Philadelphia University
Department of Basic Sciences and Mathematics

Academic Year:	2016–2017	**Course Name:**	Applied Math
Semester:	Second Semester	**Course Number:**	250473
Exam:	Final Exam	**Instructor Name:**	Feras Awad
Exam Date:	11/06/2017	**Student Name:**	______
Exam Day:	Sunday	**University ID:**	______
Mark:	[40]	**Serial:**	______

1. Evaluate $\dfrac{d^{100}}{dx^{100}}\left[x^2e^{-x}\right]$ **when** $x = 100$.

2. Find the **exact value** of $\displaystyle\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$

3. Find the **norm** of the function $f(x) = xe^{-x/2}$ on $(0, \infty)$.

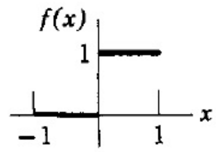
4. Evaluate $\lim_{x \to 0} \left[\frac{1}{x} J_1(x) \right]$.

5. Show that $\sqrt{\frac{\pi x}{2}} J_{-\frac{1}{2}}(x) = \cos x$. $\left[\ \textbf{Hint:}\ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = \cos x \text{ and } \Gamma\left(n + \frac{1}{2}\right) = \frac{(2n)!}{4^n\, n!}\sqrt{\pi}\ \right]$

6. Prove that $\displaystyle\int_{-1}^{1} P_n^2(x)\, dx = \frac{2}{1+2n}$.

$\Big[$ **Hint:** you may use the recursion relation $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$ $\Big]$

7. Expand in a **Legendre series**, the function $f(x)$ given by the graph.



[**Hint:** $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$, $P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$]

8. By a method similar to that we used to show that the P_n's are an orthogonal set of functions on $(-1,1)$, show that the solutions of

$$y_n'' = -n^2 y_n \qquad ; \qquad y(-\pi) = y(\pi) = 0,$$

are an orthogonal set on $(-\pi,\pi)$.