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Computer Logic Design

By

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Chapter 1:
Digital Computers and digital systems

What is digital logic?

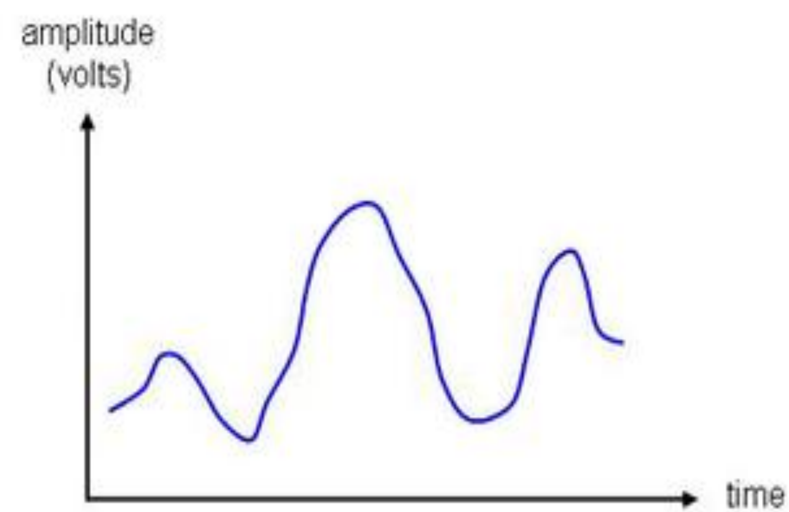
- Most modern electronic devices such as mobile telephones and computers depend on digital electronics. In fact, most electronics about the home and in industry depend on digital electronics to work.
- Generally a name given to electronic circuits that use discrete voltage levels and modeled by logic expressions. These circuits are the building blocks of computers and other devices.

Signals

- signals are used to transmit information, usually through electric signals.
- Two types : **Analog** and **digital.**
- In both these technologies, the information, such as any audio or video, is transformed into electric signals.

Analog signal

- An **Analog signal** is continuous signal for which the time varying feature of the signal is a representation of some other time varying quantity.
- Microphones and speaker are perfect examples of analog devices.
- **Analog technology** is cheaper but there is a limitation of size of data that can be transmitted at a given time.



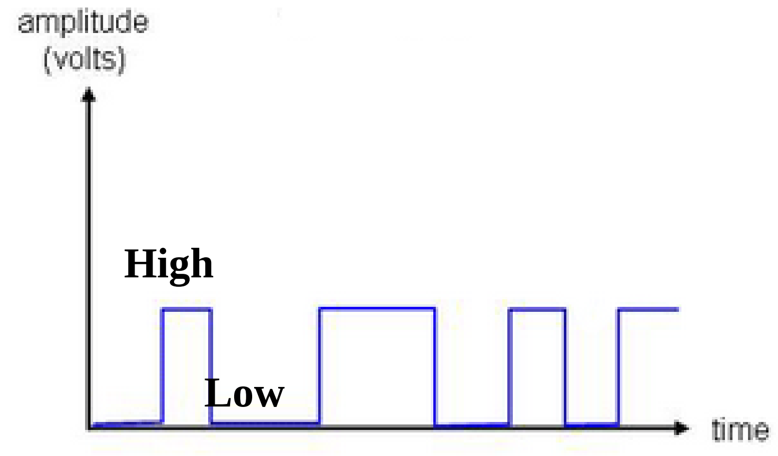
digital signal

- A digital signal uses discrete (discontinuous) values.
- **In Digital technology** Data is converted into binary code and then reassembled back into original form at reception point.
- Examples: Computers, CDs, DVDs.
- Digital systems use digital circuits that process digital signals which can take on one of two values, we call:

  0 and 1 (digits of the binary number system)

