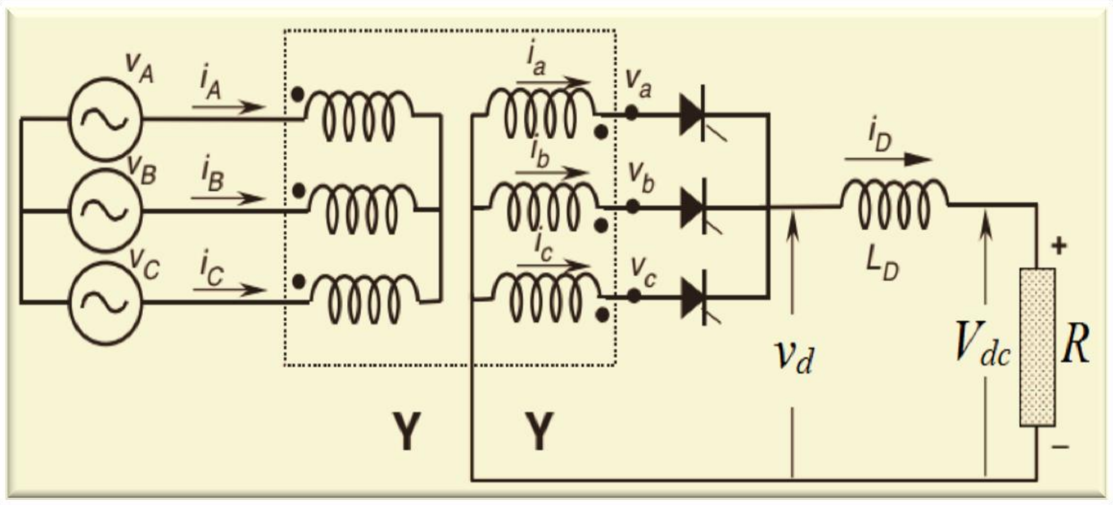
Power Electronics
Three Phase Controlled Rectifiers

Dr. Firas Obeidat

Table of contents

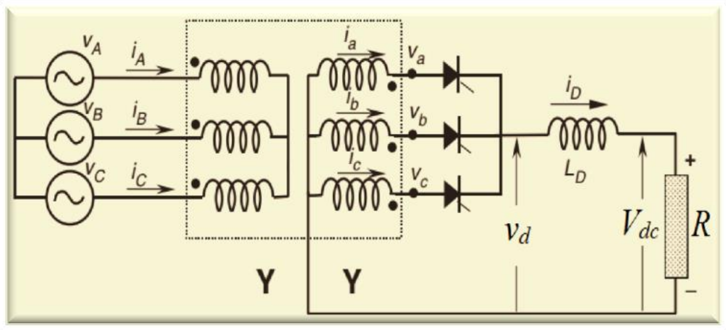
Controlled Three Phase Half Wave Rectifiers



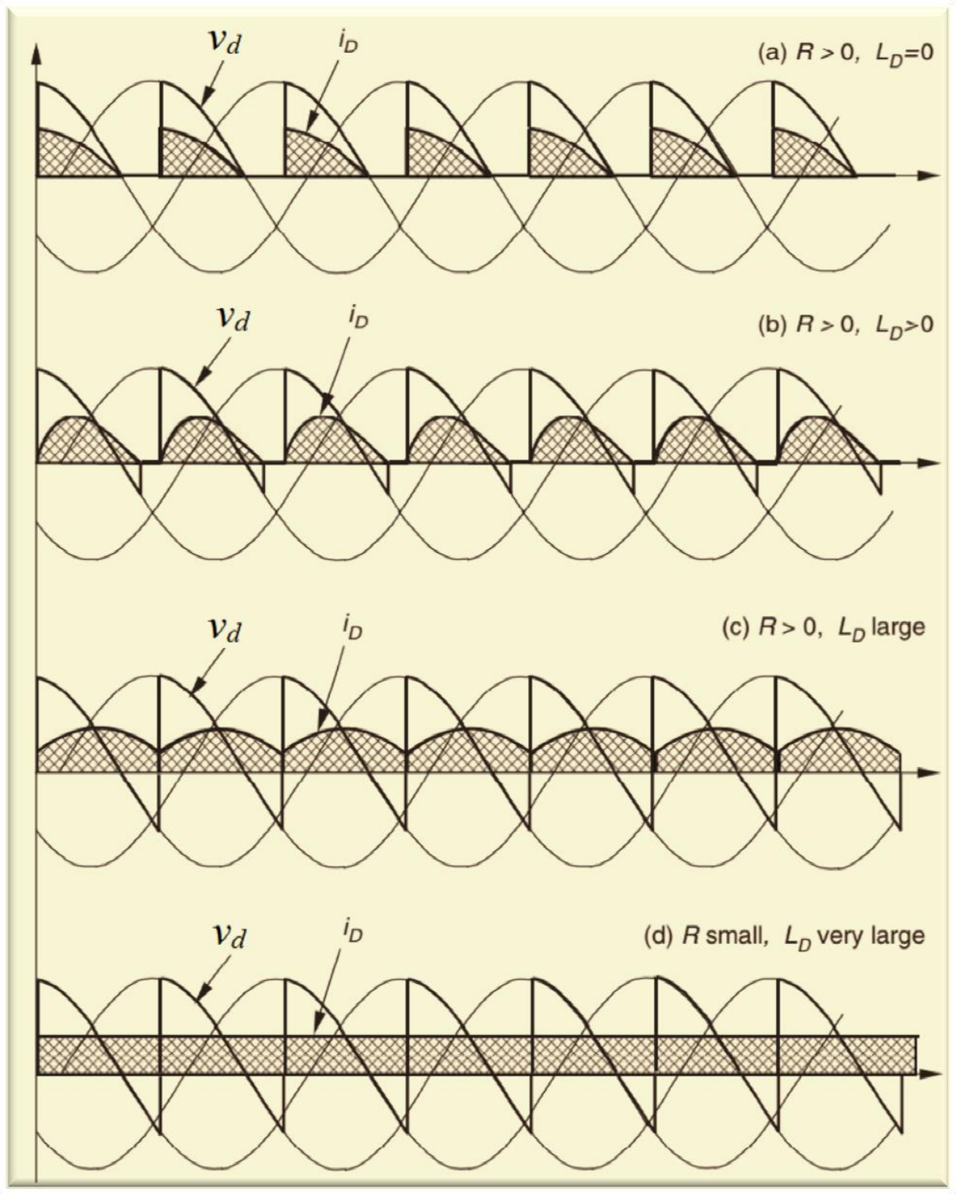
- The thyristor will conduct (ON state), when the anode-to-cathode voltage is positive and a firing current pulse is applied to the gate terminal. Delaying the firing pulse by an angle α controls the load voltage.

- The possible range for gating delay is between α = 0◦ and α = 180◦, but because of commutation problems in actual situations, the maximum firing angle is limited to around 160◦.

Controlled Three Phase Half Wave Rectifiers



- When the load is resistive, current id has the same waveform of the load voltage. As the load becomes more and more inductive, the current flattens and finally becomes constant. The thyristor goes to the non-conducting condition (OFF state) when the following thyristor is switched ON, or the current, tries to reach a negative value.



Controlled Three Phase Half Wave Rectifiers

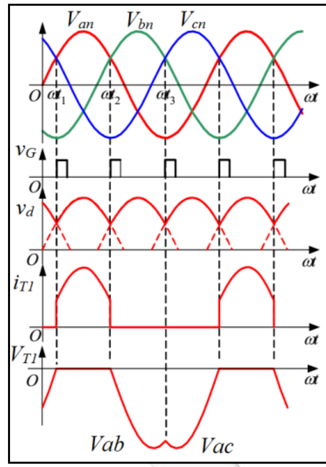
Continuous & Dicscontinuous Conduction in Three-Phase Controlled Rectifier

For resistive load

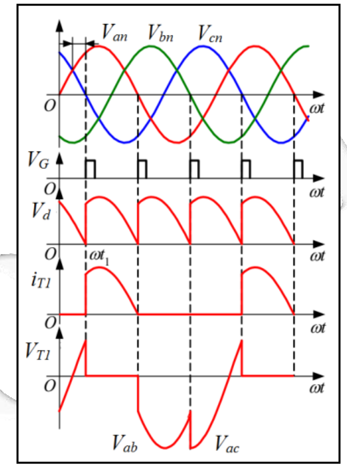
- 0°<=α<=30º, output voltage is continuous.
- 30°<=α<=120º, output voltage is discontinuous and has some intervals in which output voltage is zero.
- α >150°, output voltage is zero.

For Inductive load

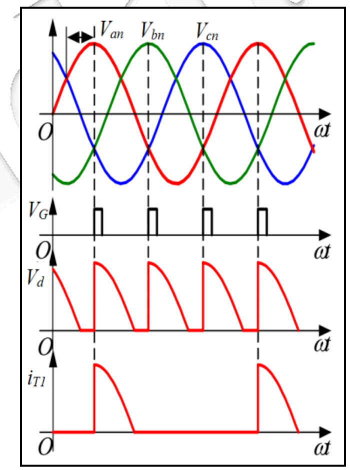
- There is no discontinuous conduction mode for three-phase controlled rectifier if L>>R.
- But if L ≈ R or firing angle is very large, discontinuities can be seen in output as output voltage can become zero in certain intervals (those intervals in which inductor has quickly dissipated its energy and firing angle hasn't reached).



