Electric Circuits I
Thevenin Theorem

Dr. Firas Obeidat
Thevenin Theorem

- Thevenin’s theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

- One of the main uses of Thévenin’s theorem is the replacement of a large part of a circuit, often a complicated and uninteresting part, with a very simple equivalent. The new, simpler circuit enables us to make rapid calculations of the voltage, current, and power which the original circuit is able to deliver to a load. It also helps us to choose the best value of this load resistance.
Thevenin Theorem

According to Thevenin’s theorem, the linear circuit in fig.(a) can be replaced by that in fig.(b) (The load in the figure may be a single resistor or another circuit.). The circuit to the left of the terminals in fig.(b) is known as the Thevenin equivalent circuit.

- **Thevenin’s theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $V_{Th}$ in series with a resistor $R_{Th}$, where $V_{Th}$ is the open-circuit voltage at the terminals and $R_{Th}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.

---

*Dr. Firas Obeidat – Philadelphia University*
Thevenin Theorem

How to find the Thevenin equivalent voltage ($V_{Th}$) and resistance ($R_{Th}$)

- Suppose the two circuits in fig.(a)&(b) are equivalent. Two circuits are said to be equivalent if they have the same voltage-current relation at their terminals.
- Terminals $a-b$ are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals in fig(a) must be equal to the voltage source in fig(b), Thus $V_{Th}$ is the open-circuit voltage across the terminals, that is, $V_{Th} = V_{oc}$.
- Turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals $a-b$ in fig.(a) must be equal to $R_{Th}$ in fig.(b) because the two circuits are equivalent. Thus, $R_{Th}$ is the input resistance at the terminals when the independent sources are turned off. that is, $R_{Th} = R_{in}$.
To find the Thevenin resistance $R_{Th}$, two cases must be considered:

- **Case 1:** If the network has no dependent sources, turn off all independent sources. $R_{Th}$ is the input resistance of the network looking between terminals $a-b$.

- **Case 2:** If the network has dependent sources, we turn off all independent sources. Dependent sources are not to be turned off because they are controlled by circuit variables. Apply a voltage source at terminals $a-b$ and determine the resulting current $i_o$. Then, $R_{Th}=v_o/i_o$. Or insert a current source at terminals $a-b$ as shown in fig. (b) and find the terminal voltage $v_o$. Then, $R_{Th}=v_o/i_o$. The two approaches will give the same result. We may use $v_o=1V$ or $i_o=1A$. 

Dr. Firas Obeidat – Philadelphia University
Thevenin Theorem

Example *(the circuit has only independent sources)*: Find the Thevenin equivalent circuit of the circuit in shown figure, to the left of the terminals a-b. Then find the current through $R_L=6, 16$ and $36\Omega$.

Find $R_{Th}$ by turning off the $32V$ voltage source (replacing it with a short circuit) and the $2A$ current source (replacing it with an open circuit). The circuit becomes as shown in fig.(a).

$$R_{Th} = \frac{4||12+1}{16} = \frac{4 \times 12}{16} + 1 = 4\Omega$$

To find $V_{Th}$ consider the circuit in fig.(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \quad i_2 = -2A$$

Solving for $i_1$, we get

$$i_1 = 0.5A$$
Thevenin Theorem

\[ v_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30V \]

\[ i_L = \frac{v_{Th}}{R_{Th} + RL} = \frac{30}{4 + RL} \]

When \( R_L = 6 \)  
\[ i_L = \frac{30}{4+6} = 3A \]

When \( R_L = 16 \)  
\[ i_L = \frac{30}{4+16} = 1.5A \]

When \( R_L = 36 \)  
\[ i_L = \frac{30}{4+36} = 0.75A \]

**Example** *(the circuit has only independent sources):* Find the Thevenin equivalent circuit of the circuit in shown figure at terminals *a-b.*

Find \( R_{Th} \) by turning off the voltage source, the short-circuit replacement of the voltage source \( E \) provides a direct connection between \( c \) and \( c' \) as shown in fig.(a). Then rearranging the circuit in fig. (a) to get the circuit in fig.(b)
Thevenin Theorem

\[ R_{Th} = \frac{R_1}{R_2 + R_3} \frac{R_2}{R_3} = \frac{6\Omega}{3\Omega + 4\Omega} \frac{4\Omega}{\Omega} = 2\Omega + 3\Omega = 5\Omega \]

