Power Electronics
Single Phase AC-AC Converters

Dr. Firas Obeidat
Introduction

An ac voltage controller is a converter that controls the voltage, current, and average power delivered to an ac load from an ac source.

The phase-controlled ac voltage controller has several practical uses including light-dimmer circuits and speed control of induction motors.

In a switching scheme called phase control, switching takes place during every cycle of the source, in effect removing some of the source waveform before it reaches the load.

Integral-cycle control, the source is connected and disconnected for several cycles at a time.
The Single Phase AC Voltage Controller

Basic Operation

For the single phase AC voltage controller shown, electronic switches are shown as parallel thyristors (SCRs). This SCR arrangement makes it possible to have current in either direction in the load. This SCR connection is called antiparallel or inverse parallel because the SCRs carry current in opposite directions. A triac is equivalent to the antiparallel SCRs. Other controlled switching devices can be used instead of SCRs.

- **Load current contains both positive and negative half-cycles.** An analysis identical to that done for the controlled half-wave rectifier can be done on a half cycle for the voltage controller. Then, by symmetry, the result can be extrapolated to describe the operation for the entire period.
- **$S_1$** conducts if a gate signal is applied during the positive half-cycle of the source. $S_1$ conducts until the current in it reaches zero.
- A gate signal is applied to $S_2$ during the negative half-cycle of the source, providing a path for negative load current.
The Single Phase AC Voltage Controller

Basic Operation

Basic observations about this controller

- The SCRs cannot conduct simultaneously.

- The load voltage is the same as the source voltage when either SCR is on. The load voltage is zero when both SCRs are OFF.

- The switch voltage $v_{sw}$ is zero when either SCR is ON and is equal to the source voltage when neither is ON.

- The average current in the source and load is zero if the SCRs are on for equal time intervals. The average current in each SCR is not zero because of unidirectional SCR current.

- The rms current in each SCR is $1/\sqrt{2}$ times the rms load current if the SCRs are on for equal time intervals.
The Single Phase AC Voltage Controller - Resistive Load

Let the voltage source be

\[ v_s(\omega t) = V_m \sin\omega t \]

Output voltage is

\[ v_s(\omega t) = \begin{cases} V_m \sin\omega t & \alpha < \omega t < \pi \text{ and } \pi + \alpha < \omega t < 2\pi \\ 0 & \text{otherwise} \end{cases} \]

The \textit{rms} load voltage is

\[ V_{o,rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} (V_m \sin\omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}} \left( \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right) \]
The Single Phase AC Voltage Controller - Resistive Load

The $rms$ current in the load and the source is

\[
I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{V_m}{R\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}
\]

The power factor of the load is

\[
pf = \frac{P}{S} = \frac{V_{o,rms}^2}{V_{s,rms}I_{s,rms}} = \frac{V_{o,rms}^2}{V_{s,rms}(V_{o,rms}/R)} = \frac{V_{o,rms}}{V_{s,rms}} = \frac{V_m}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}
\]

\[
pf = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}
\]

The $pf=1$ for $\alpha=0$, which is the same as for an uncontrolled resistive load, and the power factor for $\alpha>0$ is less than 1.

The average source current is zero because of half-wave symmetry.

\[
I_{s,Avg} = I_{o,Avg} = 0
\]
The Single Phase AC Voltage Controller - Resistive Load

The average SCR current is

\[ I_{SCR,avg} = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m \sin \omega t}{R} d\omega t = \frac{V_m}{2\pi R} (1 + \cos \alpha) \]

The \textit{rms} SCR current is

\[ I_{SCR,rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} \left( \frac{V_m \sin \omega t}{R} \right)^2 d\omega t} = \frac{V_m}{2R} \sqrt{\frac{1}{\pi} - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \]

\[ I_{SCR,rms} = \frac{V_m}{\sqrt{2\sqrt{2}R}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} = \frac{I_{o,rms}}{\sqrt{2}} \]
The Single Phase AC Voltage Controller - Resistive Load

Example: The single-phase ac voltage controller has a 120-V \(\text{rms}\) 60-Hz source. The load resistance is 15 \(\Omega\). Determine (a) the delay angle required to deliver 500 W to the load, (b) the \(\text{rms}\) source current, (c) the \(\text{rms}\) and average currents in the SCRs, (d) the power factor.

