})}{449} = \frac{30}{449} = 0.0668$$

$$v_2(\text{ft/s}) = \frac{Q(\text{ft}^3/\text{s})}{A(\text{ft}^2)}$$

$$A(\text{ft}^2) = \frac{\pi}{4}(D\ \text{ft})^2 = \frac{\pi}{4}\left(\frac{1}{12}\ \text{ft}\right)^2 = 0.00546\ \text{ft}^2$$

$$v_2 = \frac{0.0668}{0.00546}\frac{\text{ft}^3/\text{s}}{\text{ft}^2} = 12.2\ \text{ft/s}$$

$$\frac{v_2^2}{2g} = \frac{(12.2\ \text{ft/s})^2}{64.4\ \text{ft/s}^2} = 2.4\ \text{ft}$$

BERNOULLI'S EQUATION: example

On final substitution, we have

$$\frac{p_2}{\gamma} = 732 - 2.4 - 50 = 679.2 \text{ ft}$$

Solving for p_2 yields

$$p_2(\text{lb/ft}^2) = (679.2 \text{ ft})\ \gamma\ (\text{lb/ft}^3)$$

where $\gamma = (\text{SG})\ \gamma_{\text{water}}$,

$\gamma = (0.9)(62.4) = 56.2 \text{ lb/ft}^3$,

$p_2 = (679.2)(56.2) = 38{,}200 \text{ lb/ft}^2$.

Changing to units of psi yields

$$p_2 = \frac{38{,}200}{144} = 265 \text{ psig}$$

The answer, 265 obtained for p_2 is a gage pressure since a gage pressure value of zero was used in the energy equation for p_1.

ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM

Energy

In the SI system, the joule (J) is the work done when a force of 1 N acts through a distance of 1 m. Since work equals force times distance, we have

$$1\ \text{J} = 1\ \text{N} \times 1\ \text{m} = 1\ \text{N} \cdot \text{m}$$

Thus, we have

$$\text{energy (J)} = F(\text{N}) \times S(\text{m}) \qquad \textbf{(3–33)}$$

ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM

Power

Power is the rate of doing work. In the SI system, 1 watt (W) of power is the rate of 1 J of work per second:

$$\text{power} = \frac{\text{work}}{\text{time}}$$

$$1\ \text{W} = \frac{1\ \text{J}}{\text{s}} = 1\text{N} \cdot \text{m/s}$$

Thus, we have

$$\text{power (W)} = \frac{\text{work(N} \cdot \text{m)}}{\text{time(s)}} \quad \textbf{(3-34)}$$

$$\text{hydraulic power (W)} = p(\text{N/m}^2) \times Q(\text{m}^3/\text{s}) \quad \textbf{(3-35)}$$

ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM

In the SI metric system all forms of power are expressed in watts. Appendix E, which provides tables of conversion factors, shows that 1 hp = 746 W = 0.746 kW.

Pump head H_p in units of meters can be related to pump power in units of watts by using Eq. (2-9): $p = \gamma H$. Thus, we have $p = \gamma H_p$. Substituting $p = \gamma H_p$ into Eq. (3-35) yields

$$H_p(\text{m}) = \frac{\text{pump hydraulic power (W)}}{\gamma(\text{N/m}^3) \times Q(\text{m}^3/\text{s})} \qquad \textbf{(3-36)}$$

Equation (3-36) can be used to solve for the pump head for use in the energy equation. The motor head can also be calculated using Eq. (3-36), where the H_p term is replaced by H_m. The pump hydraulic power is replaced by the motor hydraulic power and Q represents the motor flow rate.

ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM

The mechanical output power (brake power or torque power) delivered by a hydraulic motor can be found from Eq. (3-37), where T is torque and ω or N is angular speed.

$$\text{power(kW)} = \frac{T(\text{N}\cdot\text{m}) \times \omega(\text{rad/s})}{1000} = \frac{T(\text{N}\cdot\text{m}) \times N(\text{rpm})}{9550} \qquad \textbf{(3-37)}$$

Equation (3-37) is derived in Appendix H.

