Philadelphia University
Department of Basic Sciences and Mathematics

Final Exam | **Linear Algebra 2** | **15-1-2013**

Name: _______________ **Number:** _______________ **Serial:** _______________ **Section: (1)**

1. (5 points) Determine whether the statement is true (**T**) or false (**F**):

   (a) [ ] A positive definite matrix is invertible.

   (b) [ ] If $\mathbf{A}$ is positive definite, then $-\mathbf{A}$ is negative definite.

   (c) [ ] If $\mathbf{A}$ is a square matrix, then $\mathbf{A^TA}$ and $\mathbf{AA^T}$ are orthogonally diagonalizable.

   (d) [ ] If $\mathbf{A}$ is both invertible and orthogonally diagonalizable, then $\mathbf{A}^{-1}$ is orthogonally diagonalizable.

   (e) [ ] The matrix $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ is orthogonal.

2. (3 points) Express the quadratic form $6x_1^2 + 4x_2^2 - 7x_3^2 - 2x_1x_2 + 4x_1x_3 + x_2x_3$ in the matrix notation $\mathbf{x^TAx}$, where $\mathbf{A}$ is symmetric matrix.

3. (4 points) Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors such that $\langle \mathbf{u}, \mathbf{v} \rangle = 3$, $\|\mathbf{u}\| = 5$, and $\|\mathbf{v}\| = 3$, evaluate $\langle \mathbf{u} - \mathbf{v}, \mathbf{u} + \mathbf{v} \rangle$.

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January 3, 2013

4. (4 points) Let $\mathbf{f} = 1 - x^2$ and $\mathbf{g} = 3 + 12x - 4x^2$. Use the inner product

$$\langle \mathbf{f}, \mathbf{g} \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2$$

on $\mathbf{P_2}$ to compute the cosine of the angle between $\mathbf{f}$ and $\mathbf{g}$.

5. (4 points) What conditions must a and b satisfy for the matrix $\mathbf{A} = \begin{bmatrix} a+b & b-a \\ a-b & b+a \end{bmatrix}$ to be orthogonal.

6. (5 points) Prove that: there is no vector space consisting of exactly two elements.

7. (15 points) Find a matrix $\mathbf{P}$ that orthogonally diagonalizes

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$