Power Electronics

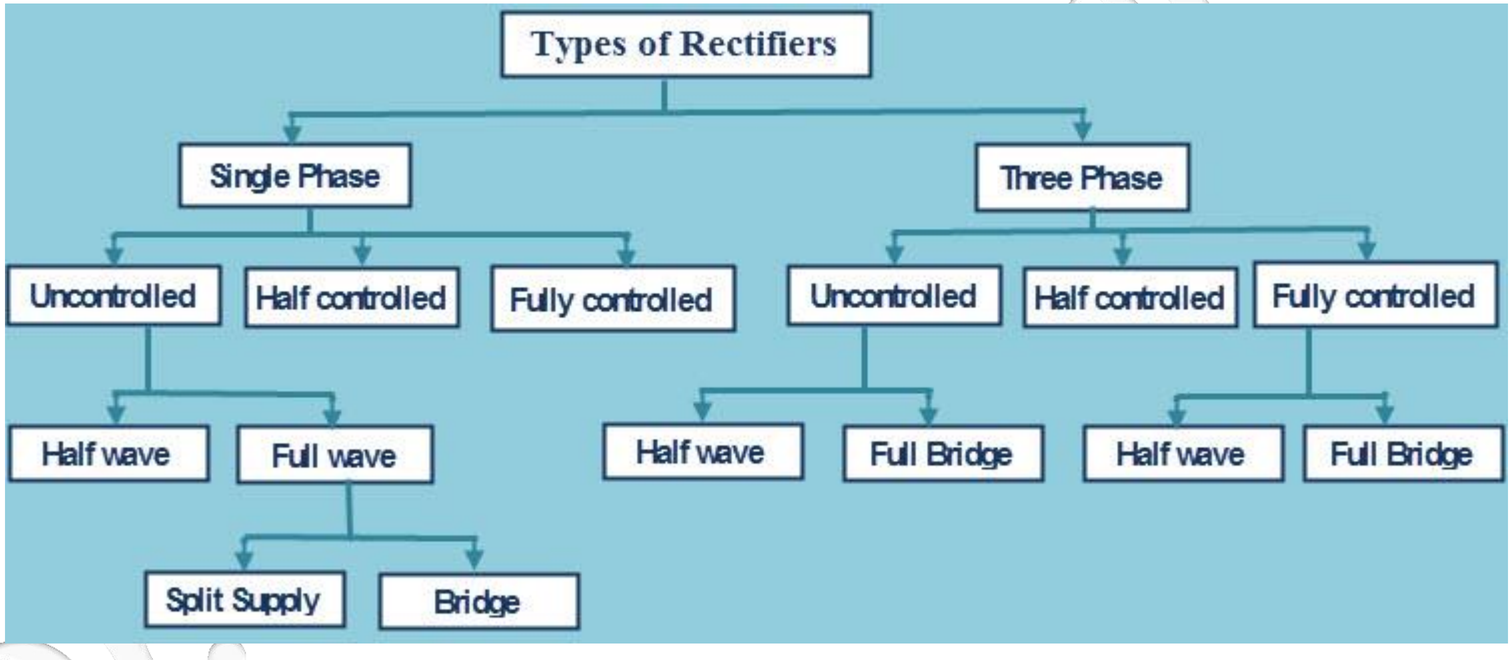
Single Phase Uncontrolled Half Wave Rectifiers

Dr. Firas Obeidat

Table of contents

Introduction

A rectifier is an electrical device that converts alternating current (AC) to direct current (DC), which flows in only one direction. The process is known as rectification.



