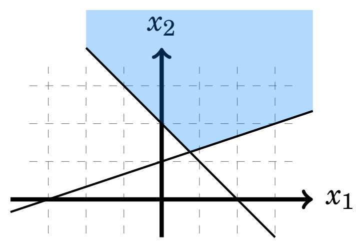
Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2016–2017	**Course Name:**	Linear Programming
Semester:	Summer Semester	**Course Number:**	250373
Exam:	Final Exam	**Instructor Name:**	Feras Awad
Exam Date:	23/08/2017	**Student Name:**	________
Exam Day:	Wednesday	**University ID:**	________
Exam Mark:	[40]	**Serial:**	________

Question ONE [10 Points] : Write the symbol of the correct answer in the **blank** beside the question number.

1. [ ] The **shaded region** in the figure is the **solution region** for the system of linear inequalities



(A) $-x_1 + 3x_2 \le 3\,,\; x_1 + x_2 \ge 2$ (B) $-x_1 + 3x_2 \ge 3\,,\; x_1 + x_2 \le 2$
(C) $-x_1 + 3x_2 \ge 3\,,\; x_1 + x_2 \ge 2$ (D) $-x_1 + 3x_2 \le 3\,,\; x_1 + x_2 \le 2$

2. [ ] An LP has **4 variables** and **2 constraints**, then its **dual** problem has

(A) 2 constraints , 4 variables (B) 4 constraints , 2 variables
(C) 4 constraints , 4 variables (D) 2 constraints , 2 variables

3. [ ] A constraint that **does not affect the feasible region** is a

(A) redundant constraint (B) non-negativity constraint
(C) standard constraint (D) slack constraint

4. [ ] All linear programming problems have all of the following properties **EXCEPT**

(A) alternative optimal solutions
(B) a linear objective function that is to be maximized or minimized
(C) a set of linear constraints
(D) variables that are all restricted to non-negative values

5. [ ] **Slack**

(A) exists for each variable in a linear programming problem

(B) is the amount by which the left side of a ≥ constraint is larger than the right side

(C) is the difference between the left and right sides of a constraint

(D) is the amount by which the left side of a ≤ constraint is smaller than the right side

Question TWO [6 Points] : You are given the tableau shown below for a **maximization** problem. Give conditions on the unknowns a_1, a_2, a_3, b, and c that make the following statements **true**.

z	x_1	x_2	x_3	x_4	x_5	RHS
Row 0	$\boxed{-c}$	2	0	0	0	10
x_3	-1	$\boxed{a_1}$	1	0	0	4
x_4	$\boxed{a_2}$	-4	0	1	0	1
x_5	$\boxed{a_3}$	3	0	0	1	$\boxed{b}$

1. The current solution is optimal.
2. The current solution is optimal, and there are alternative optimal solutions.
3. The LP is unbounded (in this part, assume that $b \geq 0$)

Question THREE [5+4 Points] : The following is the primal LP and its optimal tableau.

Maximize $z = 2x_1 + 5x_2$

Subject to

$x_1 + 2x_2 \leq 16$

$x_1 - x_2 \leq 12$

$x_1, x_2 \geq 0$

z	x_1	x_2	s_1	s_2	RHS
Row 0	1/2	0	5/2	0	40
x_2	1/2	1	1/2	0	8
s_2	3/2	0	1/2	1	20

1. Suppose we change the objective function coefficient of x_2 from 5 to $5 + \Delta$. For what values of Δ will the current set of basic variables remain optimal?

2. Find the optimal solution to the LP if we add the constraint $2x_1 + x_2 \geq 6$.

z		RHS
Row 0		
Row 0		

Question FOUR [5 Points] : Solve the following LP using the **Generalized Simplex** method.

$$\begin{aligned} \text{Maximize} \quad & z = -2x_1 + x_2 \\ \text{Subject to} \quad & x_1 + x_2 \geq 5 \\ & x_1 - 2x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

z		RHS
Row 0		
Row 0		
Row 0		
Row 0		

Question FIVE [2+4+4 Points] : Consider the following **primal** LP.

$$\begin{aligned} \text{Maximize} \quad & z = 4x_1 + x_2 \\ \text{Subject to} \quad & 3x_1 + 2x_2 \le 6 \\ & 6x_1 + 3x_2 \le 10 \\ & x_1,\, x_2 \ge 0 \end{aligned}$$

1. Find the dual problem of this LP.

2. Suppose that in solving this problem, row 0 of the optimal tableau is found to be
$$z + 2x_2 + s_2 = \frac{20}{3}.$$
Use the **Dual Theorem** to prove that the computations must be incorrect.

3. Use the **complementary slackness** method to find the **optimal dual** solution knowing that the optimal solution to the primal is $x_1 = \frac{5}{3}$, $x_2 = 0$, $s_1 = 1$, and $s_2 = 0$.