Electric Circuits I

Superposition

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Linearity Property

- **Linearity is the property of an element describing a linear relationship between cause and effect.**
- **The property is a combination of both the *homogeneity* (*scaling*) property and the *additivity* property.**
- **The *homogeneity* property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.**
- **For a resistor, for example, Ohm's law relates the input *i* to the output *v*.**

$$v = iR$$

If the current is increased by a constant k, then the voltage increases correspondingly by k; that is,

$$kiR = kv$$

Linearity Property

- **The *additivity* property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.**
- **Using the voltage-current relationship of a resistor, if**

  $v_1 = i_1 R$ **and** $v_2 = i_2 R$

  Then applying $(i_1 + i_2)$ **gives**

  $$v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

- **We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.**
- **In general, a circuit is linear if it is both additive and homogeneous. A linear circuit consists of only linear elements, linear dependent sources, and independent sources.**

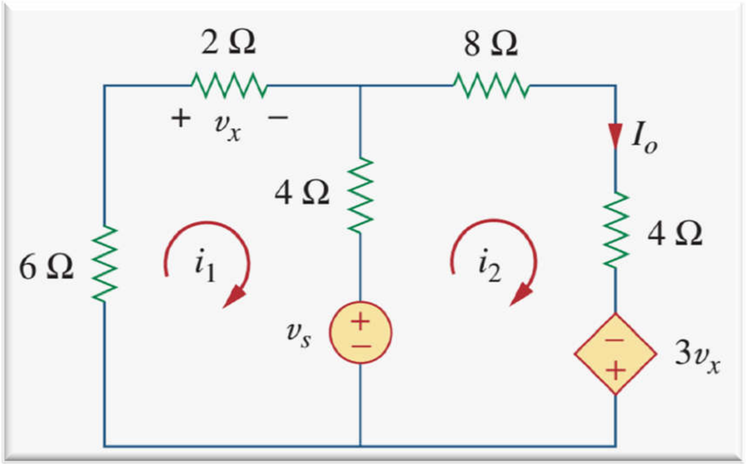
Linearity Property

- **A *linear circuit* is one whose output is linearly related (or directly proportional) to its input.**

- **The relationship between power and voltage (or current) is nonlinear.**
- **For example, when current i_1 flows through resistor R, the power is $p_1=Ri_1^2$, and when current i_2 flows through R, the power is $p_2=Ri_2^2$. If current i_1+i_2 flows through R, the power absorbed is $p_3=R(i_1+i_2)^2=Ri_1^2+Ri_2^2+2Ri_1i_2 \neq p_1+p_2$. Thus, the power relation is nonlinear.**

Linearity Property

Example: For the circuit shown in the figure. Find I_o when v_s=12V and v_s=24V.



Applying KVL to the two loops.

$$12i_1 - 4i_2 + v_s = 0 \qquad (1)$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad (2)$$

but $\quad v_x = 2i_1$

Equation (2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \qquad (3)$$

Adding eq.1 and eq.3 yields

$$2i_1 + 12i_2 = 0$$

$$i_1 = -6i_2$$

Substituting i_1 in eq.1

$$12 \times (-6) - 4i_2 + v_s = 0$$

$$i_2 = \frac{v_s}{76}$$

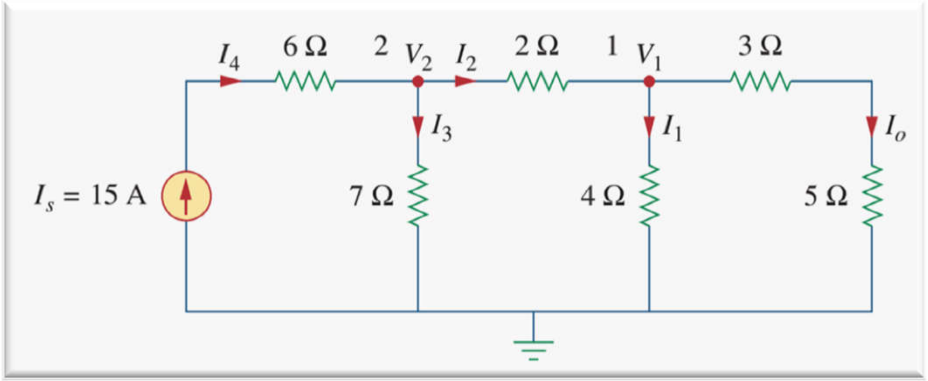
when v_s=12V $\qquad i_2 = \frac{12}{76}$A

when v_s=24V $\qquad i_2 = \frac{24}{76}A$

showing that when the source value is doubled, i_2 doubles.