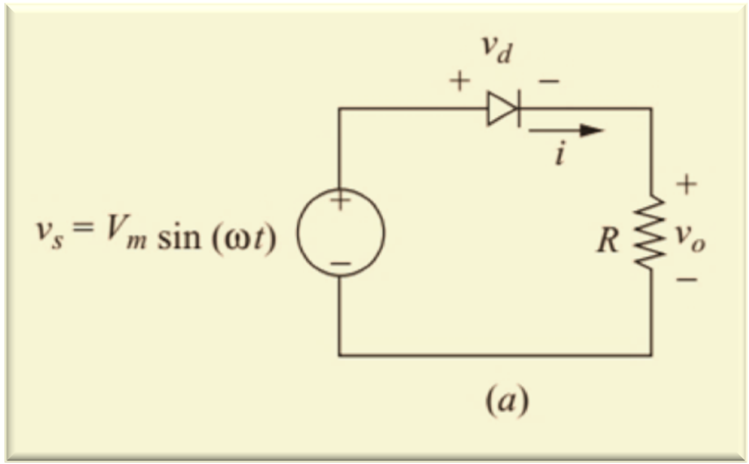
There are many applications for rectifiers. Some of them are: variable speed dc drives, battery chargers, DC power supplies and Power supply for a specific application like electroplating.

Resistive Load

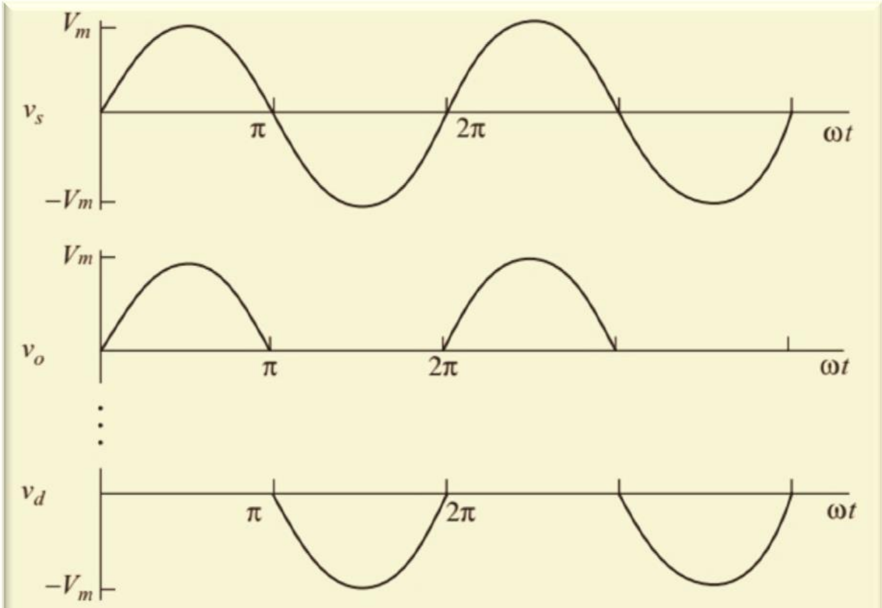
- A basic half-wave rectifier with a resistive load is shown in fig. a. The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only.

- For the positive half-cycle of the source in this circuit, the diode is on (forward-biased). Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive.

- For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.



(a)



(b)

Resistive Load

$$v_s = V_m \sin(\omega t)$$

The dc component V_o of the output voltage is the average value of a half-wave rectified sinusoid

$$V_{dc} = V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}$$

The dc component of the current for the purely resistive load is

$$I_{dc} = I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}$$

The *rms* values of V_o and I_o can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_m}{2R}$$

Resistive Load

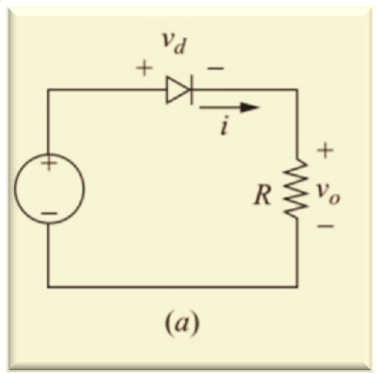
The Average output dc power is

$$p_{dc} = V_{dc} I_{dc} = {I_{dc}}^2 R = \frac{{V_{dc}}^2}{R} = \frac{{V_m}^2}{\pi^2 R}$$

The *rms* output dc power is

$$p_{ac} = V_{rms} I_{rms} = {I_{rms}}^2 R = \frac{{V_{rms}}^2}{R} = \frac{{V_m}^2}{4R}$$

Example: For the shown half-wave rectifier, the source is a sinusoid of 120 Vrms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (a) the average load current, (b) the dc and ac power absorbed by the load and (c) the power factor of the circuit.



