**Philadelphia University**
**Department of Basic Sciences and Mathematics**

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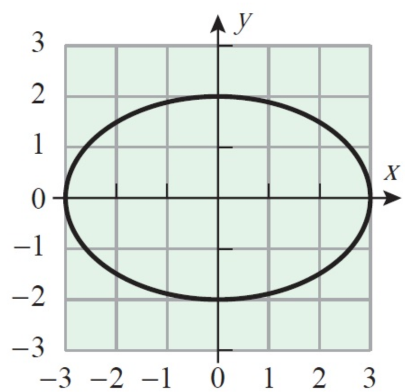
| **Academic Year:** | 2016–2017 | **Course Name:** | Calculus 2. |
| **Semester:** | First Semester | **Course Number:** | 250102 |
| **Exam:** | Final Exam | **Student Name:** | ________ |
| **Exam Date:** | 28/01/2017 | **University ID:** | ________ |
| **Exam Day:** | Saturday | **Section:** | ________ |
| **Mark:** | [ 40 ] | **Serial:** | ________ |

**Question ONE : (18 points)** Write the symbol of the correct answer in the blank.

1. [ ] The equation of the conic, shown in the figure below, is




(A) $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$ (B) $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ (C) $\dfrac{x^2}{16}-\dfrac{y^2}{16}=1$ (D) $\dfrac{y^2}{4}-\dfrac{x^2}{4}=1$

2. [ ] The directrix of the parabola $(y-3)^2=8(x-2)$ is given by

(A) $y=0$ (B) $y=1$ (C) $x=0$ (D) $x=1$

3. [ ] The equation $x^2+y^2-6y=0$ can be expressed in polar coordinates as

(A) $r=-6\sin\theta$ (B) $r=6\cos\theta$ (C) $r=-6\cos\theta$ (D) $r=6\sin\theta$

4. [ ] The curve given by the parametric equations $x(t)=\cos t;\ y(t)=\cos t+\sin^2 t$ is

(A) parabola (B) hyperbola (C) ellipse (D) line

**Time: 120 Minutes**

5. [ ] The rectangular coordinates of the point whose polar coordinates are $\left[12, \frac{\pi}{6}\right]$ equal

(A) $\left(4, 4\sqrt{3}\right)$ (B) $\left(4\sqrt{3}, 4\right)$ (C) $\left(6, 6\sqrt{3}\right)$ (D) $\left(6\sqrt{3}, 6\right)$

6. [ ] The arc length of the curve given in polar coordinates by $r(\theta) = e^{\theta}$ on $\left[0, \ln 3\right]$ equals

(A) $3\sqrt{2}$ (B) $2\sqrt{2}$ (C) $5\sqrt{2}$ (D) $4\sqrt{2}$

7. [ ] Suppose that $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum $S_n = 5 - \frac{2}{n^2}$, then $a_3 =$

(A) $\frac{25}{36}$ (B) $-\frac{25}{36}$ (C) $\frac{5}{18}$ (D) $-\frac{5}{18}$

8. [ ] If $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for $x \in \mathbb{R}$, then $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{2^{2n}(2n+1)!}$ converges to

(A) $2$ (B) $1$ (C) $-1$ (D) $0$

9. [ ] Which one of the following sequences converges ?

(A) $\left(1 - \frac{4}{n}\right)^{n^2}$ (B) $\sin\left(\frac{\pi n}{2}\right)$ (C) $1 + (-1)^n$ (D) $\frac{2n+1}{1 - 3\sqrt{n}}$

10. [ ] Given $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, then the Taylor series of $g(x) = e^{-2x}$ about $x = 1$ is

(A) $\frac{1}{e^2} \sum_{n=0}^{\infty} (-1)^n \frac{(x-2)^n}{n!}$ (B) $\frac{1}{e^2} \sum_{n=0}^{\infty} \frac{(x+2)^n}{n!}$

(C) $\frac{1}{e^2} \sum_{n=0}^{\infty} (-1)^n \frac{2^n (x-1)^n}{n!}$ (D) $\frac{1}{e^2} \sum_{n=0}^{\infty} \frac{2^n (x+1)^n}{n!}$

11. [ ] Given that $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\ln\left|\frac{x-a}{x-b}\right| + C$, then the improper integral $\int_4^{\infty} \frac{1}{(x-2)(x-3)}\,dx$ converges to

(A) $\frac{1}{2}\ln 3$ (B) $\ln 2$ (C) $\ln\left(\frac{4}{3}\right)$ (D) $\ln\left(\frac{3}{2}\right)$

12. [ ] If $P(x)$ is a polynomial of degree 2 such that $\int x^2 e^x\,dx = P(x)e^x + C$, then $P(x) =$

(A) $x^2-2x+2$ (B) $(x-1)^2$ (C) $x^2$ (D) $x^2-2$

**Question TWO : (6 points)** Find the center, foci, and vertices for the hyperbola

$$9x^2 - 16y^2 + 18x = 135.$$

**Question THREE : (5 points)** Determine whether the series $\sum_{n=3}^{\infty}(-1)^n\frac{\ln n}{n}$ is absolutely convergent, conditionally convergent, or divergent series.

**Question FOUR : (7 points)** Find the interval and radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2}$.

**Question FIVE : (4 points)** Evaluate the indefinite integral $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$.