To get \( v_{Th} \), the circuit is redrawn as in fig.(c). The absence of a direct connection between \( a \) and \( b \) results in a circuit with three parallel branches. The voltages \( V_1 \) and \( V_2 \) can therefore be determined using the voltage divider rule:

\[ V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{6 \times 7}{6 + 3} = \frac{42}{9} = 48 \text{ V} \]

\[ V_1 = \frac{R_2 E}{R_2 + R_4} = \frac{12 \times 7}{12 + 4} = \frac{84}{16} = 5.25 \text{ V} \]

applying KVL to the top loop

\[ v_{Th} = V_2 - V_1 = 54 - 48 = 6 \text{ V} \]
Thevenin Theorem

Example (the circuit has independent and dependent sources): Find the Thevenin equivalent of the circuit in shown figure at terminals a-b.

To find $R_{Th}$ set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, excite the network with a voltage source connected to the terminals as indicated in fig.(a). You may set $v_o=1V$.

Applying mesh analysis to loop 1 in the circuit of fig(a) results in.

$-2v_x + 2(i_1 - i_2) = 0$, \text{or} \quad v_x = i_1 - i_2$

But $v_x = -4i_2 = i_1 - i_2$

$i_1 = -3i_2$

For loop 2 and 3, applying KVL produces

$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$

$6(i_3 - i_2) + 2i_3 + 1 = 0$
Thevenin Theorem

Solving these equations gives
\[ i_3 = -\frac{1}{6} A \]

But \[ io = -i_3 = \frac{1}{6} A \]

\[ R_{Th} = \frac{1}{io} = 6 \Omega \]

To get \( v_{Th} \), find \( v_o \) in the circuit of fig.(b).

Applying mesh analysis, we get.

\[ i_1 = 5 \ A \quad (1) \]

\[ -2v_x + 2(i_3 - i_2) = 0, \quad \text{or} \quad vx = i_3 - i_2 \quad (2) \]

\[ 4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0 \]

\[ 12i_2 - 4i_1 - 2i_3 = 0 \]

But \( v_x = 4(i_1 - i_2) \), solving these equations leads to

\[ i_2 = \frac{10}{3} A \]

\[ v_{Th} = v_{oc} = 6i_2 = 20 V \]
Thevenin Theorem

Example (the circuit has independent and dependent sources): Determine the Thevenin equivalent of the circuit in the shown circuit at terminals $a-b$.

To find $R_{Th}$ excite the circuit with either voltage source or current source.

Applying mesh analysis for fig. (b)

Assuming $v_o=1kV$.

\[2000i_1 + 3000i_2 + 1000 = 0\]

\[2i_1 + 3i_2 = -1 \quad (1)\]

\[-i_1 + i_2 = \frac{v_x}{4000} \]

But $v_x = v_o = 1000$

\[-i_1 + i_2 = \frac{v_x}{4000} = \frac{1000}{4000} = \frac{1}{4} \quad (2)\]

Use elimination technique to solve eq.(1) and eq.(2) to get $i_2$. 

\[\text{Diagram of circuit (a) and (b)}\]
Thevenin Theorem

\[2i_1 + 3i_2 = -1 \quad (1)\]

\[-i_1 + i_2 = \frac{1}{4} \quad (2)\]

Multiply eq.(2) by 2 then add the resulting equation to eq.(1)

\[2i_1 + 3i_2 = -1 \quad (1)\]

\[-2i_1 + 2i_2 = \frac{2}{4}\]

\[5i_2 = -\frac{1}{2} \quad \Rightarrow \quad i_2 = -0.1A\]

\[i_o = -i_2 = 0.1A\]

\[R_{Th} = \frac{v_o}{i_o} = \frac{1000}{0.1} = 10k\Omega\]

To get \(v_{Th}\), apply KVL around the outer loop in fig.(a)

\[-4 + 2 \times 10^3 \left( -\frac{v_x}{4000} \right) + 3 \times 10^3 (0) + v_x = 0\]

\[v_x = 8V = v_{Th}\]

Thévenin equivalent of fig.(c) is obtained.
Thevenin Theorem

Example (the circuit has only dependent sources): Determine the Thevenin equivalent of the circuit in the shown circuit at terminals \( a-b \).