Linearity Property

Example: Assume I_o=1A and use linearity to find the actual value of I_o in the circuit of the shown figure.



if $Io = 1A,$ **then** $V_1 = (3 + 5)I_o = 8V$

$$I_1 = \frac{V_1}{4} = 2A$$

Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3A$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14V$$

$$I_3 = \frac{V_2}{7} = \frac{14}{7} = 2A$$

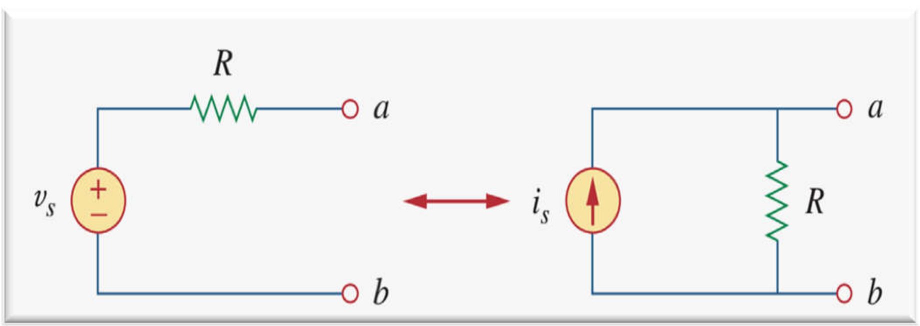
Applying KCL at node 2 gives

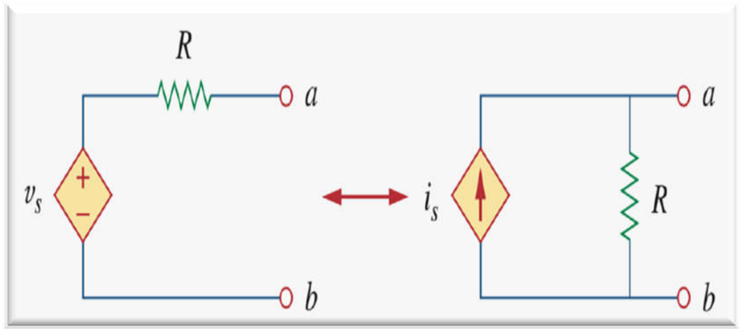
$$I_4 = I_3 + I_2 = 5A$$

Therefore, I_s=5A. This shows that assuming I_o=1A gives I_s=5A, the actual source current of 15A will give I_o=3A as the actual value.

Source Transformation

- Source transformation is another tool for simplifying circuits.
- ***source transformation*** is the process of replacing a voltage source in series with a resistor by a current source in parallel with a resistor, or vice versa.





- The above circuits have the same voltage-current relation at terminals ***a-b***.
- It is easy to show that they are indeed equivalent. In both circuits ***R***, i_s and v_s are the same.

Source Transformation

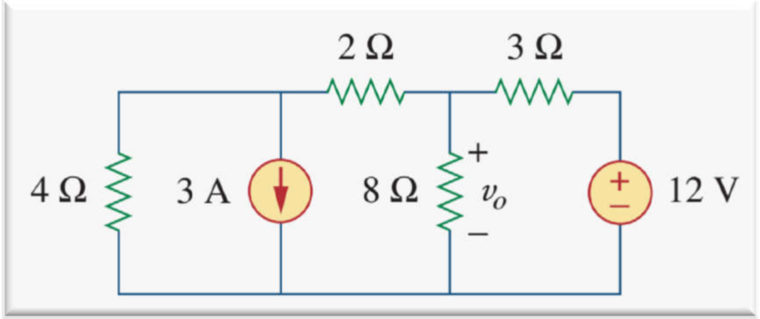
- **In order for the two circuits to be equivalent. Hence, source transformation requires that.**

$$\boldsymbol{v_s = i_s R} \qquad \text{or} \qquad \boldsymbol{i_s = \frac{v_s}{R}}$$

- **Keep the following points in mind when dealing with source transformation.**
  1- The arrow of the current source is directed toward the positive terminal of the voltage source.
  2- Source transformation is not possible when *R=0*, which is the case with an ideal voltage source. Similarly, an ideal current source with *R=∞* cannot be replaced by a finite voltage source.