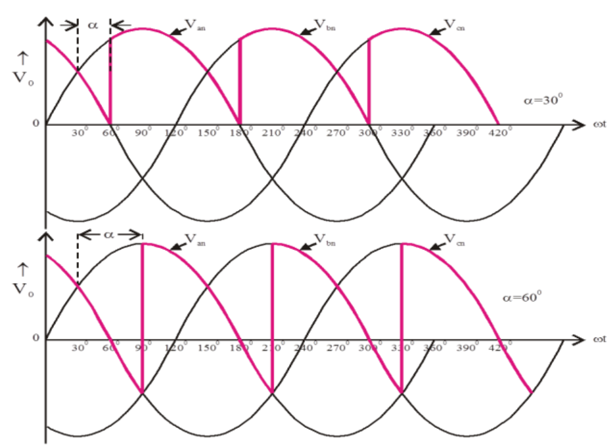
Resistive load
Firing angle α = 0º

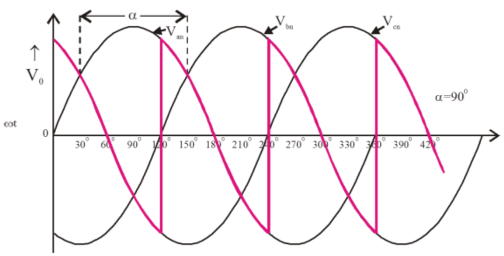


Resistive load
Firing angle α = 30º



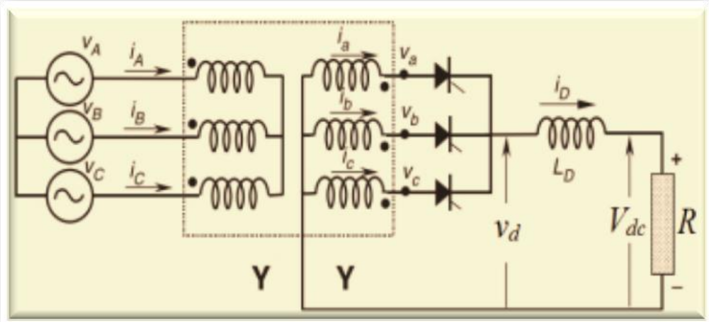
Resistive load
Firing angle 30º<=α <=150°
(α= 60º)

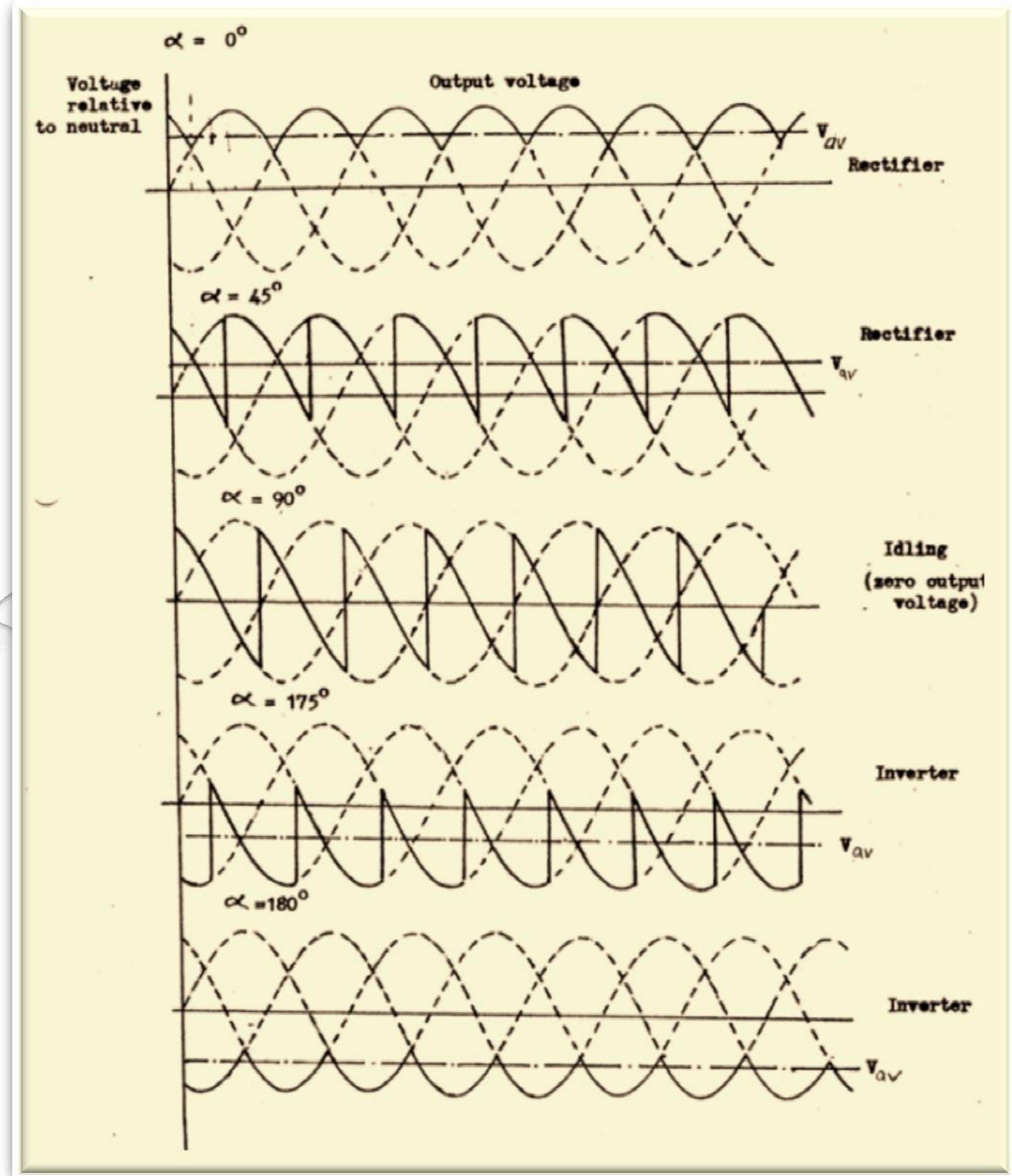




RL load

Controlled Three Phase Half Wave Rectifiers

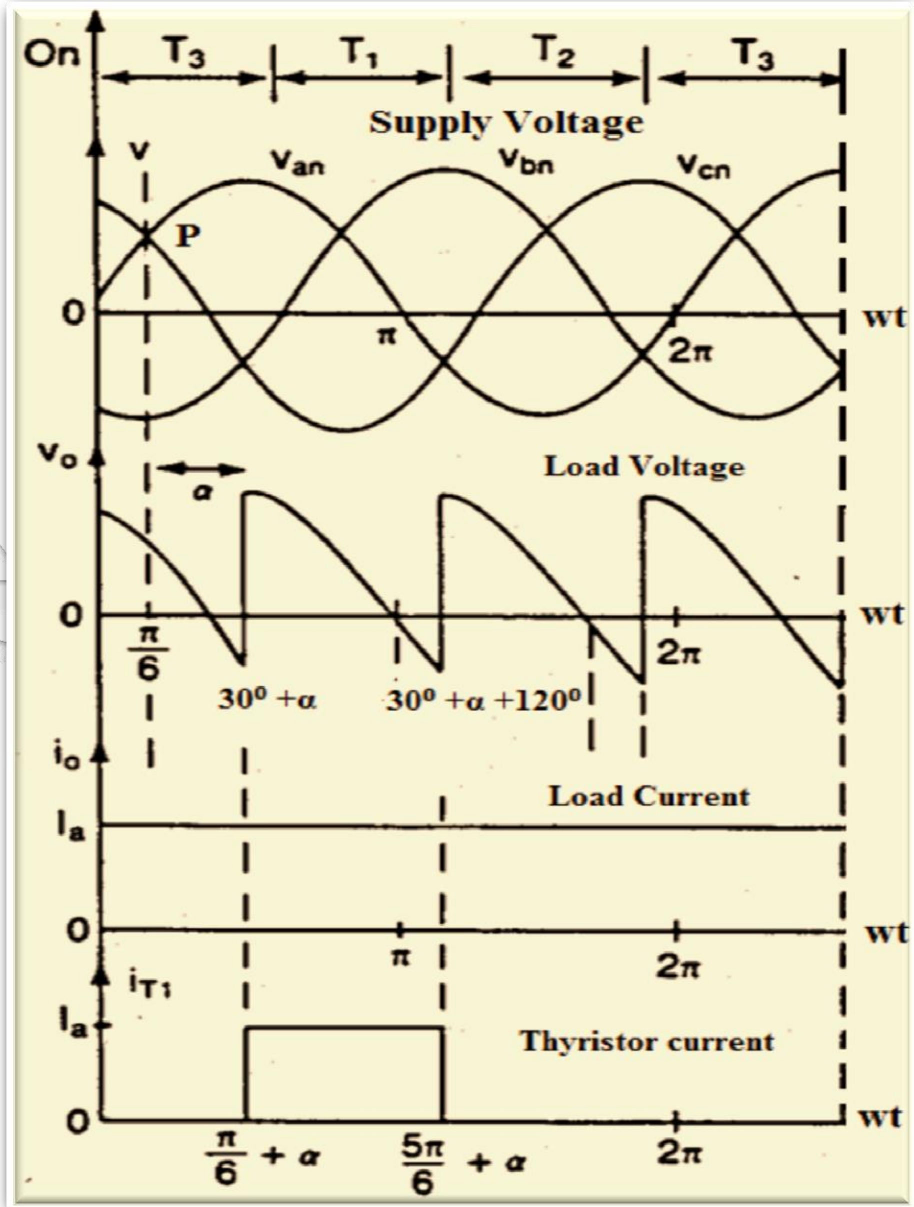




- The RL load voltage is modified by changing firing angle α. When α < 90◦, V_{dc} is positive and when α > 90◦, the average dc voltage becomes negative. In such a case, the rectifier begins to work as an inverter and the load needs to be able to generate power reversal by reversing its dc voltage.