(a) \[V_{o,rms} = \frac{V_m}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} = \frac{\sqrt{2} \times 120}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}\]

\[P = \frac{V_{o,rms}^2}{R} \rightarrow V_{o,rms}^2 = PR = 500 \times 15 = 7500\]

\[V_{o,rms}^2 = 120^2 \left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)\]

\[\therefore 7500 = 14400 \left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)\]

\[\frac{\alpha}{\pi} - \frac{\sin(2\alpha)}{2\pi} = 0.479\]

\[2\alpha - \sin(2\alpha) = 3.01\]

\[\alpha = 1.54 \text{ rad} = 88.1^\circ\]
The Single Phase AC Voltage Controller - Resistive Load

(b) From (a) $V_{o,\text{rms}}^2 = 7500 \quad \rightarrow V_{o,\text{rms}} = 86.6$

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R} = \frac{86.6}{15} = 5.77 \text{ A}$$

(c) $I_{\text{SCR,Avg}} = \frac{V_m}{2\pi R} (1 + \cos \alpha) = \frac{\sqrt{2} \times 120}{2\pi 15} (1 + \cos 88.1) = 1.86 \text{ A}$

$$I_{\text{SCR,rms}} = \frac{I_{o,\text{rms}}}{\sqrt{2}} = \frac{5.77}{\sqrt{2}} = 4.08 \text{ A}$$

(d) $\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} = \frac{500}{120 \times 5.77} = 0.722$
When a gate signal is applied to $S_1$ at $\omega t=\alpha$ in single phase AC voltage controller with RL load, Kirchhoff’s voltage law for the circuit is expressed as:

$$V_m \sin \omega t = R i_o(t) + L \frac{di_o(t)}{dt}$$

The solution for current in this equation (as obtained in the controlled half wave rectifier section) is

$$i_o(\omega t) = \begin{cases} \frac{V_m}{Z} \left[ \sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega \tau} \right] & \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2}$$
$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The extinction angle $\beta$ is the angle at which the current returns to zero, when $\omega t=\beta$,

$$i_o(\beta) = \frac{V_m}{Z} \left[ \sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega \tau} \right]$$
The above equation must be solved numerically for $\beta$.

A gate signal is applied to $S_2$ at $\omega t=\pi+\alpha$, and the load current is negative but has a form identical to that of the positive half-cycle.

The angle ($\beta-\alpha$) is called the conduction angle $\gamma$.

$$\gamma = \beta - \alpha$$

In the interval between $\pi$ and $\beta$ when the source voltage is negative and the load current is still positive, $S_2$ cannot be turned on because it is not forward biased. The gate signal to $S_2$ must be delayed at least until the current in $S_1$ reaches zero, at $\omega t=\beta$. The delay angle is therefore at least $\beta - \pi$.

$$\alpha \geq \beta - \pi$$
The Single Phase AC Voltage Controller - RL Load

When \( \alpha=\theta \), equation (2) becomes

\[
\sin(\beta - \theta) = 0
\]

which has a solution

\[
\beta - \theta = \pi
\]

Therefore

\( Y = \pi \) \quad \text{When} \quad \alpha=\theta

When \( Y=\pi \), one SCR is always conducting, and the voltage across the load is the same as the voltage of the source. The load voltage and current are sinusoids for this case, and the circuit is analyzed using phasor analysis for AC circuits. The power delivered to the load is continuously controllable between the two extremes corresponding to full source voltage and zero.

The expression of \textit{rms} load current is

\[
I_{o,rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\beta} i_o^2(\omega t) \, d\omega t} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\beta} \left[ \frac{V_m}{Z} \sin(\omega t - \theta) - \frac{V_m}{Z} \sin(\alpha - \theta)e^{(a-\omega t)/\omega t} \right]^2 \, d\omega t}
\]
The Single Phase AC Voltage Controller - \textbf{RL Load}

The average output voltage can be found as

\[ V_{o,rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\beta} (V_m \sin \omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} (\beta - \alpha - \frac{1}{2} \sin 2\beta + \frac{1}{2} \sin 2\alpha)} \]

An other way to find the expression of \textit{rms} load current is

\[ I_{o,rms} = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} (\beta - \alpha - \frac{1}{2} \sin 2\beta + \frac{1}{2} \sin 2\alpha)} \]