Source Transformation

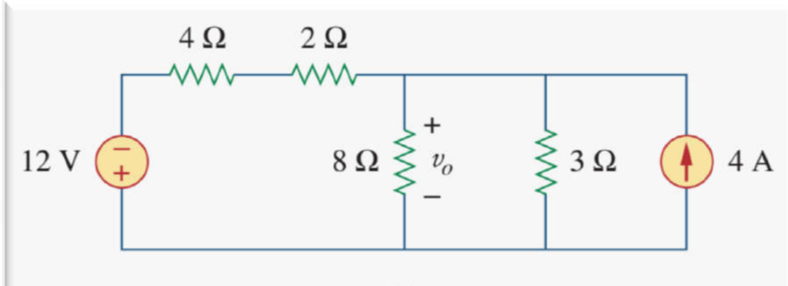
Example: Use source transformation to find v_o in the circuit of shown figure.



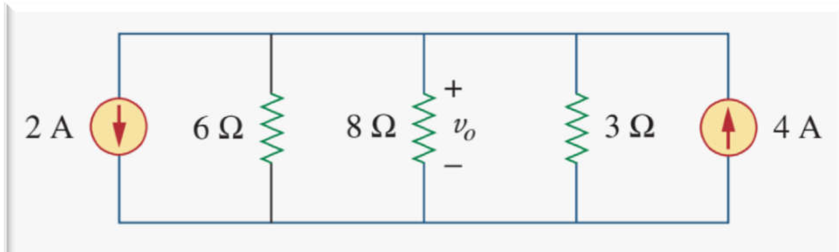
1- transform the current and voltage sources to obtain the circuit in figure (a).

2- Combining the 4Ω and 2Ω resistors in series and transforming the 12V voltage source gives us figure (b).

3- combine the 3Ω and 6Ω resistors in parallel to get 2Ω. Also combine the 2A and 4A current sources to get a 2A source to obtain figure (c).



(a)

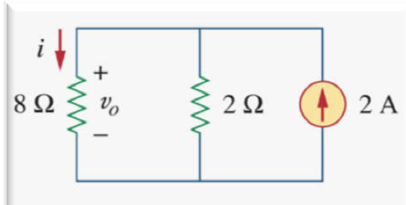


(b)

Use current division in figure (c) to get

$$i = \frac{2}{2+8} \times 2 = 0.4A$$

$$\boxed{v_o = 8i = 8 \times 0.4 = 3.2V}$$



(c)

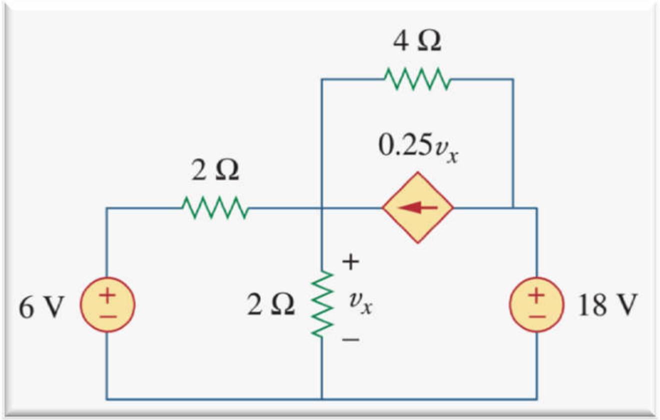
Source Transformation

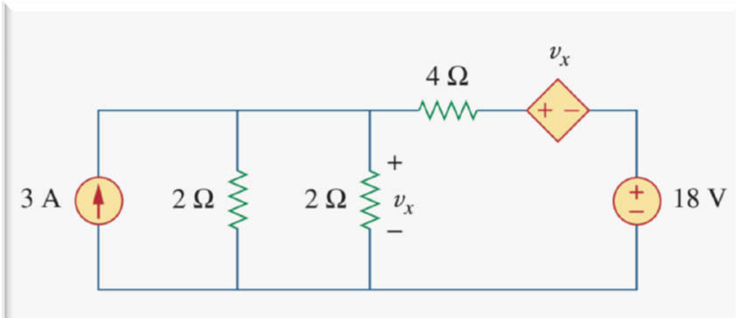
Example: **find v_x in the shown figure using source transformation.**

1- Transform the dependent current source as well as the 6V independent voltage source as shown in figure (a).

