Electric Circuits I

Step Response of an RC Circuit

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Step Response of an RL Circuit

Assume an initial voltage on the capacitor.

$$v(0^-) = v(0^+) = V_0$$

Applying KCL, we have.

$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}u(t)$$

Where *v* is the voltage across the capacitor. For *t>0*

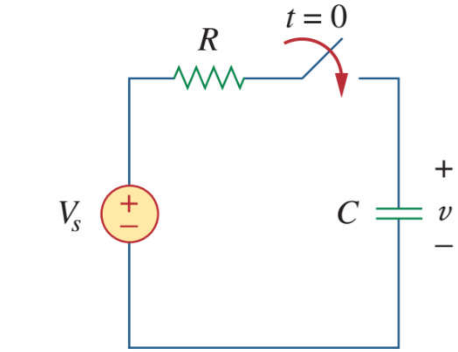
$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}$$

Rearranging terms gives

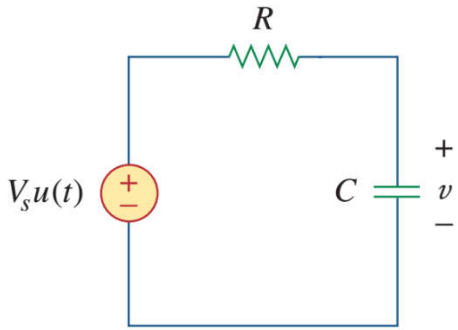
$$\frac{dv}{dt} = -\frac{v - V_s}{RC}$$

Or

$$\frac{dv}{v - V_s} = -\frac{dt}{RC}$$



(a)



(b)

Step Response of an RL Circuit

Integrating both sides and introducing the initial conditi

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

Or

$$\ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

Taking the exponential of both sides

$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}, \qquad \tau = RC$$

$$v - V_s = (V_0 - V_s)e^{-t/\tau}$$

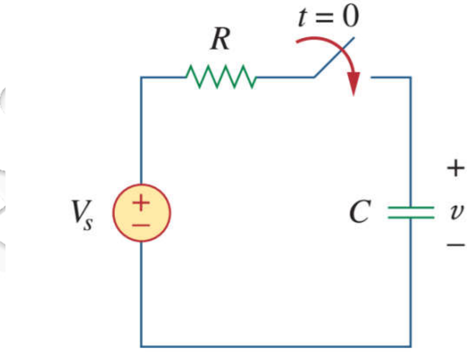
Or

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \qquad t > 0$$

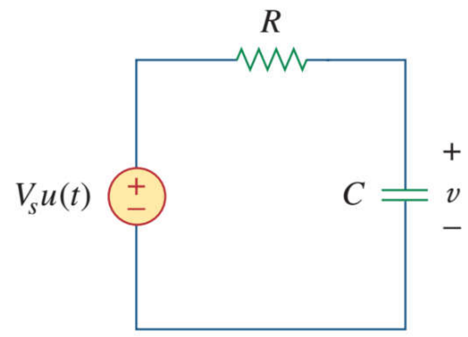
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Thus

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$



(a)



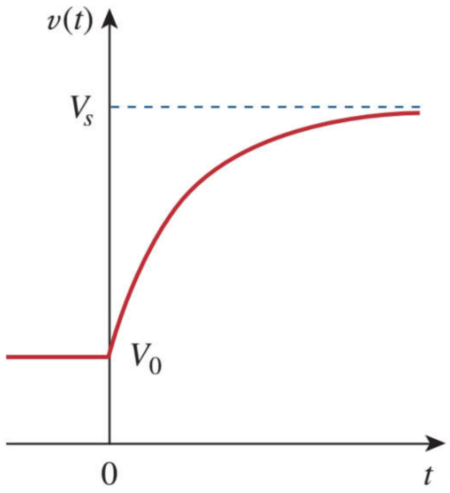
(b)

if the switch changes position at time $t=t_o$ instead of $t=0$

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau}$$

Step Response of an RL Circuit

- **This is known as the *complete response* (or total response) of the RC circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged.**
- **Assuming that $V_s > V_o$ a plot of v(t) is shown in the figure.**