Excite the circuit with either a 1V voltage source or a 1A current source.

writing the nodal equation at \( a \) in fig.(b)
Assuming \( i_o=1A \).

\[
2i_x+(v_o-0)/4+(v_o-0)/2+(-1)=0 \tag{1}
\]

\[
i_x=-v_o/2 \tag{2}
\]

Substitute eq.(1) in eq.(2) yields

\[
2(-v_o/2)+(v_o-0)/4+(v_o-0)/2+(-1)=0 \tag{2}
\]

\[
-1+1/4+1/2)v_o-1=0 \quad v_o=-4V
\]

\[
R_{Th}=v_o/i_o=-4/1=-4\Omega
\]
Maximum Power Transfer

- The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.
- If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in the figure, the power delivered to the load is

\[ p = i^2 R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L \]

- For a given circuit, \( V_{Th} \) and \( R_{Th} \) are fixed. By varying the load resistance the power delivered to the load varies as sketched in the show figure.

- If \( R_L \) varies from 0 to \( \infty \), the maximum power delivered to the load occurs when \( R_L \) is equal to \( R_{Th} \). This is known as the maximum power theorem.
Maximum Power Transfer

**Maximum power** is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

**The Proof**

\[ p = i^2 R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L \]

differentiate \( p \) in the above equation with respect to \( R_L \) and set the result equal to zero.

\[
\frac{dp}{dR_L} = V_{Th}^2 \left[ \frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right]
= V_{Th}^2 \left[ \frac{R_{Th} + R_L - 2R_L}{(R_{Th} + R_L)^3} \right] = V_{Th}^2 \left[ \frac{R_{Th} - R_L}{(R_{Th} + R_L)^3} \right] = 0
\]

This implies that \( R_{Th} = R_L \)

Substitute the result in power equation to get the maximum power.

\[
p_{max} = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L
\]

\[
p_{max} = \left( \frac{V_{Th}}{R_{Th}} \right)^2 R_{Th}
\]

\[
p_{max} = \frac{V_{Th}^2}{(2R_{Th})^2} R_{Th}
\]

\[
p_{max} = \frac{V_{Th}^2}{4R_{Th}^2} R_{Th}
\]
**Example:** Find the value of for maximum power transfer in the circuit of the shown figure. Find the maximum power.

At maximum power $R_{Th} = R_L$, so, we need to find the Thevenin resistance and the Thevenin voltage across the terminals $a-b$.

$$R_{Th} = 2 + 3 + 6/12 = 5 + (6 \times 12)/18 = 9 \Omega$$

To get $V_{Th}$ use mesh analysis

$$-12 + 18i_1 - 12i_2 = 0, \quad i_2 = -2A$$

Solve the above equations to get $i_1 = -2/3A$

Applying KVL around the outer loop to get $V_{Th}$ across terminals $a-b$.

$$-12 + 6i_1 + 3i_2 + 2 \times 0 + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22V$$

$$R_L = R_{Th} = 9\Omega$$

$$p_{max} = \frac{22^2}{4 \times 9} = 13.44W$$
Maximum Power Transfer

Example: Find the value of for maximum power transfer in the circuit of the shown figure. Find the maximum power.

Since $v_\pi = 0$, the dependent current source is an open circuit, and $R_{TH} = 1\, k\Omega$

$$v_{oc} = -0.03v_\pi(1000) = -130v_\pi$$

$$v_\pi = (2.5 \times 10^{-3} \sin 440t) \left( \frac{3864}{300 + 3864} \right)$$

$$(a)$$

$$v_{oc} = -69.6\sin 440t \, mV$$

$$R_L = R_{Th} = 1\, k\Omega$$

$$p_{\text{max}} = \frac{V_{Th}^2}{4R_{Th}} = 1.211\sin^2 440t \, \mu W$$

(b)