Philadelphia University
Department of Basic Sciences and Mathematics

Final Exam | Probability Theory | 4-2-2016

Name: ____________ Number: ____________ Serial: ____________ Section: (1)

Question ONE : (10 points) Write the symbol of the correct answer.

1. [ ] How many distinct permutations are there of the letters in the word "**statistics**" that begin and end with the letter "**s**" ?

   (A) $\frac{10!}{3!\,3!\,2!}$ (B) $\frac{8!}{3!\,2!}$ (C) $\frac{10!}{3!\,2!}$ (D) $\frac{8!}{3!\,3!\,2!}$

2. [ ] If A and B are mutually exclusive, $P(A) = 0.37$, and $P(B) = 0.44$, find $P(A^c \cap B^c)$.

   (A) 0.0 (B) 0.63 (C) 0.81 (D) 0.19

3. [ ] Four candidates are seeking a vacancy on a school board. If A is twice as likely to be elected as B, and B and C are given about the same chance of being elected, while C is twice as likely to be elected as D, who will win the vacancy ?

   (A) Candidate D (B) Candidate C (C) Candidate B (D) Candidate A

4. [ ] A coin is loaded so that the probabilities of heads and tails are 0.52 and 0.48, respectively. If the coin is tossed three times, what are the probabilities of getting all heads ?

   (A) 0.140608 (B) 0.110592 (C) 0.119808 (D) 0.129792

5. [ ] If the joint probability distribution of X and Y is given by

$$f(x,y) = c\left(x^2 + y^2\right) \quad \text{for} \quad x = -1,0,1,3 \quad ; \quad y = -1,2,3$$

   find the value of c.

   (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{89}$ (D) $\frac{1}{100}$

Mr. Feras Awad
January 22, 2016

Question TWO : (3 points) If X is the number of heads and Y the number of heads minus the number of tails obtained in three flips of a balanced coin, construct a table showing the values of the joint probability distribution of X and Y.

Question THREE : (6 points) If the joint density of X and Y is given by

$$f(x, y) = \begin{cases} 6e^{-3x-2y} & \text{for } x > 0\,,\ y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

find the probability density of $Z = X + Y$.

Question FOUR : (3 + 1 points) If $E[X]=1$ and $\sigma_X^2=6$, find

(a) $E\left[(2+X)^2\right]$

(b) $\mathrm{Var}[4+3X]$

Question FIVE : (2 + 3 points) Given the moment−generating function $M_X(t)=e^{3t+8t^2}$ for a random variable X. Let $Z=\dfrac{X-3}{4}$. Find

(a) the moment generating function of Z.

(b) the mean and variance of Z.

Question SIX : (4 points) The joint and marginal probability of X and Y are as follows.

		x			
		0	**1**	**2**	
y	**0**	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{7}{12}$
	1	$\frac{2}{9}$	$\frac{1}{6}$		$\frac{7}{18}$
	2	$\frac{1}{36}$			$\frac{1}{36}$
		$\frac{5}{12}$	$\frac{1}{2}$	$\frac{1}{12}$	

Find the covariance of X and Y.

Question SEVEN : (4 points) If X, Y, and Z are independent and have the means $\mu_X = 4$, $\mu_Y = 9$, and $\mu_Z = 3$ and the variances $\sigma_X^2 = 3$, $\sigma_Y^2 = 7$, and $\sigma_Z^2 = 5$, find the mean and the variance of the random variable $W = 2X - 3Y + 4Z$.

Question EIGHT : (4 points) Let X be a continuous random variable follows the exponential distribution function with probability density

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Show that $E[X] = \theta$.