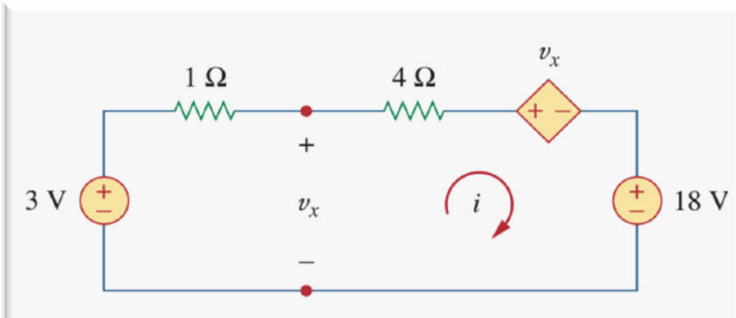
2- The two 2Ω resistors in parallel combine to give a 1Ω resistor, which is in parallel with the 3A current source.

3- The current source is transformed to a voltage source as shown in figure (b).





(a)



(b)

Source Transformation

Applying KVL around the loop in figure (b).

$$-3 + 5i + v_x + 18 = 0 \qquad (1)$$

Applying KVL around the loop containing only the 3V voltage source, the 1Ω resistor and v_x.

$$-3 + 1i + v_x = 0$$

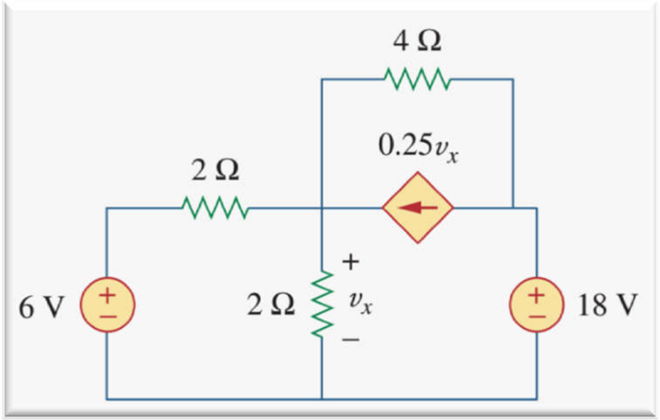
$$v_x = 3 - i \qquad (2)$$

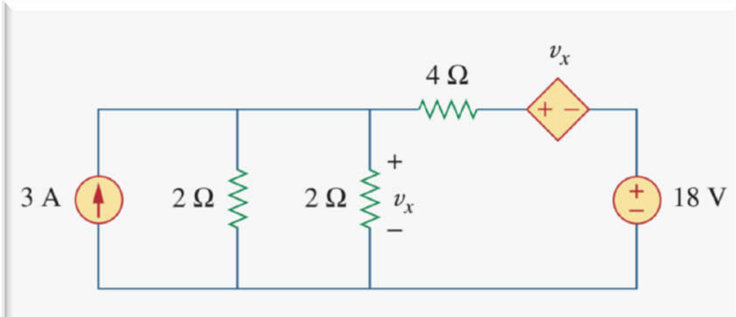
Substituting (2) in (1)

$$15 + 5i + 3 - i = 0 \qquad (1)$$

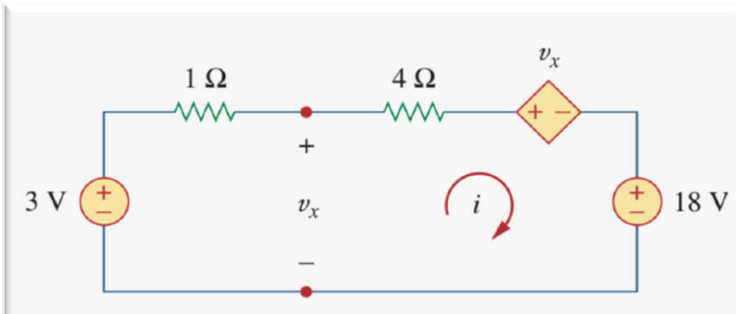
$$i = -4.5A$$

$$v_x = 3-(-4.5) = 7.5V$$





(a)



(b)