  or LOW and HIGH

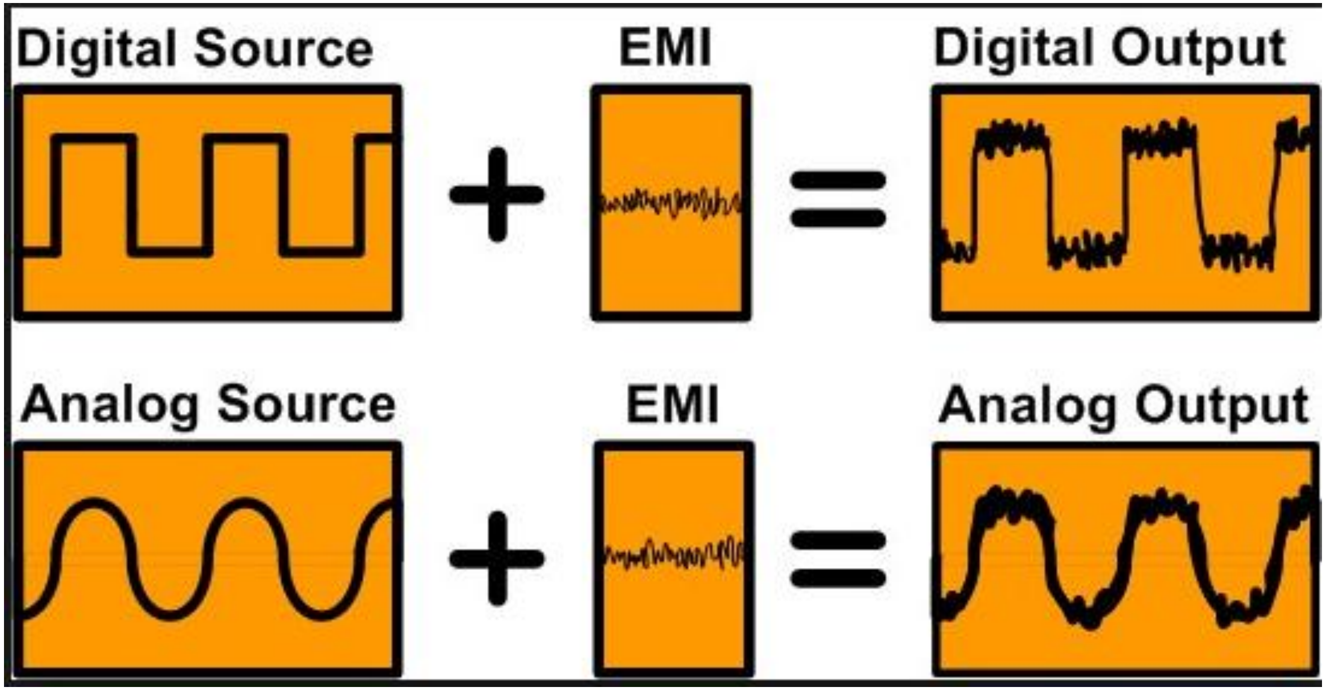
  or FALSE and TRUE



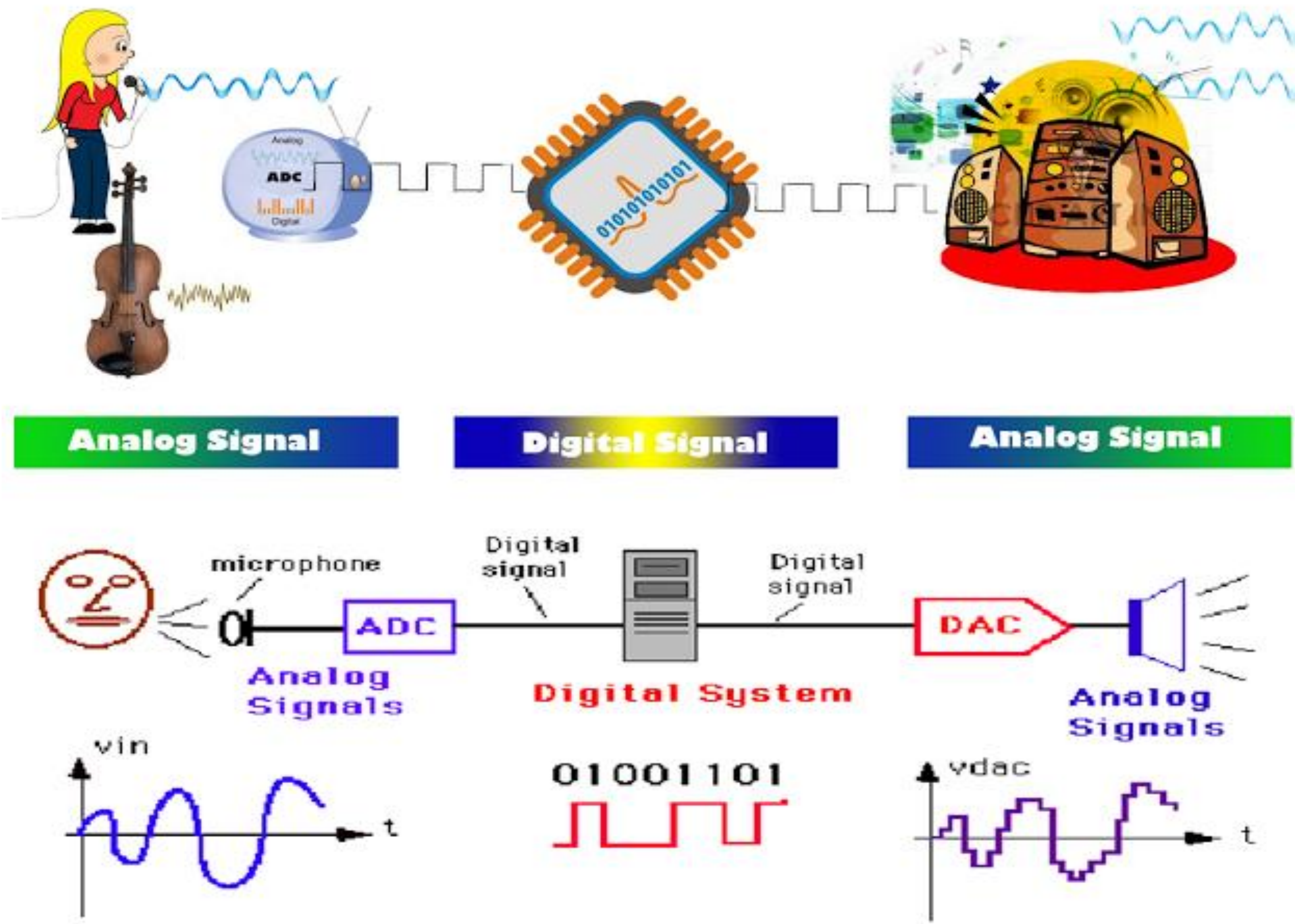
Advantages of Digital Systems Over Analog Systems