Controlled Three Phase Half Wave Rectifiers

For RL Load



Let

$$V_{an} = V_m \sin\omega t$$

$$V_{bn} = V_m \sin(\omega t - 2\pi/3)$$

$$V_{cn} = V_m \sin(\omega t - 4\pi/3)$$

$$T_1 \text{ is triggered at } \omega t = \left(\frac{\pi}{6} + \alpha\right) = \left(30^0 + \alpha\right)$$

$$T_2 \text{ is triggered at } \omega t = \left(\frac{5\pi}{6} + \alpha\right) = \left(150^0 + \alpha\right)$$

$$T_3 \text{ is triggered at } \omega t = \left(\frac{7\pi}{6} + \alpha\right) = \left(270^0 + \alpha\right)$$

Each thytistor conducts for 120^0 or $\frac{2\pi}{3}$ radians

Controlled Three Phase Half Wave Rectifiers

For RL Load

Load current is always continuous. The <u>dc component</u> of the output voltage is the average value, and load current is the resistor voltage divided by resistance.

$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \, sin\omega t \; d\omega t = \frac{3\sqrt{3}V_m}{2\pi} cos\,\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R} cos\,\alpha$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} (V_m sin\omega t)^2 d\omega t} = \sqrt{3}V_m\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\pi}cos2\alpha}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2+(\omega L)^2}} = \frac{\sqrt{3}V_m}{\sqrt{R^2+(\omega L)^2}}\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\pi}cos2\alpha}$$

Controlled Three Phase Half Wave Rectifiers

For Resistive Load

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T_1 is triggered at ωt=(30°+α) and T_1 conducts up to ωt=180°. When the phase supply voltage decreases to zero, the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from ωt=(30° + α) to (180°).

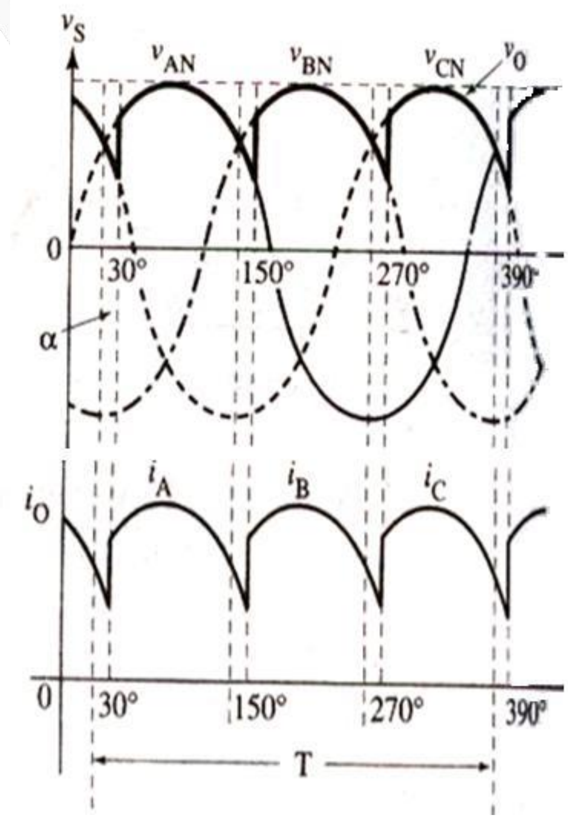
1- *when α≤30°*

$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t \, d\omega t = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R}\cos\alpha$$

$$V_{rms} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}(V_m \sin\omega t)^2 d\omega t} = \sqrt{3}V_m\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$

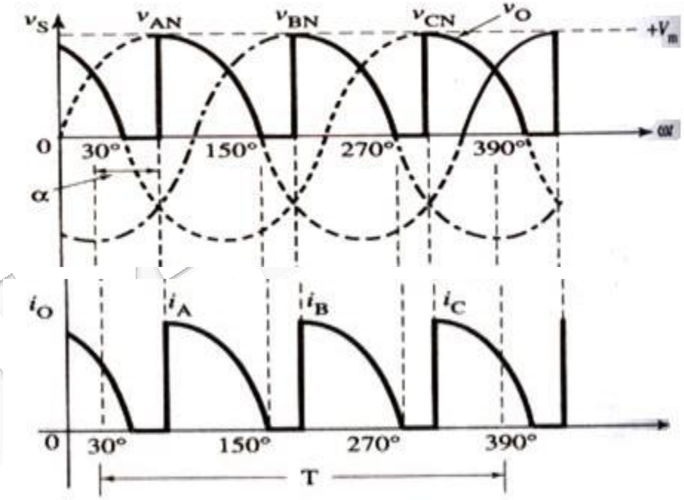
$$I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{3}V_m}{R}\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$



Controlled Three Phase Half Wave Rectifiers

For Resistive Load

2- *when α≥30º*



$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m\, sin\omega t\; d\omega t = \frac{3V_m}{2\pi}\left(1+\cos\left(\frac{\pi}{6}+\alpha\right)\right)$$

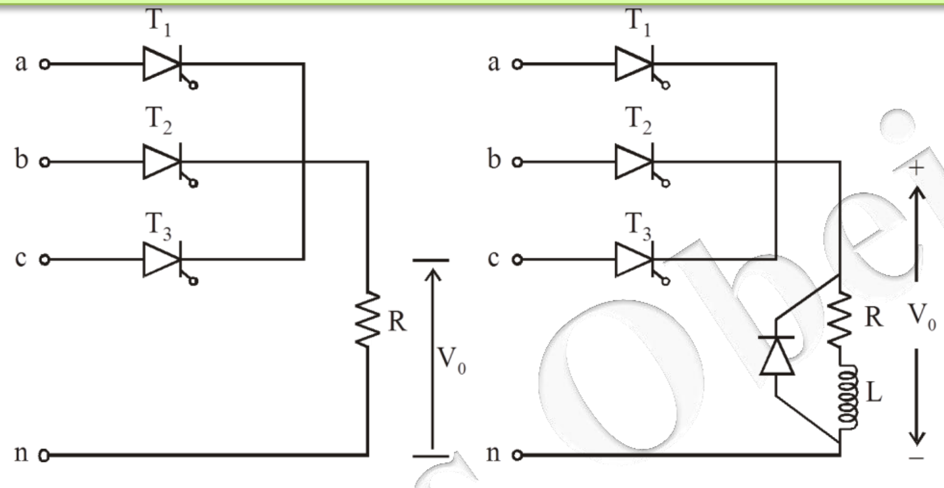
$$I_{dc} = \frac{V_{dc}}{R} = \frac{3V_m}{2\pi R}\left(1+\cos\left(\frac{\pi}{6}+\alpha\right)\right)$$

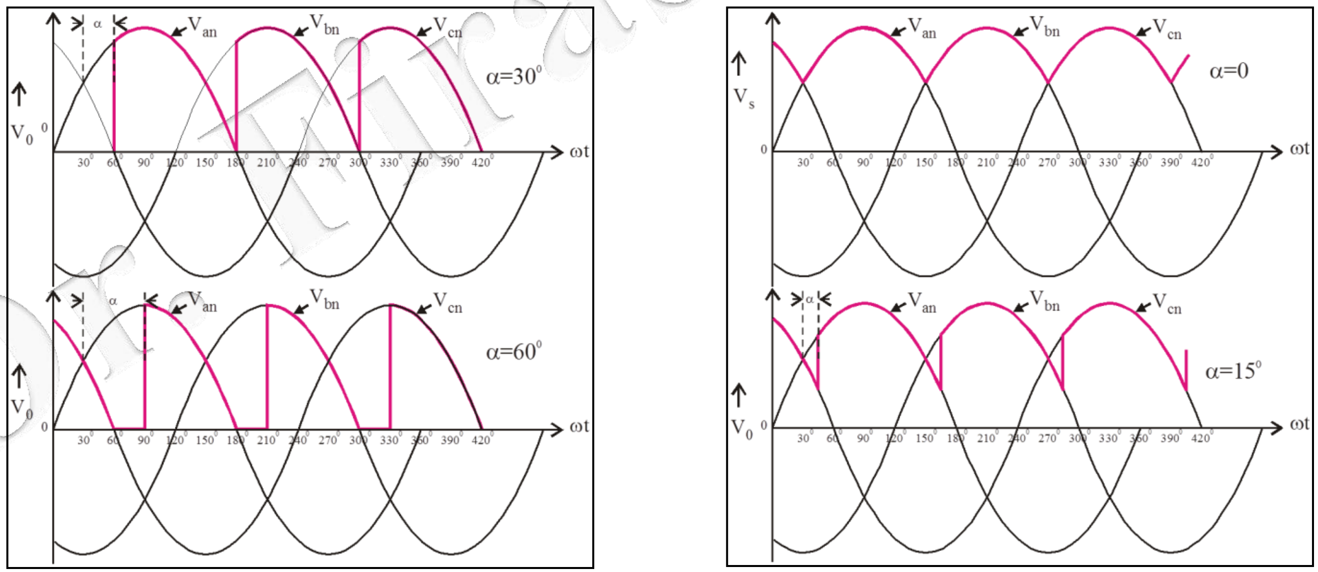
$$V_{rms} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} (V_m sin\omega t)^2 d\omega t} = \sqrt{\frac{3V_m{}^2}{4\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} (1-cos2\omega t)d\omega t}$$

$$V_{rms} = V_m\sqrt{\frac{3}{4\pi}\left(\frac{5\pi}{6}-\alpha+\frac{1}{2}\sin\left(\frac{\pi}{3}+2\alpha\right)\right)} =$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R}\sqrt{\frac{3}{4\pi}\left(\frac{5\pi}{6}-\alpha+\frac{1}{2}\sin\left(\frac{\pi}{3}+2\alpha\right)\right)}$$