(a)

(a)

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{5\pi} = 10.8 \text{ A}$$

Resistive Load

(b)

$$p_{dc} = \frac{{V_m}^2}{\pi^2 R} = \frac{169.7^2}{\pi^2 \times 5} = 583.57$$

$$V_{\text{rms}} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9 \text{ V}$$

$$p_{ac} = \frac{{V_{rms}}^2}{R} = \frac{84.9^2}{5} = 1441.6$$

(c)

The *rms* current in the resistor is

$V_m/(2R) = 17.0$ A

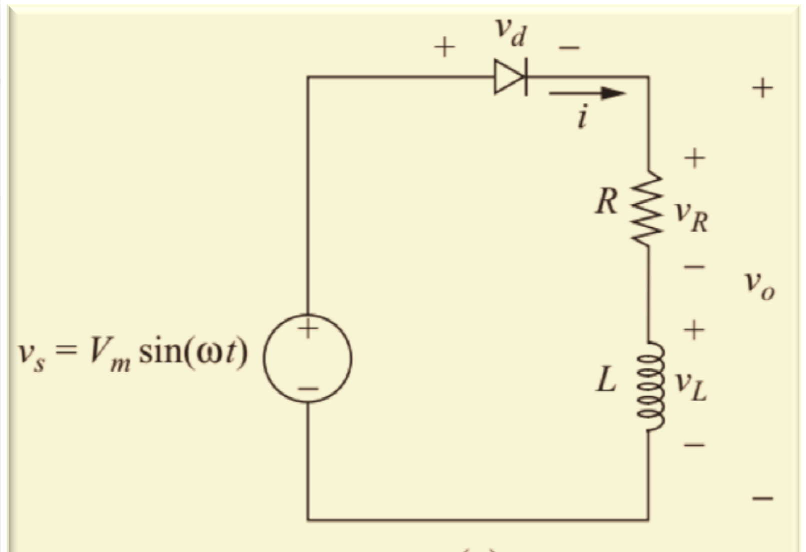
The power factor is

$$pf = \frac{p}{S} = \frac{p_{ac}}{V_{rms} I_{rms}} = \frac{1441.6}{120 * 17} = 0.707$$

R-L Load

- **Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit of fig. a, the diode becomes forward-biased. The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is**

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} \qquad (1)$$



(a)

The <u>dc component</u> of the output voltage is

$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\beta} sin\omega t \; d\omega t = \frac{V_m}{2\pi}(1 - cos\beta)$$

The <u>dc component</u> of the output current is

$$I_{dc} = \frac{V_m}{2\pi R}(1 - cos\beta)$$

The solution of equation (1) can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = i_f(t) + i_n(t)$$