Superposition

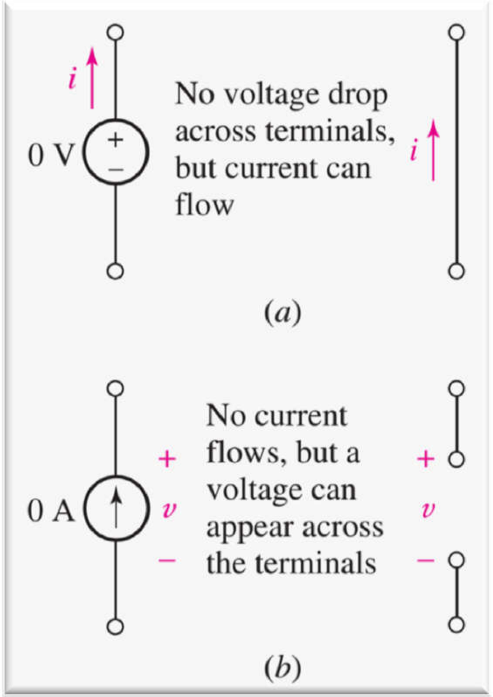
- Nodal and Mesh analysis are used to determine the value of a specific variable (voltage or current).
- Another way is to determine the contribution of each independent source to the variable and then add them up. This approach is known as the ***superposition.***

The ***superposition*** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Superposition

To apply the superposition principle, we must keep two things in mind:

1. **We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by *0V* (or a short circuit), and every current source by *0A* (or an open circuit). This way we obtain a simpler and more manageable circuit.**
2. **Dependent sources are left intact because they are controlled by circuit variables.**



(a) A voltage source set to zero acts like a short circuit. (b) A current source set to zero acts like an open circuit.

Number of networks to be analyzed =	Number of independent sources

Superposition

Steps to Apply Superposition Principle:

1. **Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using any technique.**

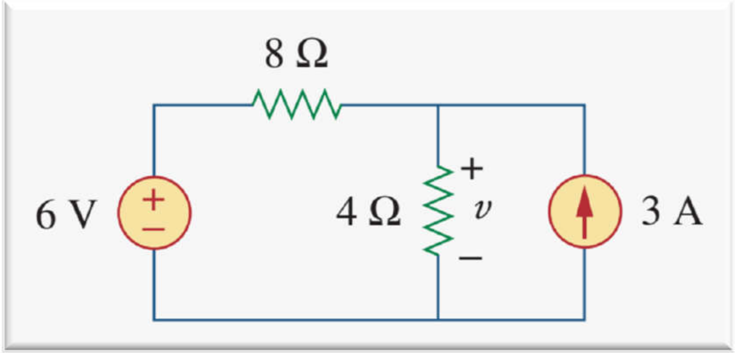
2. **Repeat step 1 for each of the other independent sources.**

3. **Find the total contribution by adding algebraically all the contributions due to the independent sources.**

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Superposition

Example: Use the superposition theorem to find *v* in the circuit of shown figure.



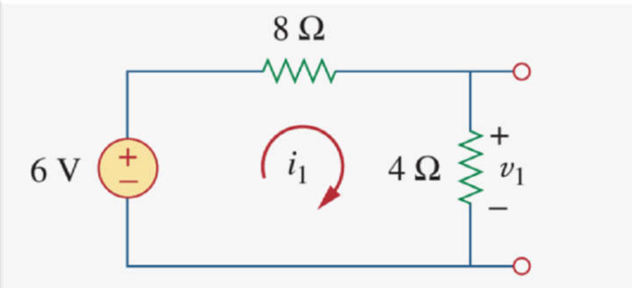
Since there are two sources, let

$v = v_1|_{6V} + v_2|_{3A}$

To obtain $v_1|_{6V}$ we set the current source to zero, as shown in fig(a). Applying KVL to the loop fig(a) gives

$12i_1 - 6 = 0 \quad \Rightarrow i_1 = 0.5A$

$v_1|_{6V} = 4i_1 = 4 \times 0.5 = 2V$



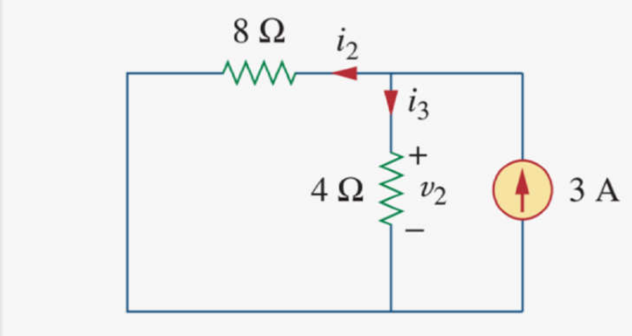
(a)