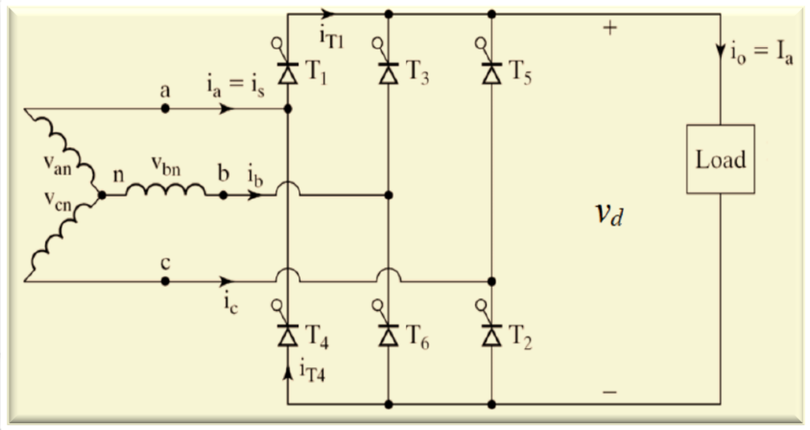
Controlled Three Phase Half Wave Rectifiers with Freewheeling Diode





Controlled Three Phase Full Wave Rectifiers

- Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals.



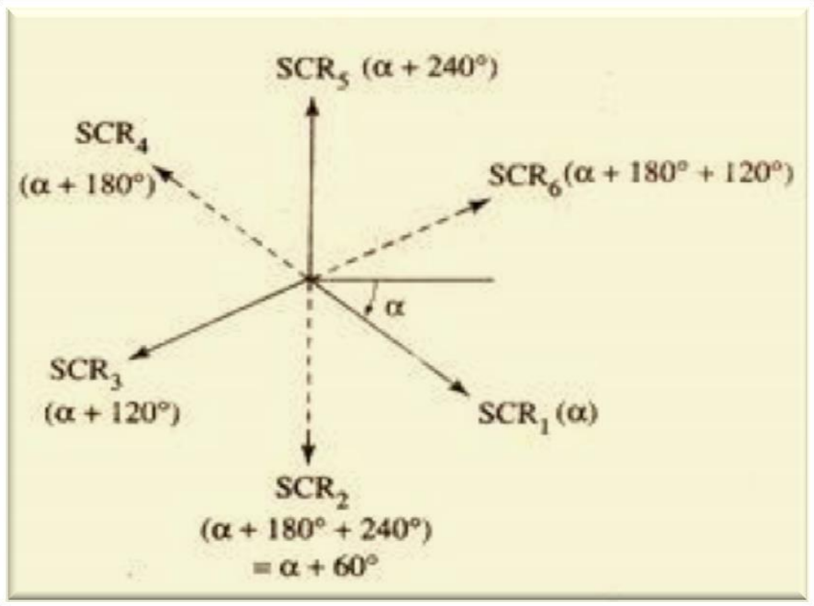
- The three thyristors (T_1,T_3 andT_5) will not work together at the same time or two of them also will not work together at the same time.
- The three thyristors (T_2,T_4 andT_6) will not work together at the same time or two of them also will not work together at the same time.
- (T_1 and T_4), (T_3 and T_6) or (T_5 and T_2) will not work together at the same time.
- Each thyristor is triggered at an interval of $2\pi/3$.
- Each thyristors pair ((T_6&T_1), (T_1&T_2), (T_2&T_3), (T_3&T_4), (T_4&T_5), (T_5&T_6)) is triggered at an interval of $\pi/3$.
- The frequency of output ripple voltage is $6f_S$.

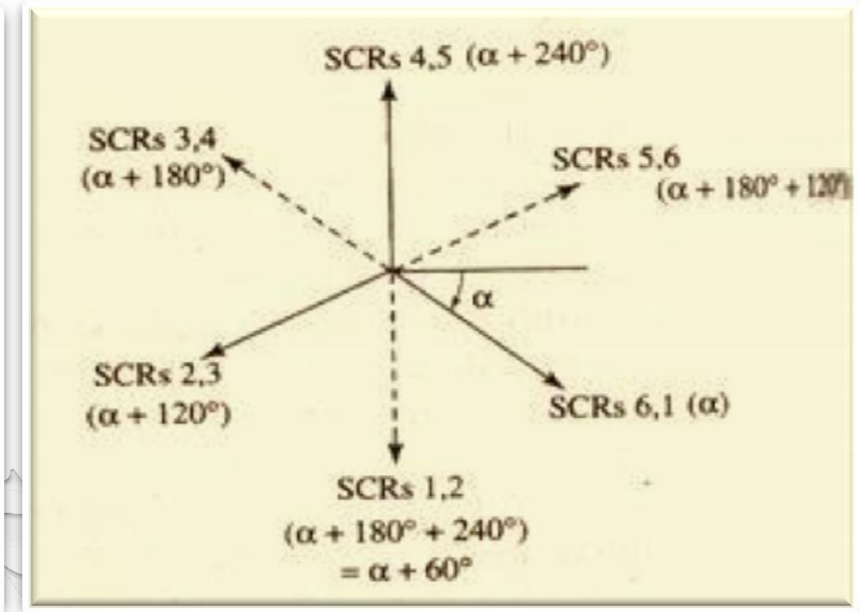
Controlled Three Phase Full Wave Rectifiers

- **If T_1 is triggered at (30 + α), T_3 will be triggered at (30 + α+120) and T_5 will be triggered at (30 + α+240). T_4 will be triggered at (30 + α+180), T_6 will be triggered at (30 + α+120+180) and T_2 will be triggered at (30 + α+240+180).**

Firing Angle	T_1	T_2	T_3	T_4	T_5	T_6
0º	30º	90	150º	210	270º	330
30º	60º	120º	180º	240º	300º	360º
60º	90º	150º	210º	270º	330º	390º
90º	120º	180º	240º	300º	360º	420º

Controlled Three Phase Full Wave Rectifiers





Single-Pulse Firing Scheme ***Double-Pulse Firing Scheme***

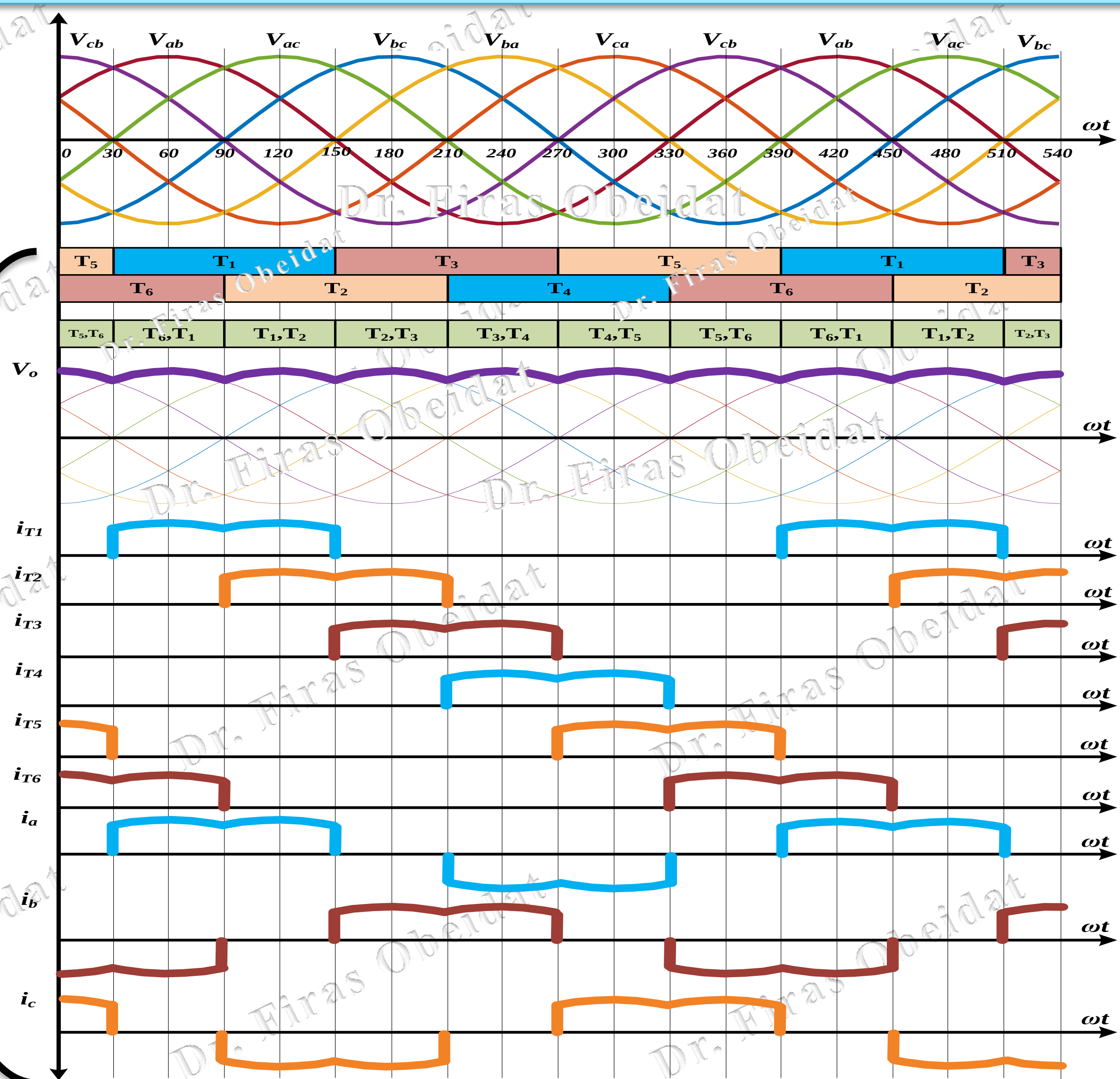
- **Thyristors are numbered in the order in which they are triggered.**
- **The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34, ……**