R-L Load

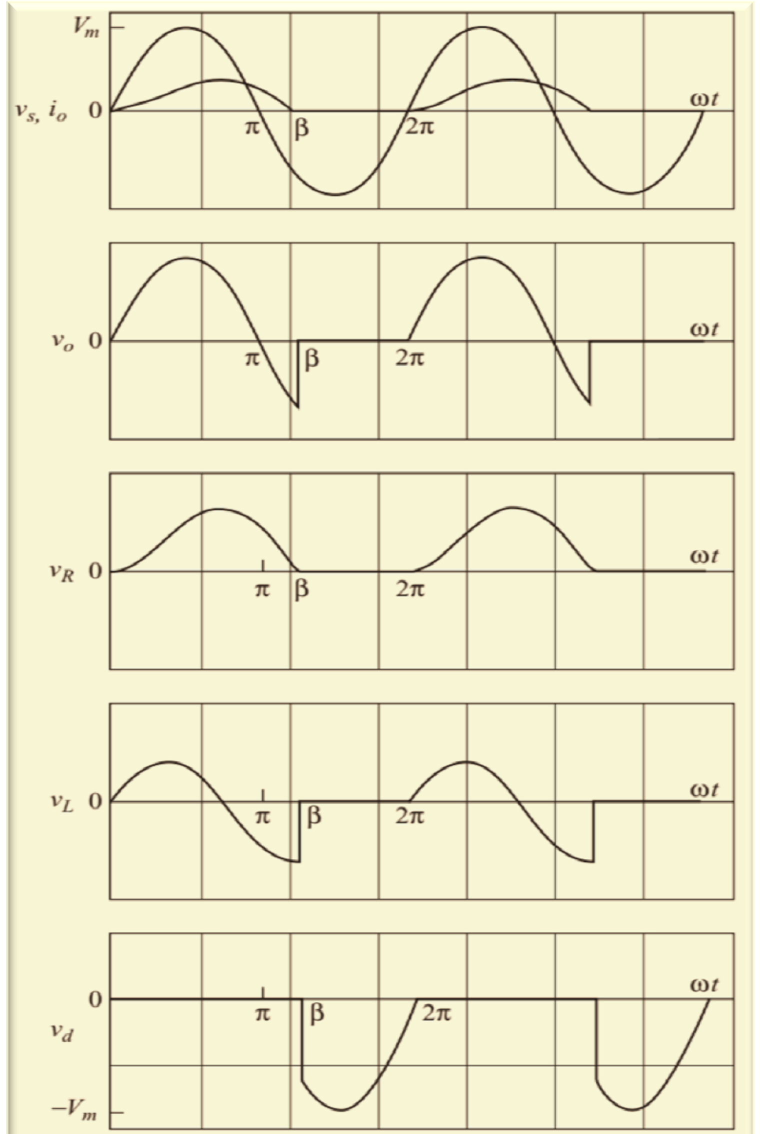
- The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present.
- This steady-state current can be found from phasor analysis, resulting in

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta)$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

- The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode.



(b)

R-L Load

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

For this first-order circuit, the natural response has the form

$$i_n(t) = Ae^{-t/\tau}$$

Where $\tau = \dfrac{L}{R}$ A= constant

Adding the forced and natural responses gets the complete solution.

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau} \qquad (2)$$

The constant *A* is evaluated by using the initial condition for current:
t=0, i(ωt)=0.
Using the initial condition and equation (2) to evaluate *A* yields

$$i(0) = \frac{V_m}{Z}\sin(0 - \theta) + Ae^0 = 0$$

$$A = -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin\theta$$

R-L Load

Substituting for A in equation (2) gives

$$i(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)e^{-t/\tau}$$

$$= \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-t/\tau}\right]$$

The final current equation can be written as

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] \qquad (3)$$

The point when the current reaches zero in Eq. (3-12) occurs when the diode turns off. The first positive value of ωt in equation (3) that results in zero current is called the extinction angle β.

To find β, substitute ωt= β in equation (3)

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}\right] = 0$$

Which reduces to $\quad \sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau} = 0$

There is no closed-form solution for β, and some numerical method is required.

R-L Load

To summarize, the current in the half-wave rectifier circuit with RL load is expressed as

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The <u>dc component</u> of the output current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\, d(\omega t)$$

Or it can be found as

$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\beta} sin\omega t\, d\omega t = \frac{V_m}{2\pi}(1 - cos\beta)$$

$$I_{dc} = I_o = \frac{V_m}{2\pi R}(1 - cos\beta)$$

R-L Load

The *rms* value of I_o can be written as

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d(\omega t)}$$

Or it can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\beta}(V_m sin\omega t)^2\,d\omega t} = \sqrt{\frac{{V_m}^2}{4\pi}(\beta - \frac{1}{2}sin2\beta)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{{V_m}^2}{4\pi}(\beta - \frac{1}{2}sin2\beta)}$$

R-L Load

Example: For the RL half-wave rectifier, R=100Ω, L=0.1 H, ω=377 rad/s, and V_m=100 V. Determine (a) an expression for the current in this circuit, (b) the average current, (c) the rms current, (d) the power absorbed by the RL load, and (e) the power factor.

$$Z = [R^2 + (\omega L)^2]^{0.5} = 106.9\,\Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 20.7^\circ = 0.361 \text{ rad}$$
$$\omega t = \omega L/R = 0.377 \text{ rad}$$

(a)

$$i(\omega t) = 0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377} \quad \text{A} \qquad \text{for } 0 \le \omega t \le \beta$$

$$\sin(\beta - 0.361) + \sin(0.361)e^{-\beta/0.377} = 0$$

Using a numerical root-finding program, β is found to be 3.50 rad, or 201°.