Power absorbed by the load is determined from

\[ P = I_{o,rms}^2 R \]

The power factor of the load is

\[ pf = \frac{P}{S} = \frac{V_{o,rms} I_{o,rms}}{V_{s,rms} I_{s,rms}} = \frac{V_{o,rms}}{V_{s,rms}} = \sqrt{\frac{1}{\pi} (\beta - \alpha - \frac{1}{2} \sin 2\beta + \frac{1}{2} \sin 2\alpha)} \]
The Single Phase AC Voltage Controller - *RL Load*

The *rms* current in each SCR is

\[ I_{\text{SCR,rms}} = \frac{I_{o,rms}}{\sqrt{2}} \]

The average load current is zero,

\[ I_{o,\text{Avg}} = 0 \]

Each SCR carries one-half of the current waveform, making the average SCR current

\[ I_{\text{SCR,Avg}} = \frac{1}{2\pi} \int_{\alpha}^{\beta} i_o(\omega t) \, d\omega t = \frac{1}{2\pi} \int_{\alpha}^{\beta} \frac{V_m}{Z} \left[ \sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha-\omega t)/\omega t} \right] d\omega t \]
The Single Phase AC Voltage Controller - RL Load

Example: For the single-phase voltage controller with RL load, the source is 120V at 60 Hz, and the load is a series RL combination with R=20Ω and L=50mH. The delay angle $\alpha$ is 90°. Determine (a) an expression for load current for the first half-period, (b) the $rms$ load current, (c) the $rms$ SCR current, (d) the average SCR current, (e) the power delivered to the load, and (f) the power factor.

(a) $Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(20)^2 + [(377)(0.05)]^2} = 27.5 \Omega$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{377(0.05)}{20}\right) = 0.756 \text{ rad}$

$\omega \tau = \omega \left(\frac{L}{R}\right) = 377 \left(\frac{0.05}{20}\right) = 0.943 \text{ rad}$

$\frac{V_m}{Z} = \frac{120\sqrt{2}}{27.5} = 6.18 \text{ A}$

$\alpha = 90° = 1.57 \text{ rad}$

$\frac{V_m}{Z} \sin (\alpha - \theta) e^{\alpha/\omega \tau} = 23.8 \text{ A}$

$i_0(\omega t) = 6.18 \sin (\omega t - 0.756) - 23.8e^{-\omega t/0.943} \text{ A for } \alpha \leq \omega t \leq \beta$

The extinction angle is determined from the numerical solution of $i(\beta)=0$ in the above equation.
\[ \beta = 3.83 \text{ rad} = 220^\circ \]

\[ (b) \quad I_{o, \text{rms}} = \sqrt{\frac{1}{\pi}} \int_{1.57}^{3.83} \left[ 6.18 \sin(\omega t - 0.756) - 23.8e^{-\omega t/0.943} \right]^2 d(\omega t) = 2.71 \text{ A} \]

Or

\[ I_{o, \text{rms}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \frac{V_m}{\sqrt{2}} \frac{1}{\pi} \left( \beta - \alpha - \frac{1}{2} \sin 2\beta - \frac{1}{2} \sin 2\alpha \right) \]

\[ I_{o, \text{rms}} = \frac{1}{27.5} \times 120 \times \sqrt{\frac{1}{\pi} (3.83 - 1.57 - \frac{1}{2} \sin 440 + 0)} = 3.27 \text{ A} \]

(c) \[ I_{\text{SCR, rms}} = \frac{I_{o, \text{rms}}}{\sqrt{2}} = \frac{2.71}{\sqrt{2}} = 1.92 \text{ A} \]

(d) \[ I_{\text{SCR, avg}} = \frac{1}{2\pi} \int_{1.57}^{3.83} \left[ 6.18 \sin(\omega t - 0.756) - 23.8e^{-\omega t/0.943} \right] d(\omega t) = 1.04 \text{ A} \]

(e) \[ P = I_{o, \text{rms}}^2 R = (2.71)^2 (20) = 147 \text{ W} \]

(f) \[ \text{pf} = \frac{P}{S} = \frac{P}{V_{s, \text{rms}} I_{s, \text{rms}}} = \frac{147}{(120)(2.71)} = 0.45 \]