If $V_s=0$

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

Which can be written alternatively as

$$v(t) = V_s(1 - e^{-t/\tau})u(t)$$

The current through the capacitor is obtained as

$$i(t) = C\frac{dv}{dt} = \frac{C}{\tau}V_s e^{-t/\tau}, \qquad \tau = RC, \qquad t > 0$$

Or

$$i(t) = \frac{V_s}{R}e^{-t/\tau}u(t)$$

Step Response of an RL Circuit

Complete response = natural response + forced response
stored energy (natural response); independent source (forced response)

$$v = v_n + v_f$$

$$v_n = V_o e^{-t/\tau}$$

$$v_f = V_s(1 - e^{-t/\tau})$$

The *transient response* is the circuit's temporary response that will die out with time.

The *steady-state response* is the behavior of the circuit a long time after an external excitation is applied.

Complete response = transient response + steady-state response
temporary part (transient response); permanent part (steady-state response)

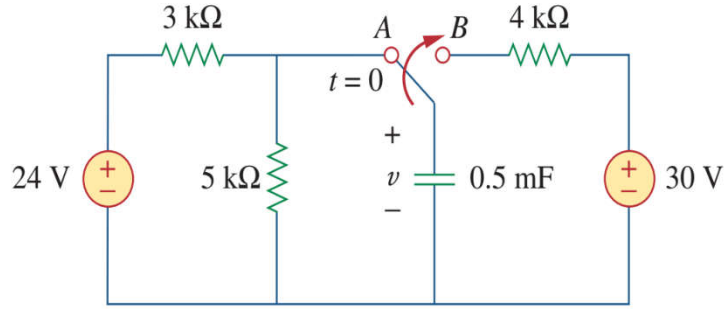
$$v = v_t + v_{ss}$$

$$v_t = (V_o - V_s)e^{-t/\tau}$$

$$v_{ss} = V_s$$

Step Response of an RL Circuit

Example: The switch in shown figure has been in position *A* for a long time. At *t=0* the switch moves to *B*. Determine *v(t)* for *t>0* and calculate its value at 1s and 4s.



For $t < 0$, the switch is at position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across the 5-kΩ resistor. Hence, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$, the switch is in position B. The Thevenin resistance connected to the capacitor is $R_{Th} = 4$ kΩ, and the time constant is

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30$ V. Thus,

$$\begin{aligned} v(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\ &= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V} \end{aligned}$$

At $t = 1$,

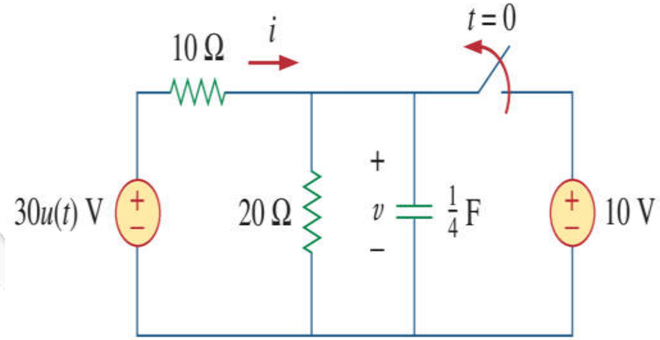
$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

At $t = 4$,

$$v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

Step Response of an RL Circuit

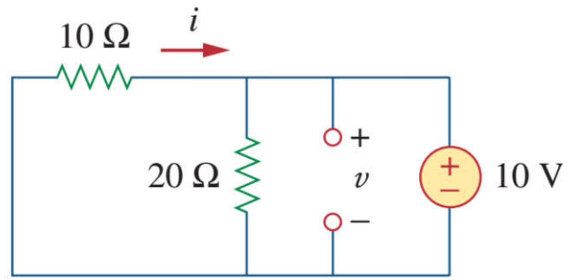
Example: In shown figure, the switch has been closed for a long time and is opened at *t=0*. Find *i* and for all time.