(b)

$$I_o = \frac{1}{2\pi}\int_0^{3.50}\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]d(\omega t) = 0.308 \text{ A}$$

(A numerical integration program is recommended.)

R-L Load

Io can be also found from

$$I_{dc} = I_o = \frac{V_m}{2\pi R}(1 - cos\beta) = \frac{100}{2\pi 100}(1 - cos201) = 0.308\ A$$

(c)

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{3.50}\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]^2 d(\omega t)} = 0.474\ \text{A}$$

I_{rms} can be also found from

$$I_{rms} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{V_m{}^2}{4\pi}\left(\beta - \frac{1}{2}sin2\beta\right)} = \frac{1}{106.9}\sqrt{\frac{100^2}{4\pi}\left(3.5 - \frac{1}{2}sin7\right)} = 0.489\ \text{A}$$

(d)

$$P = I_{\text{rms}}^2 R = (0.474)^2(100) = 22.4\ \text{W}$$

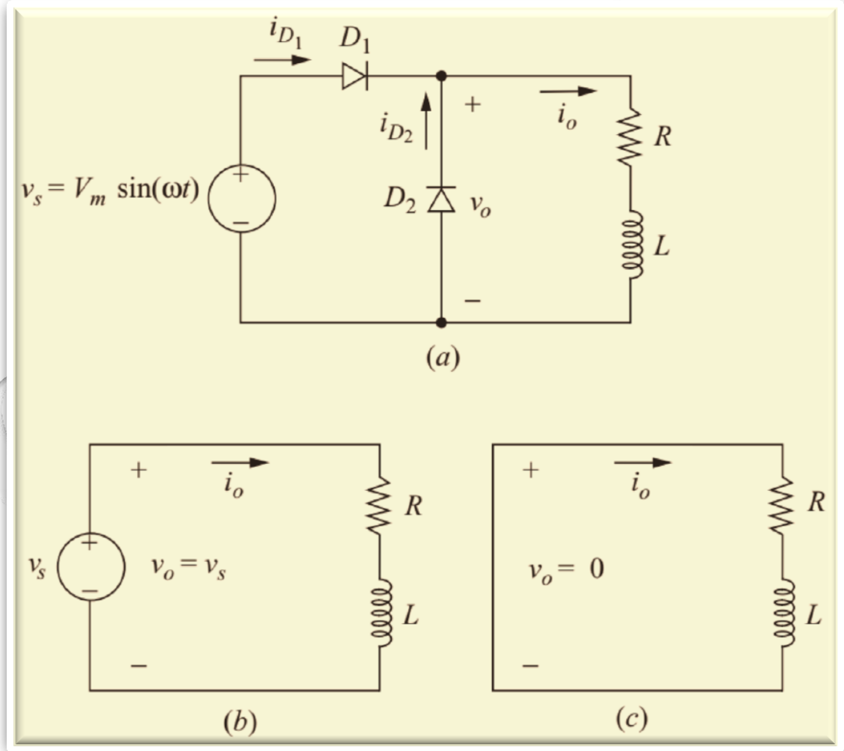
(e)

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{\text{rms}}} = \frac{22.4}{(100/\sqrt{2})0.474} = 0.67$$

Note that the power factor is not cosθ.

R-L Load with Freewheeling Diode

- A freewheeling diode D_2, can be connected across an RL load as shown in fig. a.
- Both diodes cannot be forward-biased at the same time. Diode D_1 will be ON when the source is positive, and diode D_2 will be ON when the source is negative.