Controlled Three Phase Full Wave Rectifiers

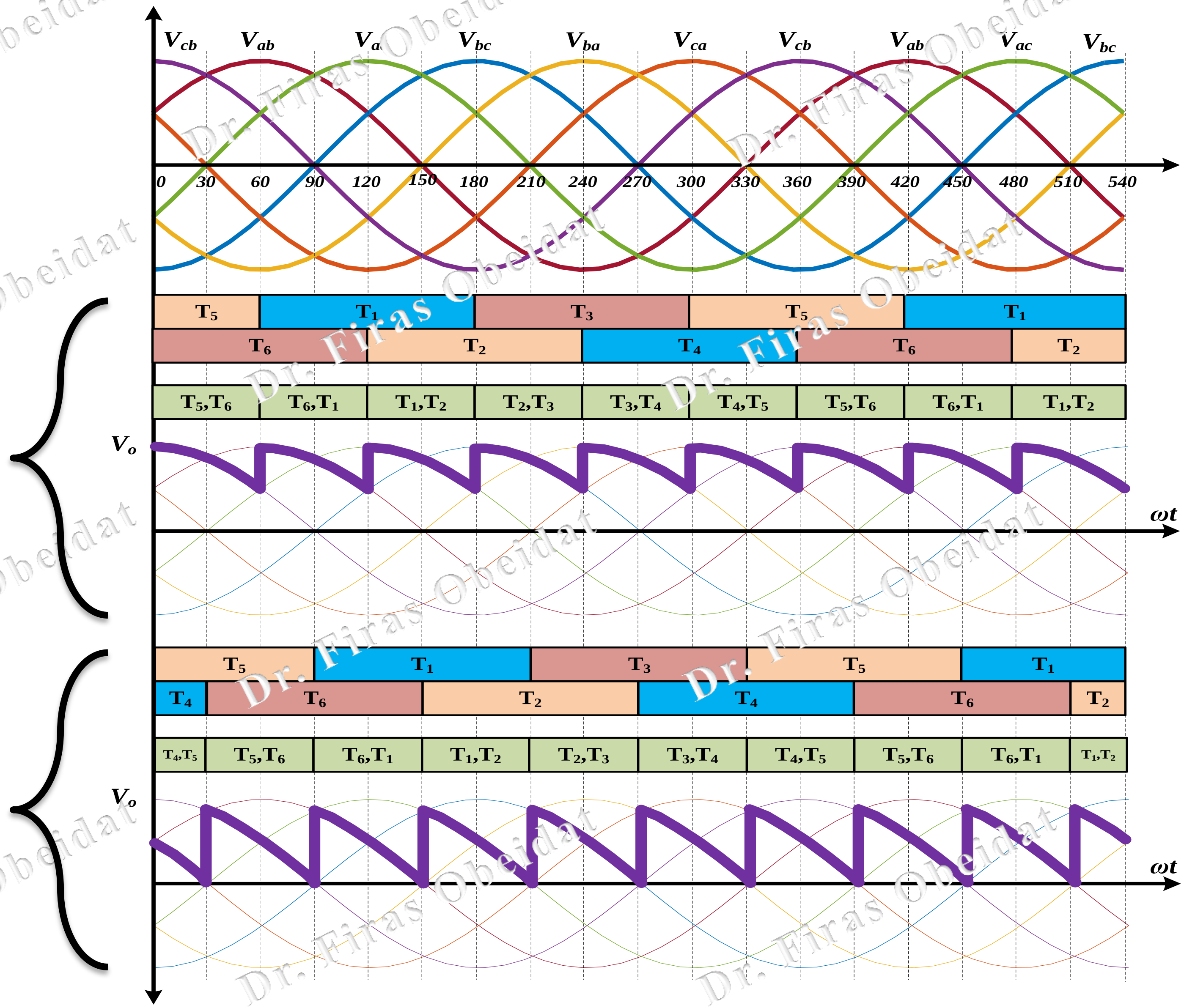
- **T_1 is triggered at $\omega t = (30 + \alpha)$, T_6 is already conducting when T_1 is turned ON.**
- **During the interval $(30 + \alpha)$ to $(90 + \alpha)$, T_1 and T_6 conduct together & the output load voltage is equal to $v_o = v_{ab} = (v_{an} - v_{bn})$.**
- **T_2 is triggered at $\omega t = (90 + \alpha)$, T_6 turns off naturally as it is reverse biased as soon as T_2 is triggered. During the interval $(90 + \alpha)$ to $(150 + \alpha)$, T_1 and T_2 conduct together & the output load voltage $v_o = v_{ac} = (v_{an} - v_{cn})$.**
- **T_3 is triggered at $\omega t = (150 + \alpha)$, T_1 turns off naturally as it is reverse biased as soon as T_3 is triggered. During the interval $(150 + \alpha)$ to $(210 + \alpha)$, T_2 and T_3 conduct together & the output load voltage $v_o = v_{bc} = (v_{bn} - v_{cn})$.**
- **T_4 is triggered at $\omega t = (210 + \alpha)$, T_2 turns off naturally as it is reverse biased as soon as T_4 is triggered. During the interval $(210 + \alpha)$ to $(270 + \alpha)$, T_3 and T_4 conduct together & the output load voltage $v_o = v_{ba} = (v_{bn} - v_{an})$.**
- **T_5 is triggered at $\omega t = (270 + \alpha)$, T_3 turns off naturally as it is reverse biased as soon as T_5 is triggered. During the interval $(270 + \alpha)$ to $(230 + \alpha)$, T_4 and T_5 conduct together & the output load voltage $v_o = v_{ca} = (v_{cn} - v_{an})$.**
- **T_6 is triggered at $\omega t = (330 + \alpha)$, T_4 turns off naturally as it is reverse biased as soon as T_6 is triggered. During the interval $(330 + \alpha)$ to $(390 + \alpha)$, T_5 and T_6 conduct together & the output load voltage $v_o = v_{cb} = (v_{cn} - v_{bn})$.**