ENERGY, POWER, AND FLOW RATE IN THE SI METRIC SYSTEM

Flow Rate

Volume flow rate within a pipeline equals the product of the pipe cross-sectional area and the fluid velocity:

$$Q(\mathrm{m^3/s}) = A(\mathrm{m^2}) \times v(\mathrm{m/s}) \qquad \textbf{(3-38)}$$

A flow rate of 1 $\mathrm{m^3/s}$ is an extremely large flow rate (1 $\mathrm{m^3/s}$ = 15,800 gpm). Thus flow rates are frequently specified in units of liters per second (Lps) or liters per minute (Lpm). Since 1 liter = 1 L = 0.001 $\mathrm{m^3}$, we have

$$Q(\mathrm{Lps}) = Q(\mathrm{L/s}) = 1000\, Q(\mathrm{m^3/s})$$

EXAMPLES USING THE SI METRIC SYSTEM

EXAMPLE 3-13

For the hydraulic jack of Figure 3-7 the following data are given:

$$A_1 = 25\ \text{cm}^2 \qquad A_2 = 100\ \text{cm}^2$$
$$F_1 = 200\ \text{N}$$
$$S_1 = 5\ \text{cm}$$

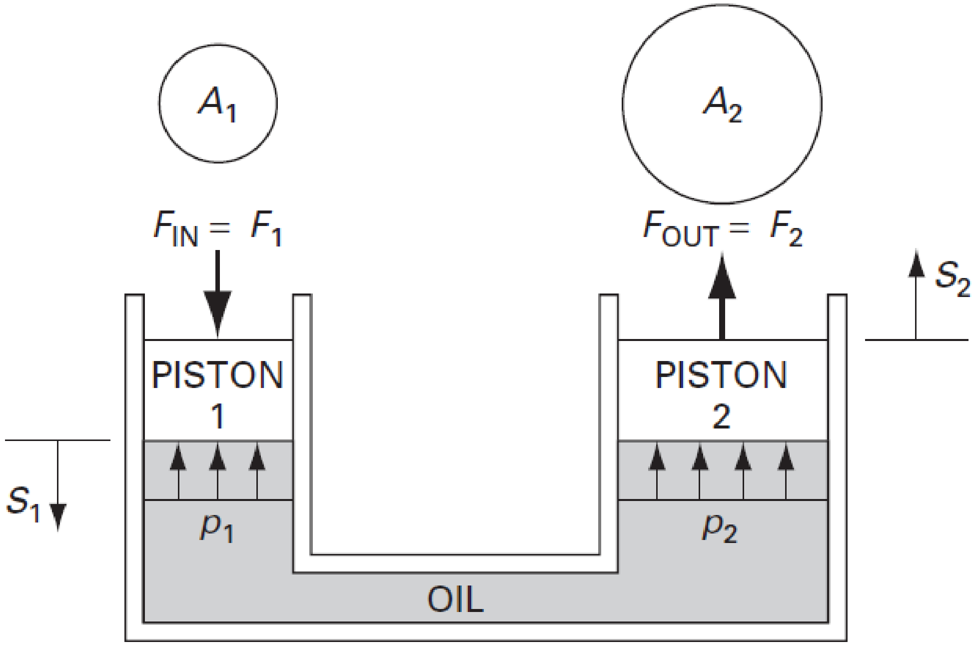
Determine

a. F_2
b. S_2

Solution

a. $$F_2 = \frac{A_2}{A_1} \times F_1 = \frac{100}{25} \times 200 = 800\ \text{N}$$

b. $$S_2 = \frac{A_1}{A_2} \times S_1 = \frac{25}{100} \times 5 = 1.25\ \text{cm}$$



EXAMPLES USING THE SI METRIC SYSTEM

EXAMPLE 3-14

Oil is flowing through a 30-mm diameter pipe at 60 liters per minute (Lpm). Determine the velocity.