For a positive source voltage,

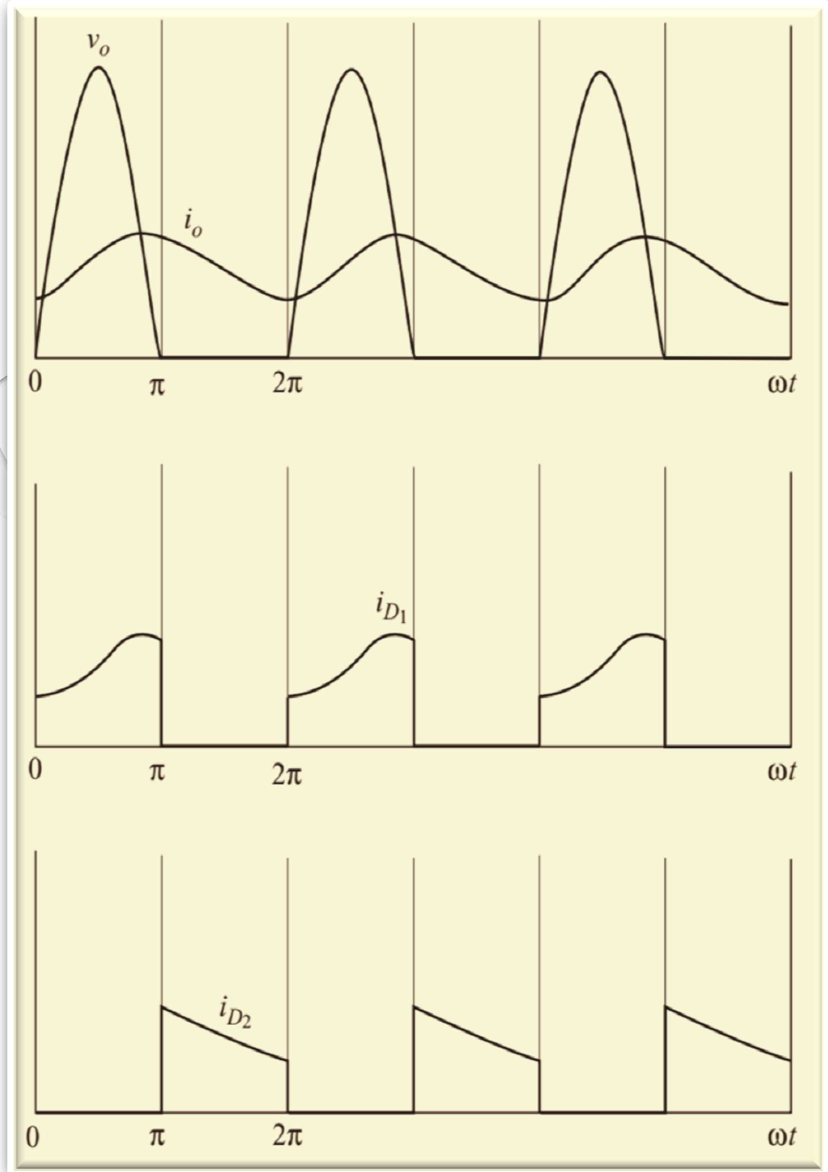
- D_1 is on.
- D_2 is off.
- The equivalent circuit is the same as that of fig. b.
- The voltage across the RL load is the same as the source.

For a negative source voltage,

- D_1 is off.
- D_2 is on.
- The equivalent circuit is the same at that of fig. c.
- The voltage across the RL load is zero

R-L Load with Freewheeling Diode

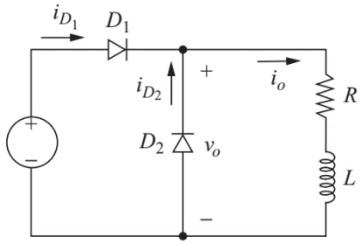
- Since the voltage across the RL load is the same as the source voltage when the source is positive and is zero when the source is negative, the load voltage is a half-wave rectified sine wave. Steady-state load, source, and diode currents are shown in the fig..



Example: Determine the average load voltage and current for the circuit, where R=2 Ω and L=25mH, V_m is 100 V, and the frequency is 60 Hz.