Controlled Three Phase Full Wave Rectifiers

When α=0



Controlled Three Phase Full Wave Rectifiers

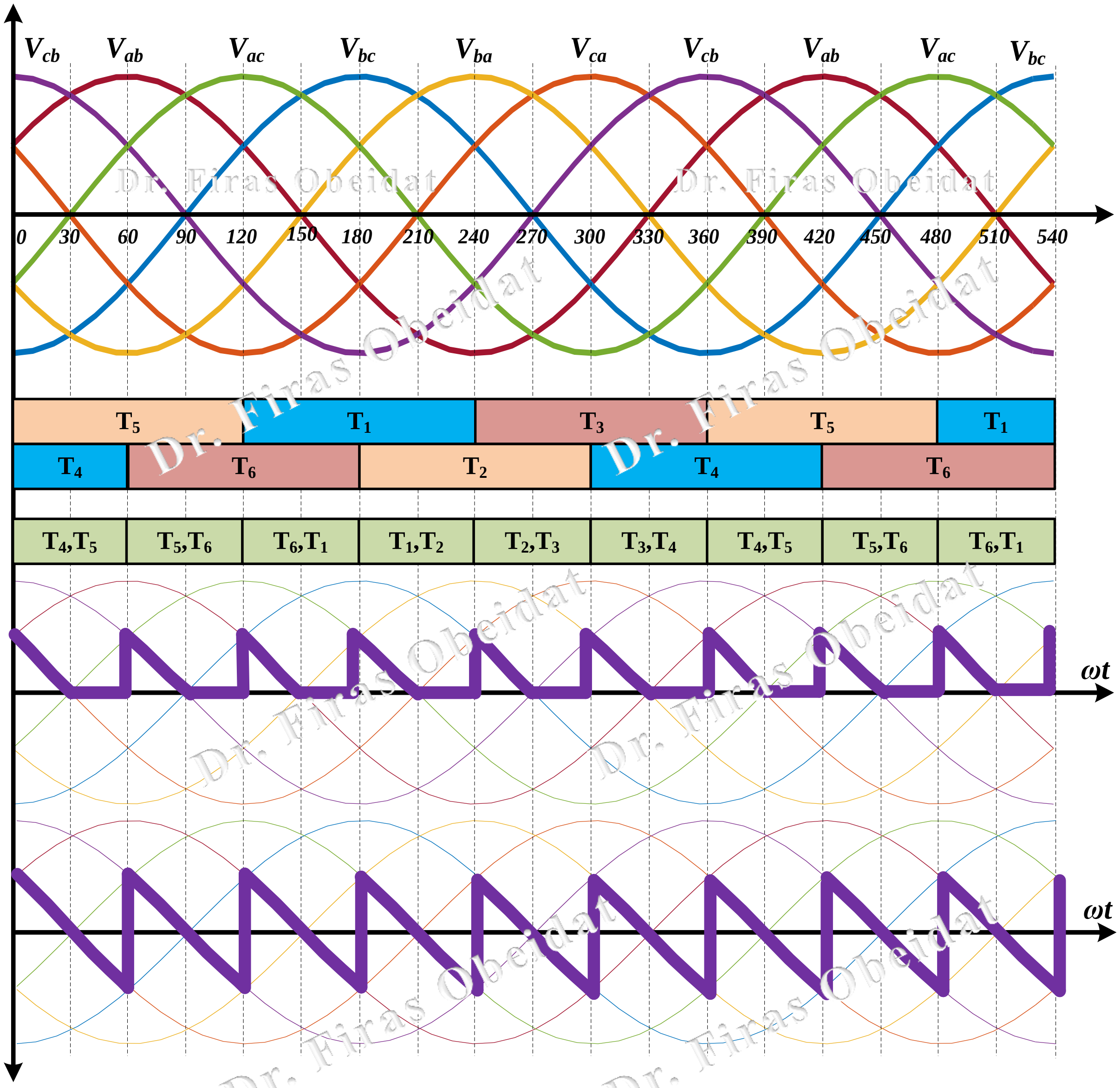


When α=30

When α=60

Controlled Three Phase Full Wave Rectifiers

When α=90

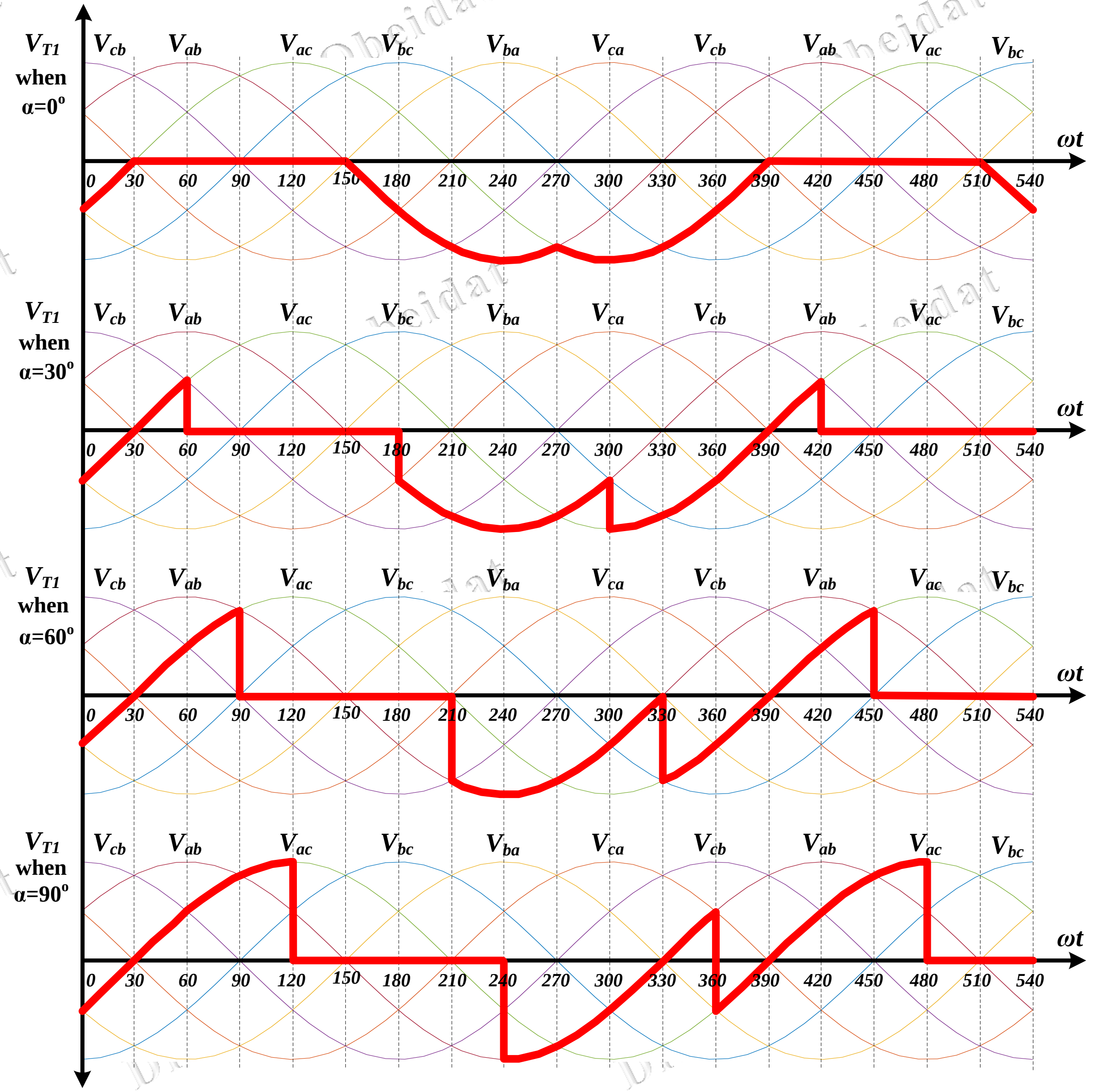


Output Voltage when α =90 for <u>Resistive load</u>

Output Voltage when α =90 for <u>RL load</u>

Controlled Three Phase Full Wave Rectifiers

Thyristor one (T_1) voltage for different firing angles.



Controlled Three Phase Full Wave Rectifiers

Let

$V_{an} = V_m \sin\omega t \qquad V_{bn} = V_m \sin(\omega t\text{-}2\pi/3) \qquad V_{bn} = V_m \sin(\omega t\text{-}4\pi/3)$

$V_{ab} = \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) \qquad V_{bc} = \sqrt{3}V_m \sin(\omega t - \frac{\pi}{2}) \qquad V_{ca} = \sqrt{3}V_m \sin(\omega t - \frac{7\pi}{6})$

The <u>dc component</u> of the output voltage and current can be found as

$$V_{dc} = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})d\omega t = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R}\cos\alpha$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha}\left(\sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})\right)^2 d\omega t} = \sqrt{3}V_m\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{\sqrt{3}V_m}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha}$$

Controlled Three Phase Full Wave Rectifiers

Special case: resistive load α>60°

The <u>dc component</u> of the output voltage and current can be found as

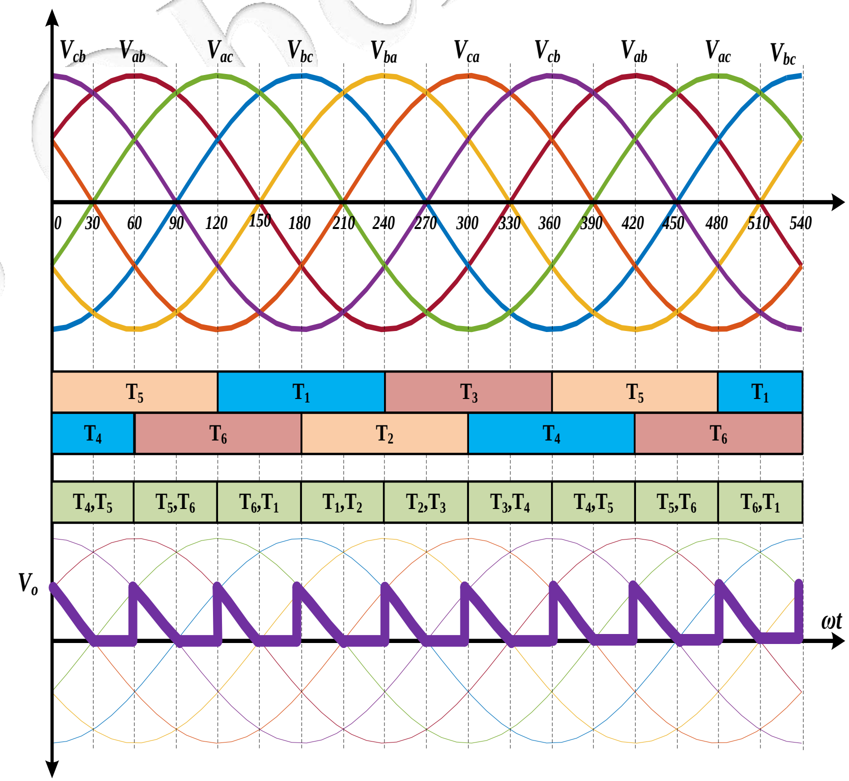
$$V_{dc}=\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi}\sqrt{3}V_m\sin(\omega t+\frac{\pi}{6})d\omega t=\frac{3\sqrt{3}V_m}{\pi}\cos(\frac{\pi}{3}+\alpha)$$

