Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2016–2017	**Course Name:**	Applied Math
Semester:	Second Semester	**Course Number:**	250473
Exam:	Second Exam	**Instructor Name:**	Feras Awad
Exam Date:	02/05/2017	**Student Name:**	_____
Exam Day:	Tuesday	**University ID:**	_____
Mark:	[20]	**Serial:**	_____

1. Evaluate $\dfrac{d^{25}}{dx^{25}}\left[x^2 \cos x\right]$ using **Leibniz Rule**.

2. Express the function $f(x) = 3x^2 + x - 1$ as a linear combinations of Legendre polynomials. $\left[\textbf{Hint: } P_0(x) = 1,\ P_1(x) = x, \text{ and } P_2(x) = \frac{1}{2}\left(3x^2 - 1\right).\right]$

3. Prove the recursion relation of Legendre polynomials

$$\left(1-x^2\right)P_n'(x) = nP_{n-1}(x) - nxP_n(x)$$

using the two recurrence relations

$$xP_n'(x) - P_{n-1}'(x) = nP_n(x) \tag{1}$$
$$P_n'(x) - xP_{n-1}'(x) = nP_{n-1}(x) \tag{2}$$

4. Solve the ordinary differential equation $y' = 2y$ by series.
$\left[\textbf{Hint: } e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}.\right]$

5. Show that $P_n(x)$ and $P_n'(x)$ are orthogonal on $(-1, 1)$.