Solution Per Eq. (3-38) we have

$$v(\text{m/s}) = \frac{Q(\text{m}^3/\text{s})}{A(\text{m}^2)}$$

where $Q(\text{m}^3/\text{s}) = Q(\text{L/min}) \times \frac{0.001\ \text{m}^3}{1\ \text{L}} \times \frac{1\ \text{min}}{60\ \text{s}} = 0.0000167\ Q\ (\text{Lpm})$

EXAMPLES USING THE SI METRIC SYSTEM

$$= 0.0000167 \times 60 = 0.0010 \text{ m}^3/\text{s}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.03 \text{ m})^2 = 0.000707 \text{ m}^2$$

Substituting values yields

$$v = \frac{0.0010}{0.000707} = 1.41 \text{ m/s}$$

EXAMPLES USING THE SI METRIC SYSTEM

EXAMPLE 3-15

A hydraulic pump delivers oil at 50 Lpm and 10,000 kPa. How much hydraulic power does the pump deliver?

Solution Per Eq. (3-35), we have

$$\text{hydraulic power (kW)} = \frac{p(\text{Pa}) \times Q(\text{m}^3/\text{s})}{1000} = p(\text{kPa}) \times Q(\text{m}^3/\text{s})$$

$$\text{where } Q(\text{m}^3/\text{s}) = Q(\text{L/min}) \times \frac{0.001\ \text{m}^3}{1\ \text{L}} \times \frac{1\ \text{min}}{60\ \text{s}} = 0.0000167Q\ (\text{Lpm})$$

$$= 0.0000167 \times 50 = 0.000835\ \text{m}^3/\text{s}$$

Substituting values yields

$$\text{Hydraulic power (kW)} = 10{,}000 \times 0.000835 = 8.35\ \text{kW}$$

EXAMPLES USING THE SI METRIC SYSTEM

EXAMPLE 3-16

Determine the torque delivered by a hydraulic motor if the speed is 1450 rpm and the mechanical output power is 10 kW.

Solution Solving Eq. (3-37) for torque yields

$$T(\mathrm{N \cdot m}) = \frac{9550 \times \text{power}(\mathrm{kW})}{N(\mathrm{rpm})}$$