To get $v_2|_{3A}$ we set the voltage source to zero, as in fig(b). Using current division,

$$i_3 = \frac{8}{4+8} \times 3 = 2A$$

$v_2|_{3A} = 4i_3 = 4 \times 2 = 8V$

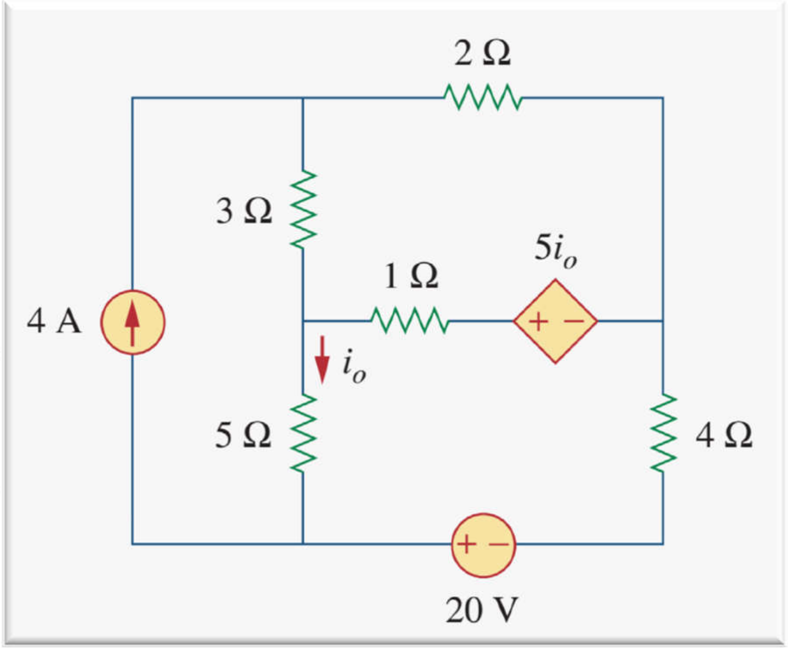
$v = v_1|_{6V} + v_2|_{3A} = 2 + 8 = 10V$



(b)

Superposition

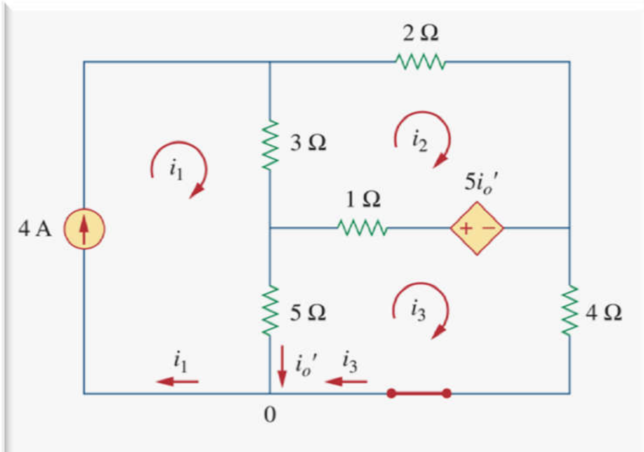
Example: Find i_o in the circuit of shown figure using superposition.



Since there are two sources, let

$$i_o = i_o|_{4A} + i_o|_{20v}$$

To obtain $i_o|_{4A}$ we set the voltage source to zero, as shown in fig(a). Applying mesh analysis gives



(a)

For loop 1

$$i_1 = 4A \qquad (1)$$

For loop 2

$$-3i_1 + 6i_2 - 1i_3 - 5i_o|_{4A} = 0 \qquad (2)$$

For loop 3

$$-5i_1 - 1i_2 + 10i_3 + 5i_o|_{4A} = 0 \qquad (3)$$

Superposition

but

$i_o = i_1 - i_3$

$i_3 = 4 - i_o \qquad (4)$

Substitute eq.(1) & eq.(4) in eq.(2) &eq.(3)