$$I_{dc}=\frac{V_{dc}}{R}=\frac{3\sqrt{3}V_m}{\pi R}cos(\frac{\pi}{3}+\alpha)$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms}=\sqrt{\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi}\left(\sqrt{3}V_m\sin(\omega t+\frac{\pi}{6})\right)^2 d\omega t}$$

$$I_{rms}=\frac{V_{rms}}{\sqrt{R^2+(\omega L)^2}}$$



α=90

Controlled Three Phase Full Wave Rectifiers

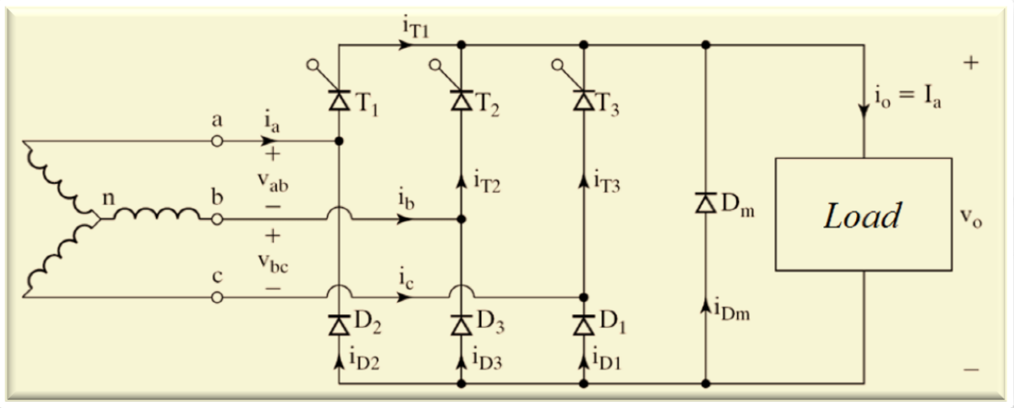
Example: A three-phase controlled rectifier has an input voltage which is 480V$_{rms}$ at 60 Hz. The load is modeled as a series resistance and inductance with R=10 Ω and L=50mH. Determine the delay angle required to produce an average current of 50 A in the load.

$$V_{dc} = I_{dc}R = 50 * 10 = 500V$$

$$\sqrt{3}V_{rms} = 480V$$

$$\alpha = cos^{-1}\left(\frac{V_{dc}\pi}{3\sqrt{3}V_m}\right) = cos^{-1}\left(\frac{500\pi}{3\sqrt{2}480}\right) = 39.5^o$$

Three Phase Full Wave Half Controlled Rectifiers



- **3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.**
- **The power factor of 3-phase semi-converter decreases as the trigger angle α increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.**

Three Phase Full Wave Half Controlled Rectifiers

- Thyristor T_1 is forward biased when the phase supply voltage v_{an} is positive and greater than the other phase voltages v_{bn} and v_{cn}. The diode D1 is forward biased when the phase supply voltage v_{cn} is more negative than the other phase supply voltages.

- Thyristor T_2 is forward biased when the phase supply voltage v_{bn} is positive and greater than the other phase voltages. Diode D_2 is forward biased when the phase supply voltage van is more negative than the other phase supply voltages.

- Thyristor T_3 is forward biased when the phase supply voltage v_{cn} is positive and greater than the other phase voltages. Diode D_3 is forward biased when the phase supply voltage v_{bn} is more negative than the other phase supply voltages.

- The frequency of the output supply waveform is $3f_S$, where f_S is the input ac supply frequency. The trigger angle α can be varied from 0 to 180^o.

Three Phase Full Wave Half Controlled Rectifiers

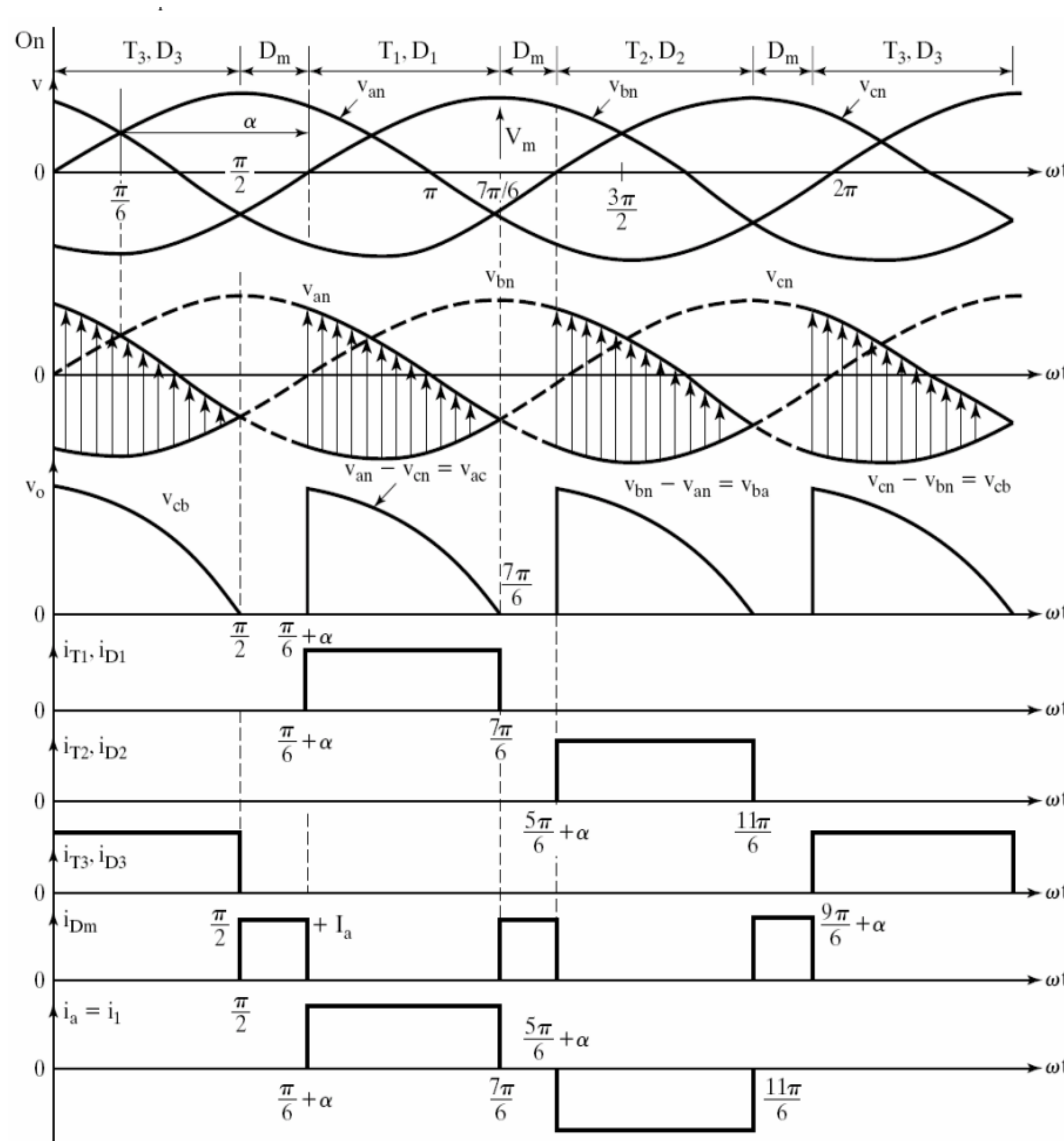
For $\alpha > 60^o$

- During the time period $\pi/6 \leq \omega t \leq 7\pi/6$ (i.e. $30^o \leq \omega t \leq 210^o$) thyristor T_1 is forward biased. If T_1 is triggered at $\omega t = \pi/6 + \alpha$, T_1 and D_1 conduct together and the line to line supply voltage v_{ac} appears across the load. At $\omega t = 7\pi/6$, v_{ac} starts to become negative and the free wheeling diode D_m turns on and conducts. The load current continues to flow through the free wheeling diode D_m and thyristor T_1 and diode D_1 are turned off.

- If the free wheeling diode D_m is not connected across the load, then T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = 5\pi/6 + \alpha$ and the free wheeling action is accomplished through T_1 and D_2, when D_2 turns on as soon as v_{an} becomes more negative at $\omega t = 7\pi/6$.

