Philadelphia University
Department of Basic Sciences and Mathematics

Final Exam Ordinary Differential Equations 31-1-2016

Name: ______________ Number: ______________ Serial: ______________ Section: (1)

Question ONE : (12 points) Write the symbol of the correct answer.

1. [ ] Any linear $n^{th}-$order initial value problem has

   (A) exactly n solutions (B) infinitely many solutions
   (C) exactly one solution (D) None of all

2. [ ] Consider the differential equation $\left(3x^2+y\right)dx+\left(x^2y-x\right)dy=0$. An integrating factor of the equation is

   (A) $\frac{2}{x}-1$ (B) $-1-\frac{2}{x}$ (C) $1-\frac{2}{x}$ (D) $1-\frac{1}{x}$

3. [ ] The Laplace transform of $f(t)=e^{2t}\cos\left(\sqrt{3}\,t\right)$ is

   (A) $\frac{p-2}{(p-2)^2+9}$ (B) $\frac{\sqrt{3}}{(p-2)^2+3}$
   (C) $\frac{p-2}{p^2+3}$ (D) $\frac{p-2}{(p-2)^2+3}$

4. [ ] Let L be defined by $L[y]:=\left(D^2-xD+2\right)[y]$. Then $L\left[x^2\right]$ equals

   (A) $2-2x^2$ (B) 2 (C) $2-x^2$ (D) $2x$

5. [ ] Which one of the following is a form of the particular solution of the second − order differential equation $y''-4y'+4y=xe^{2x}$?

   (A) Axe^{2x} (B) $x^2e^{2x}(Ax+B)$ (C) $xe^{2x}(Ax+B)$ (D) Ax^2e^{2x}

6. [ ] The differential operator that annihilates the function $xe^{-2x}+xe^{-5x}\sin 3x$ is

   (A) $(D+2)^2\left[(D+5)^2+9\right]^2$ (B) $(D+2)^2\left[(D+3)^2+25\right]^2$
   (C) $(D+2)^2\left[(D+3)^2+9\right]^2$ (D) $(D+2)^2\left[(D+5)^2+9\right]$

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Question TWO : (3 points) Find the inverse Laplace transform for the function

$$F(p) = \frac{2p-1}{p^2-4p+6}$$

Question THREE : (4 points) Solve the nonlinear equation $y'' + (y')^2 + 1 = 0$.

Question FOUR : (5 points) Solve the following initial value problem using the method of Laplace transforms.

$$y'' + 4y' + 4y = t^2 e^t \quad ; \quad y(0) = y'(0) = 0.$$

Question FIVE : (5 points) Solve the given system of differential equations by systematic elimination.

$$\frac{dx}{dt} = 2x - y$$
$$\frac{dy}{dt} = x$$

Question SIX : (5 points) Find a power series solution $\sum_{n=0}^{\infty} c_n x^n$ of the differential equation $y' = xy$.

Question SEVEN : (3 points) Solve the Cauchy–Euler equation $x^2y'' - 3xy' - 2y = 0$.

Question EIGHT : (5 points) Solve the equation $y'' + y = \sec x$ by variation of parameters.