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.8 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{31.8}{2} = 15.9 \text{ A}$$



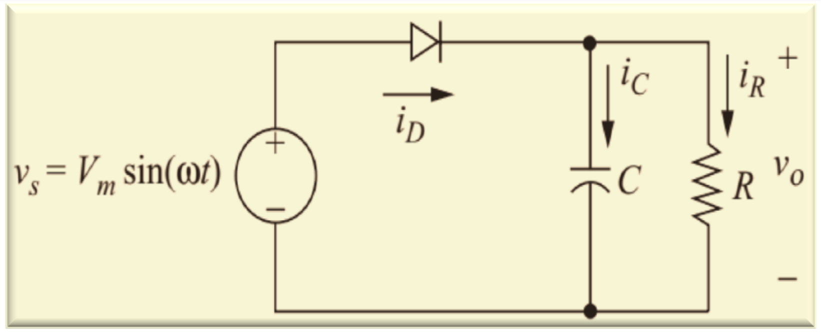
Half Wave Rectifier with a Capacitor Filter

- The purpose of the capacitor is to reduce the variation in the output voltage, making it more like dc. The resistance may represent an external load, and the capacitor may be a filter which is part of the rectifier circuit.

- Assuming the capacitor is initially uncharged and the circuit is energized at $\omega t=0$, the diode becomes forward-biased as the source becomes positive. With the diode on, the output voltage is the same as the source voltage, and the capacitor charges. The capacitor is charged to V_m when the input voltage reaches its positive peak at $\omega t=\pi/2$.

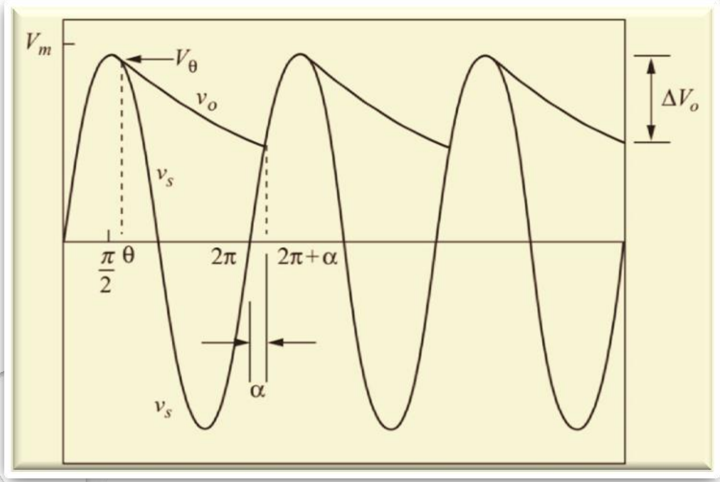
- As the source decreases after $\omega t=\pi/2$, the capacitor discharges into the load resistor. At some point, the voltage of the source becomes less than the output voltage, reverse-biasing the diode and isolating the load from the source. The output voltage is a decaying exponential with time constant RC while the diode is off.

- The angle $\omega t=\theta$ is the point when the diode turns off in the figure. The output voltage is described by



Half Wave Rectifier with a Capacitor Filter

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases} \quad (1)$$



where

$$V_\theta = V_m \sin \theta$$

The slopes of these functions are

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t)$$

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta \left(-\frac{1}{\omega RC}\right) e^{-(\omega t - \theta)/\omega RC}$$

At ωt=θ , the slopes of the voltage functions are equal:

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right) e^{-(\theta-\theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

Half Wave Rectifier with a Capacitor Filter

Solving for θ and expressing θ so it is in the proper quadrant,

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

In practical circuits where the time constant is large,

$$\theta \approx \frac{\pi}{2} \qquad \text{and} \qquad V_m \sin\theta \approx V_m$$

The angle at which the diode turns on in the second period, $\omega t = 2\pi + \alpha$, is the point when the sinusoidal source reaches the same value as the decaying exponential output:

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta) e^{-(2\pi + \alpha - \theta)/\omega RC}$$

$$\sin\alpha - (\sin\theta) e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$$

The above equation must be solved numerically for α.

