Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2015–2016	**Course Name:**	Linear Algebra (1)[1]
Semester:	Second Semester	**Course Number:**	250241
Exam:	Final Exam	**Instructor Name:**	Feras Awad
Exam Date:	12/06/2016	**Student Name:**	________
Exam Day:	Sunday	**University ID:**	________
Mark:	[40]	**Serial:**	________

1. (4 points) Use Cramer's Rule to solve the linear system $\begin{cases} x_1 + 2x_2 = 5 \\ -x_1 + x_2 = 1 \end{cases}$.

[1]Internal Examiner : Dr. Marouf Samhan

2. For the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$, find

(a) (4 points) $\mathrm{adj}(A)$

(b) (2 points) $\det(A)$

(c) (2 points) A^{-1}

3. (3 points) Let $\vec{u} = (1, 2, -3, 1)$, $\vec{v} = (0, 2, -1, -2)$, and $\vec{w} = (2, -2, 1, 3)$, find $\|2\vec{u} + \vec{w} - 3\vec{v}\|$.

4. (4 points) Let $\vec{u}_1 = (1, 3, 4)$, $\vec{u}_2 = (2, -1, 3)$, and $\vec{u}_3 = (-3, 2, -4)$. Write $\vec{v} = (-1, 7, 2)$ as a linear combination of $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$.

5. An invertible square matrix A is called *orthogonal* if $A^T = A^{-1}$.

   (a) (3 points) Determine whether the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$ is orthogonal or not.

   (b) (4 points) Prove that if A is an orthogonal matrix, then $\det(A) = \pm 1$.

6. Find an example of 2×2 matrices A, B, and C such that

   (a) (2 points) $A^2 = -I_2$.

   (b) (2 points) $BC = 0$ although no entries of B or C are zero.

7. (a) (3 points) Let A be a nonsingular $n \times n$ matrix. Assuming $n \geq 2$, prove that $\det\left(\operatorname{adj}(A)\right) = \left[\det(A)\right]^{n-1}$.

(b) (3 points) Prove that if A is $n \times n$ invertible matrix, then $\operatorname{adj}\left(A^{-1}\right) = \left[\operatorname{adj}(A)\right]^{-1}$.

8. (4 points) Suppose that $A = \begin{bmatrix} n & -1 & \cdots & -1 \\ -1 & n & \cdots & -1 \\ \vdots & \vdots & \cdots & \vdots \\ -1 & -1 & \cdots & n \end{bmatrix}$ is an invertible matrix of size $n \times n$.

Find the value for c if $A^{-1} = \dfrac{1}{n+1} \begin{bmatrix} c & 1 & \cdots & 1 \\ 1 & c & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & c \end{bmatrix}$.