$3i_2 - 2i_o|_{4A} = 8$

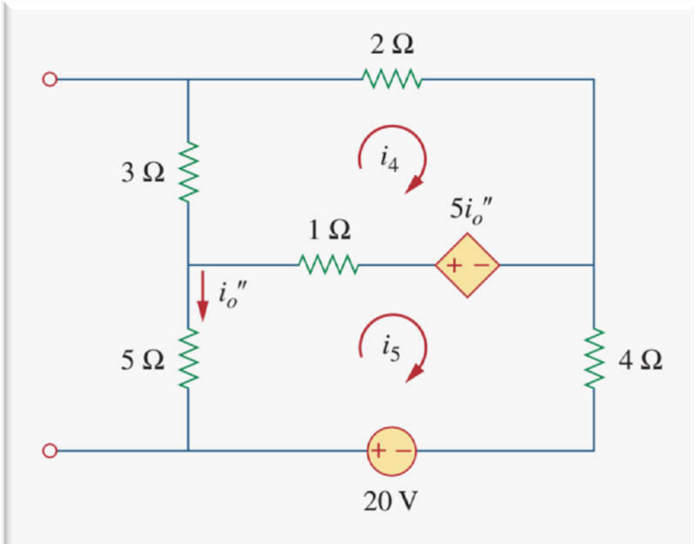
$i_2 + 5i_o|_{4A} = 20$

Which can be solved to get

$i_o\,|_{4A} = \frac{52}{17}\text{A} \qquad (5)$

To obtain $i_o|_{20\text{v}}$ eq.(1) we turn off the 4A current source so that the circuit becomes that shown in fig(b). For loop 4, KVL gives

$6i_4 - i_5 - 5i_o|_{20V} = 0 \qquad (6)$



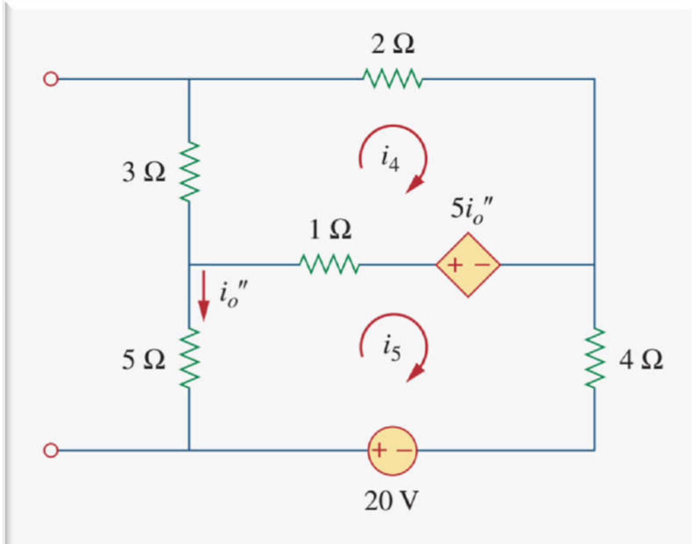
(b)

Superposition

For loop 5,

$$-i_4 + 10i_5 - 20 + 5i_o|_{20V} = 0 \qquad (7)$$

But $i_5 = -i_o|_{20V}$. Substituting this in eq.(6) &eq.(7)



(b)

$$6i_4 - 4i_o|_{20V} = 0$$

$$i_4 + 5i_o|_{20V} = -20$$

Which can be solved to get

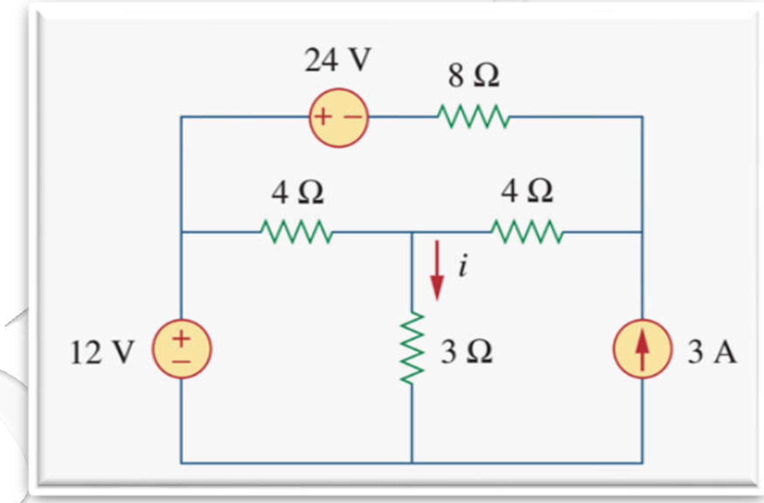
$$i_o|_{20V} = \frac{-60}{17}\text{A} \qquad (8)$$

now

$$i_o = i_o|_{4A} + i_o|_{20v} = \frac{52}{17} - \frac{60}{17} = -\frac{8}{17} = -0.4706\text{A}$$

Superposition

Example: Find *i* in the circuit of shown figure using superposition.