By definition of the unit step function,

$$30u(t) = \begin{cases} 0, & t < 0 \\ 30, & t > 0 \end{cases}$$

$$v = 10 \text{ V}, \qquad i = -\frac{v}{10} = -1 \text{ A}$$



(a)

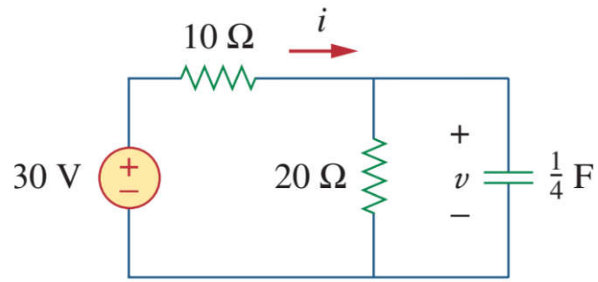
$$v(0) = v(0^-) = 10 \text{ V}$$

$$v(\infty) = \frac{20}{20 + 10}(30) = 20 \text{ V}$$

$$R_{\text{Th}} = 10 \,\|\, 20 = \frac{10 \times 20}{30} = \frac{20}{3} \,\Omega$$

$$\tau = R_{\text{Th}} C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3} \text{ s}$$

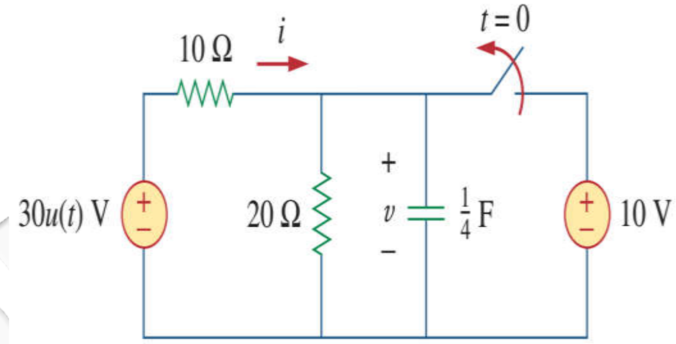
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 20 + (10 - 20)e^{-(3/5)t} = (20 - 10e^{-0.6t}) \text{ V}$$



(b)

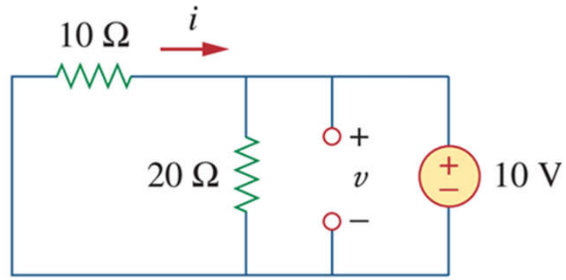
Step Response of an RL Circuit

Example: In shown figure, the switch has been closed for a long time and is opened at *t=0*. Find *i* and for all time.



$$i = \frac{v}{20} + C\frac{dv}{dt}$$

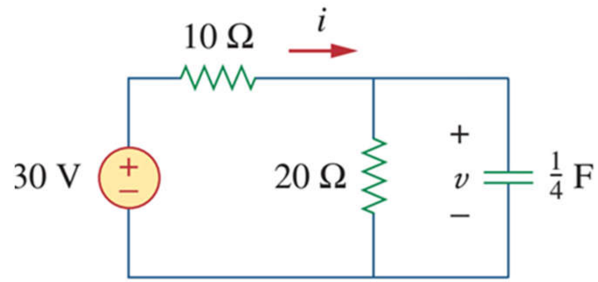
$$= 1 - 0.5e^{-0.6t} + 0.25(-0.6)(-10)e^{-0.6t} = (1 + e^{-0.6t})\ \text{A}$$



(a)

$$v = \begin{cases} 10\ \text{V}, & t < 0 \\ (20 - 10e^{-0.6t})\ \text{V}, & t \geq 0 \end{cases}$$

$$i = \begin{cases} -1\ \text{A}, & t < 0 \\ (1 + e^{-0.6t})\ \text{A}, & t > 0 \end{cases}$$



(b)

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