Three Phase Full Wave Half Controlled Rectifiers

Waveforms for α=90º

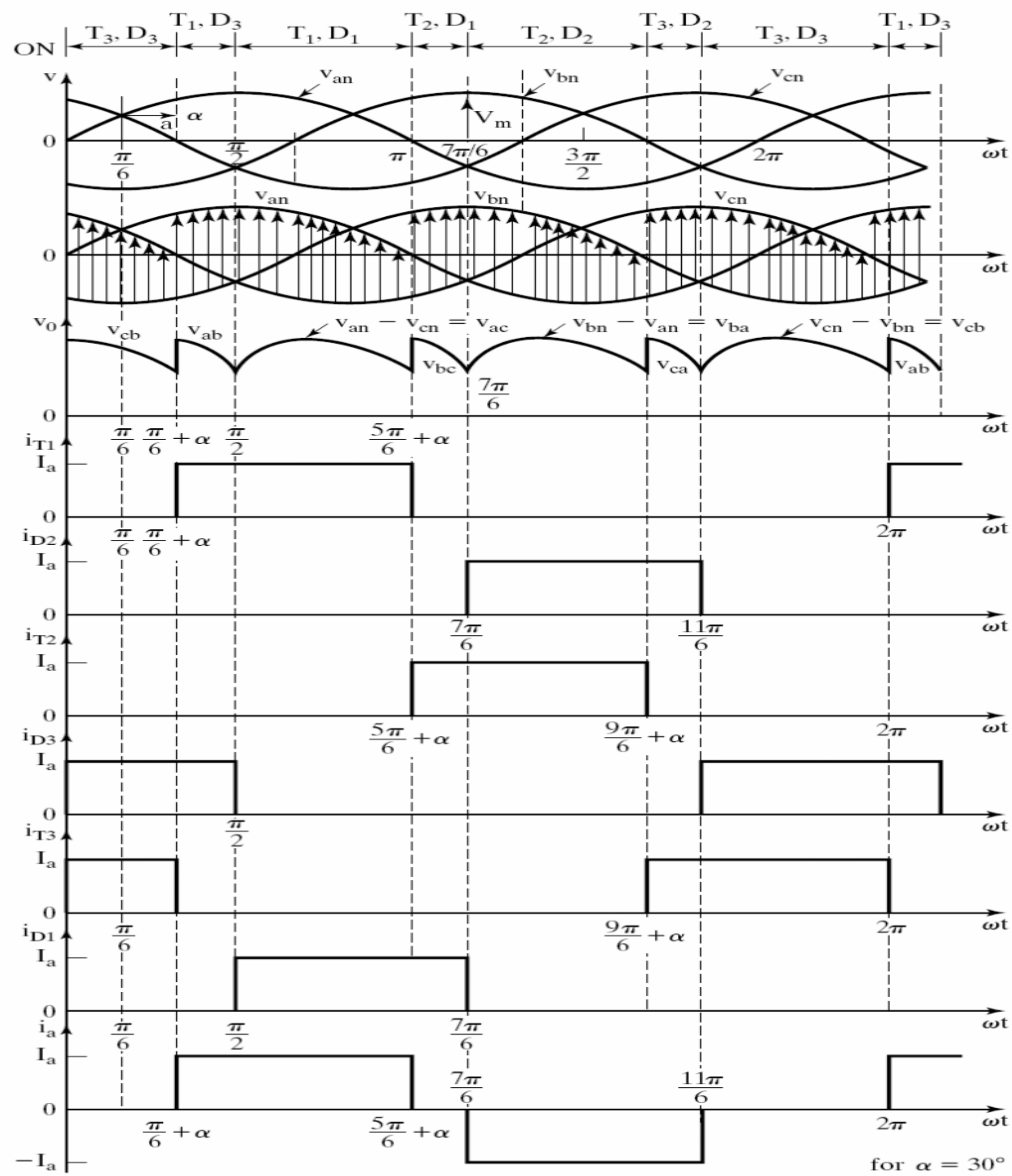


Three Phase Full Wave Half Controlled Rectifiers

For α<60º

If the trigger angle $\alpha \le \pi/3$ each thyristor conducts for $2\pi/3$ and the free wheeling diode D_m does not conduct.

Waveforms for α=30º

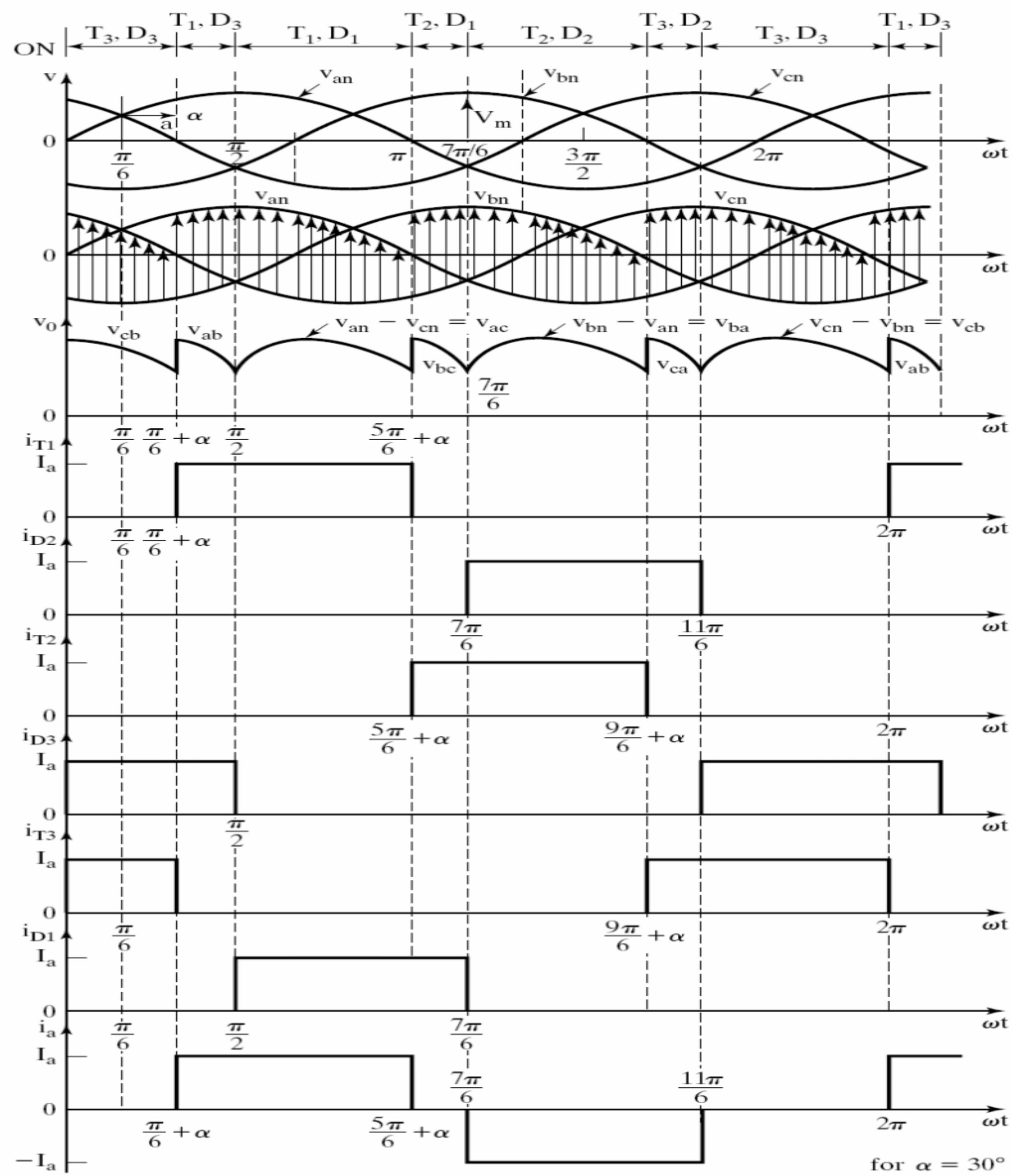


Three Phase Full Wave Half Controlled Rectifiers

For α<60º

If the trigger angle α≤π/3 each thyristor conducts for 2π/3 and the free wheeling diode D_m does not conduct.

Waveforms for α=30º



Three Phase Full Wave Half Controlled Rectifiers

Let $V_{an} = V_m \sin\omega t$ $V_{bn} = V_m \sin(\omega t\text{-}2\pi/3)$ $V_{bn} = V_m \sin(\omega t\text{-}4\pi/3)$

For α>60º and Discontinuous Output Voltage

$$V_o = V_{ac} = \sqrt{3}V_m \sin(\omega t - \frac{\pi}{6})$$

The <u>dc component</u> of the output voltage and current can be found as

$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \sqrt{3}V_m \sin(\omega t - \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R}(1 + \cos\alpha)$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \left(\sqrt{3}V_m \sin(\omega t - \frac{\pi}{6})\right)^2 d\omega t} = \frac{3V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{3V_m}{2\sqrt{R^2 + (\omega L)^2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}$$

Three Phase Full Wave Half Controlled Rectifiers

For α≤60º and Continuous Output Voltage

$$V_o = V_{ab} = \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})$$

The <u>dc component</u> of the output voltage and current can be found as

$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6}) d\omega t = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R}(1 + \cos\alpha)$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} (V_{ab})^2 d\omega t + \int_{\frac{\pi}{62}}^{\frac{5\pi}{6}} (V_{ac})^2 d\omega t\right]} = \frac{3V_m}{2}\sqrt{\frac{2}{3} + \frac{\sqrt{3}(\cos\alpha)^{\,2}}{\pi}}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{3V_m}{2\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{2}{3} + \frac{\sqrt{3}(\cos\alpha)^{\,2}}{\pi}}$$


Q
&
A