Philadelphia University
Department of Basic Sciences and Mathematics

Second Exam **Linear Algebra (2)** **16-12-2012**

Name: ______________ **Number:** ______________ **Serial:** ______________ **Section: (1)**

1. (2 points) Let $\mathbf{A}$ be a 5×9 matrix with $\text{Rank}(\mathbf{A}) = 2$. Find the dimension of the
   (a) [ ] row space of $\mathbf{A}$.
   (b) [ ] column space of $\mathbf{A}$.
   (c) [ ] null space of $\mathbf{A}$.
   (d) [ ] null space of $\mathbf{A}^T$.

2. (3 points) Determine whether the statement is true or false:
   (a) [ ] If $\mathbf{R}$ is the reduced row echelon form of $\mathbf{A}$, then those column vectors of $\mathbf{R}$ that contain the leading 1's form a basis for the column space of $\mathbf{A}$.
   (b) [ ] If $\text{Rank}\left(\mathbf{A^T}\right) = \text{Rank}(\mathbf{A})$, then $\mathbf{A}$ must be square matrix.
   (c) [ ] If $\mathbf{A}$ is a 2×3 matrix, then the domain of the transformation $T_{\mathbf{A}}$ is $\mathbb{R}^2$.

3. (4 points) Determine whether the matrix operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the equations

$$\begin{aligned} w_1 &= 9x_1 + 5x_2 \\ w_2 &= 2x_1 - 7x_2 \end{aligned}$$

is $1-1$; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2)$.

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4. Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$.

(a) (6 points) Find the **eigenvalues** and **eigenvectors** for the matrix $\mathbf{A}$.

(b) (2 points) Find the matrix $\mathbf{P}$ that diagonalizes $\mathbf{A}$, then find $\mathbf{P}^{-1}$.

(c) (2 points) Evaluate $\mathbf{A}^5$.