The current in the resistor is calculated from

$$i_R = \frac{v_o}{R}$$

The current in the capacitor is calculated from

$$i_C(t) = C\frac{dv_o(t)}{dt} \qquad \textbf{Or} \qquad i_C(\omega t) = \omega C \frac{dv_o(\omega t)}{d(\omega t)}$$

Half Wave Rectifier with a Capacitor Filter

Using v_o from equation (1) we, get

$$i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m \sin\theta}{R}\right)e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases} \qquad (2)$$

The source current, which is the same as the diode current, is

$$i_S = i_D = i_R + i_C$$

Peak capacitor current occurs when the diode turns on at ωt=2π+α. From equation (2)

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

Resistor current at ωt=2π+α is obtained from equation (1)

$$i_R(2\omega t + \alpha) = \frac{V_m \sin(2\omega t + \alpha)}{R} = \frac{V_m \sin\alpha}{R}$$

Peak diode current is

$$I_{D,\text{peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)$$

Half Wave Rectifier with a Capacitor Filter

The effectiveness of the capacitor filter is determined by the variation in output voltage. This may be expressed as the difference between the maximum and minimum output voltage, which is the peak-to-peak ripple voltage. For the half wave rectifier with a capacitor filter, the maximum output voltage is V_m. The minimum output voltage occurs at $\omega t=2\pi+\alpha$, which can be computed from $V_m \sin\alpha$. The peak-to-peak ripple is expressed as

$$\Delta V_o = V_m - V_m \sin\alpha = V_m(1-\sin\alpha)$$

If $V_\theta \approx V_m$ and $\theta=\pi/2$, then (1) evaluated at $\alpha=\pi/2$ is

$$v_o(2\pi+\alpha) = V_m e^{-(2\pi+\pi/2-\pi/2)\omega RC} = V_m e^{-2\pi/\omega RC}$$

The ripple voltage can then be approximated as

$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1-e^{-2\pi/\omega RC}\right) \qquad (3)$$

the exponential in the above equation can be approximated by the series expansion:

$$e^{-2\pi/\omega RC} \approx 1-\frac{2\pi}{\omega RC}$$

Substituting the above equation in equation (3). The peak-to-peak ripple is approximately

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

Half Wave Rectifier with a Capacitor Filter

The output voltage ripple is reduced by increasing the filter capacitor C. As C increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

Example: The half-wave rectifier with a capacitor filter has a 120-V rms source at 60 Hz, R=500 =Ω, and C=100μF. Determine (a) an expression for output voltage, (b) the peak-to-peak voltage variation on the output, (c) an expression for capacitor current, (d) the peak diode current, and (e) the value of C such that V_o is 1 percent of V_m.

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85 \text{ rad}$$

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

Using numerical solution to get α

$$\sin\alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62/18.85)} = 0$$

$$\alpha = 0.843 \text{ rad} = 48°$$

Half Wave Rectifier with a Capacitor Filter

(a) an expression for output voltage.

$$v_o(\omega t) = \begin{cases} 169.7 \sin(\omega t) & 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ 169.5 e^{-(\omega t - 1.62)/18.85} & \theta \le \omega t \le 2\pi + \alpha \end{cases}$$

(b) the peak-to-peak voltage variation on the output

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

(c) an expression for capacitor current

$$i_C(\omega t) = \begin{cases} -0.339 e^{-(\omega t - 1.62)/18.85} \quad \text{A} & \theta \le \omega t \le 2\pi + \alpha \\ 6.4 \cos(\omega t) \quad \text{A} & 2\pi + \alpha \le \omega t \le 2\pi + \theta \end{cases}$$

(d) the peak diode current

$$I_{D,\text{peak}} = \sqrt{2}(120)\left[377(10)^{-4} \cos 0.843 + \frac{\sin 8.43}{500}\right]$$
$$= 4.26 + 0.34 = 4.50 \text{ A}$$

(e) the value of C such that V_o is 1 percent of V_m.

$$C \approx \frac{V_m}{fR(\Delta V_o)} = \frac{V_m}{(60)(500)(0.01 V_m)} = \frac{1}{300} \text{F} = 3333\ \mu\text{F}$$


Q
&
A