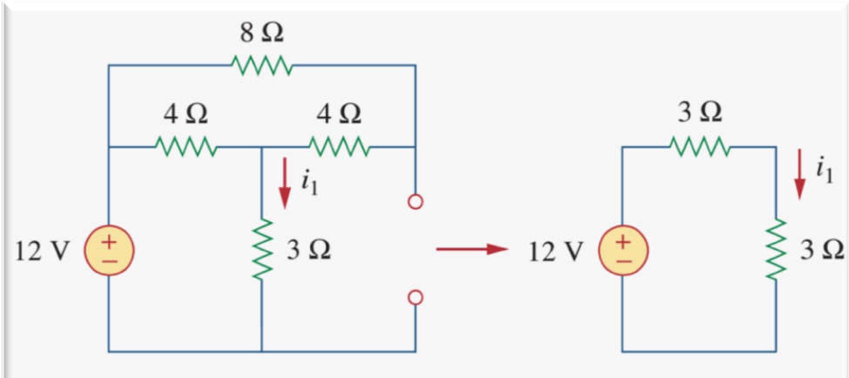
we have three sources. Let

$$\boldsymbol{i = i_1 + i_2 + i_3}$$

where i_1, i_2 and i_3 are due to the 12V, 24V, and 3A sources respectively.

To get i_1, consider fig.(a). Combining 4Ω (on the right hand side) in series with 8Ω gives 12 Ω. the 12 Ω in parallel with 4 Ω gives12×4/16=3Ω. thus,

$$\boldsymbol{i_1 = \frac{12}{6}\text{=2A}}$$



(a)

To get i_2 consider fig.(b), applying mesh analysis gives

Superposition

$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad (1)$

$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4} i_b \qquad (2)$

Substitute eq.(2) in eq.(1) gives

$i_2 = i_b = -1$

To get i_2 consider fig.(c), using nodal analysis gives

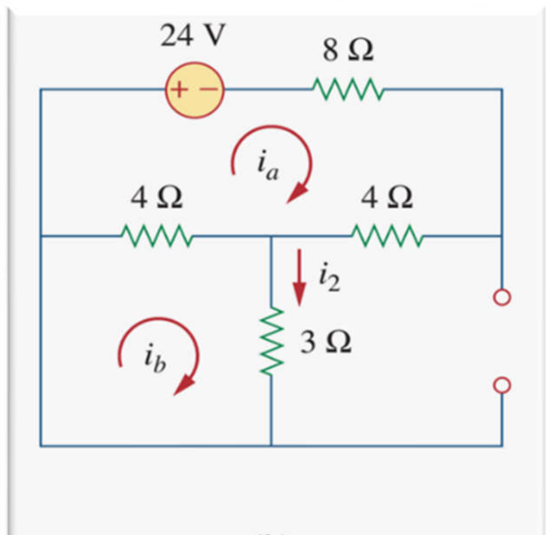
$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - v_1 \qquad (3)$

$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3} v_1 \qquad (4)$

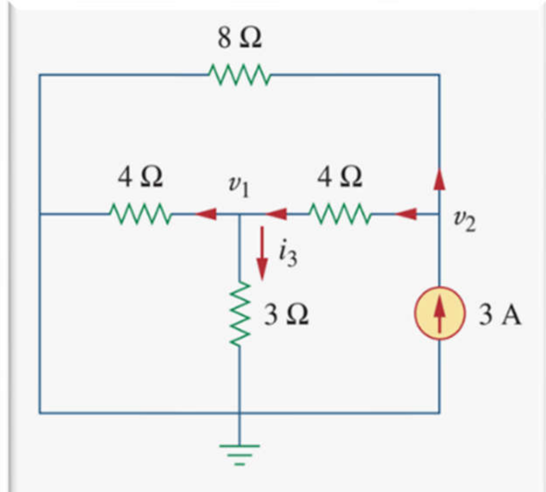
Substitute eq.(4) in eq.(3) leads to $v_1 = 3V$ and

$i_3 = \frac{v_1}{3} = 1A$

$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2A$



(b)



(c)


Q&A