$$= \frac{9550 \times 10}{1450} = 65.9\ \mathrm{N \cdot m}$$

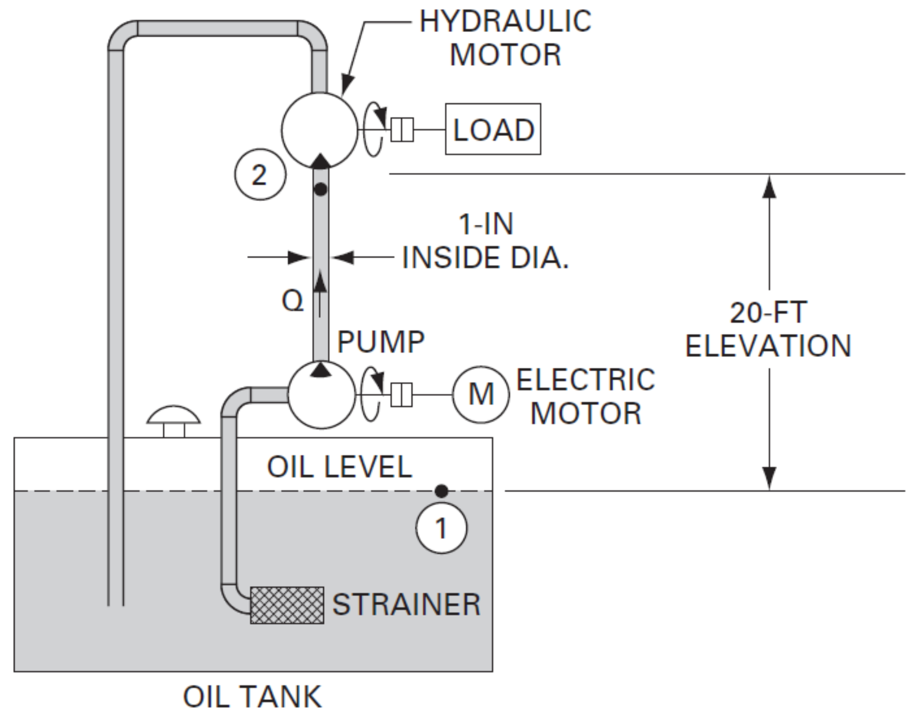
EXAMPLES USING THE SI METRIC SYSTEM

EXAMPLE 3-17

For the hydraulic system of Figure 3-29, the following SI metric data are given:

- **a.** The pump is adding 3.73 kW (pump hydraulic power = 3.73 kW) to the fluid.
- **b.** Pump flow is 0.001896 m^3/s.
- **c.** The pipe has a 0.0254-m inside diameter. Note that this size can also be represented in units of centimeters or millimeters as 2.54 cm or 25.4 mm, respectively.
- **d.** The specific gravity of the oil is 0.9.
- **e.** The elevation difference between stations 1 and 2 is 6.096 m.

Find the pressure available at the inlet to the hydraulic motor (station 2). The pressure at station 1 in the hydraulic tank is atmospheric (0 Pa or 0 N/m^2 gage). The head loss H_L due to friction between stations 1 and 2 is 9.144 m of oil.



EXAMPLES USING THE SI METRIC SYSTEM

Solution Writing the energy equation between stations 1 and 2, we have

$$Z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Since there is no hydraulic motor between stations 1 and 2, $H_m = 0$. Also, $v_1 = 0$ because the cross section of an oil tank is large. Also, $H_L = 9.144$ m and $p_1 = 0$ per the given input data.

Substituting known values, we have

$$Z_1 + 0 + 0 + H_p - 0 - 9.144 = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Solving for p_2/γ, we have

$$\frac{p_2}{\gamma} = (Z_1 - Z_2) + H_p - \frac{v_2^2}{2g} - 9.144$$

EXAMPLES USING THE SI METRIC SYSTEM

Since $Z_2 - Z_1 = 6.096$ m, we have

$$\frac{p_2}{\gamma} = H_p - \frac{v_2^2}{2g} - 15.24$$

From Eq. (3-36) we solve for the pump head:

$$H_p(\mathrm{m}) = \frac{\text{pump hydraulic power (W)}}{\gamma(\mathrm{N/m^3}) \times Q(\mathrm{m^3/s})}$$

EXAMPLES USING THE SI METRIC SYSTEM

where

$$\gamma_{\text{oil}} = (\text{SG})\,\gamma_{\text{water}} = 0.9 \times 9797\ \text{N/m}^3 = 8817\ \text{N/m}^3$$

$$H_p(\text{m}) = \frac{3730}{8817 \times 0.001896} = 223.1\ \text{m}.$$

Next, we solve for υ_2 and $\upsilon_2^2/2g$:

$$\upsilon_2(\text{m/s}) = \frac{Q(\text{m}^3/\text{s})}{A(\text{m}^2)} = \frac{0.001896}{(\pi/4)(0.0254\ \text{m})^2} = 3.74\ \text{m/s}$$

$$\frac{\upsilon_2^2}{2g} = \frac{(3.74\ \text{m/s})^2}{(2)(9.80\ \text{m/s}^2)} = 0.714\ \text{m}$$

On final substitution, we have

$$\frac{p_2}{\gamma} = 223.1 - 0.714 - 15.24 = 207.1\ \text{m}$$

Solving for p_2 yields

$$p_2(\text{N/m}^2) = (207.1\ \text{m})\gamma(\text{N/m}^3)$$

EXAMPLES USING THE SI METRIC SYSTEM

$$p_2 = (207.1)(8817) = 1{,}826{,}000 \text{ Pa} = 1826 \text{ kPa gage}$$

end