Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2016–2017	**Course Name:**	Numerical Analysis
Semester:	Second Semester	**Course Number:**	250371
Exam:	First Exam	**Instructor Name:**	Feras Awad
Exam Date:	03/04/2017	**Student Name:**	______
Exam Day:	Monday	**University ID:**	______
Mark:	[20]	**Serial:**	______

1. The iteration formula

$$x_{n+1} = x_n - (\cos x_n)(\sin x_n) + R\cos^2 x_n$$

where R is a positive constant, was obtained by applying Newton's method to some function $f(x)$. What was $f(x)$? For what value does the sequence converge?

2. Find a bound for the number of iterations needed to achieve an approximation for $\sqrt[3]{25}$ with accuracy 10^{-4} using the Bisection Algorithm on $[2,3]$.

3. Show that the function $g(x) = 2^{-x}$ has a unique fixed point on $\left[\frac{1}{3}, 1\right]$, then use fixed-point iteration to find the third approximation p_3 starting with $p_0 = 1$.

4. Let $f(x) = e^x$ where $x \in [0,2]$. Approximate $f(0.25)$ by using the second Lagrange interpolating polynomial with $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.

5. Suppose $x_i = i$, for $i = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = x + 1, \quad P_{1,2} = 3x - 1, \quad \text{and} \quad P_{1,2,3}(1.5) = 4.$$

Find $P_{0,1,2,3}(1.5)$.