Philadelphia University
Department of Basic Sciences and Mathematics

First Exam | **Linear Algebra (2)** | **11-11-2012**

Name: ______________ **Number:** ______________ **Serial:** ______________ **Section: (1)**

1. (6 points) Consider the vectors $\mathbf{v}_1 = (1, 0, 0)$, $\mathbf{v}_2 = (2, 2, 0)$, and $\mathbf{v}_3 = (3, 3, 3)$. Show that the set $\mathbf{S} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ form a basis for $\mathbb{R}^3$.

2. (4 points) Determine whether the statement is true or false:
   (a) [ ] There is a basis of 12 vectors in $\mathbb{R}^{15}$.
   (b) [ ] The zero vector space has dimension zero.
   (c) [ ] There is a vector space consisting of exactly two distinct elements.
   (d) [ ] Two subsets of a vector space $\mathbf{V}$ that span the same subspace of $\mathbf{V}$ must be equal.

Mr. Feras Awad
November 5, 2012

3. (4 points) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a vector space $\mathbf{V}$. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly independent set of vectors where $\mathbf{u}_1 = \mathbf{v}_1$, $\mathbf{u}_2 = \mathbf{v}_1 + \mathbf{v}_2$, and $\mathbf{u}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$.

4. (6 points) Consider the bases $\mathbf{B} = \{(1,0),(0,1)\}$ and $\mathbf{B}' = \{(2,1),(-3,4)\}$ for $\mathbb{R}^2$.

   (a) Find the transition matrix $\mathbf{P}$ from $\mathbf{B}$ to $\mathbf{B}'$.

   (b) Compute the coordinate vector $[\mathbf{w}]_{\mathbf{B}}$ where $\mathbf{w} = (3,-5)$, and then use it and the transition matrix $\mathbf{P}$ from part (a) to compute $[\mathbf{w}]_{\mathbf{B}'}$.