1. Digital can be encrypted so that only the intended receiver can decode it .
2. Accuracy of results
3. More reliable than analog systems due to better immunity to noise.
4. Ease of design: No special math skills needed to visualize the behavior of small digital (logic) circuits.
5. Flexibility and functionality.
6. Speed: A digital logic element can produce an output in less than 10 nanoseconds (10^{-8} seconds).
7. Economy: Digital signals can be processed by digital circuit components, which are cheap and easily produced in many components on a single chip.

Digital immunity to noise.



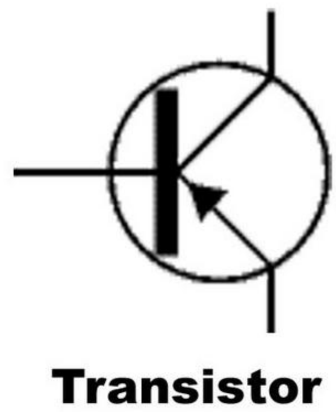
From Analog to digital

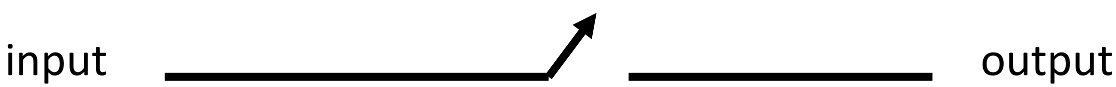


Computers as Digital Systems

- Microprocessors contain several functional and control units.
- Logic gates make up functional and control units
- Transistors are used to build logic gates.
  - **Transistor** **(** is a semiconductor device used to amplify and switch electronic signals and electrical power) Basic building block: the transistor = “on/off switch”
    - Digital signals – voltage levels high/low

Transistor





An ideal switch

Numbering Systems

Number based conversion

- A number system consists of an order set of symbols (digits) with relations defined for +,-,*, /
- The radix (or base) of the number system is the total number of digits allowed in the number system.
  - **Example, for the decimal number system:**
    - Radix, r = 10, Digits allowed = 0,1, 2, 3, 4, 5, 6, 7, 8, 9
- In positional number systems, a number is represented by a string of digits, where each digit position has an associated weight.
- The value of a number is the weighted sum of the digits.
- The general representation of an unsigned number D with whole and fraction portions number in a number system with radix r:

$$D_r = d_{p-1}\, d_{p-2} \ldots\ldots d_1\, d_0 . d_{-1}\, d_{-2} \ldots\ldots D_{-n}$$

- The number above has p digits to the left of the radix point and n fraction digits to the right.
- A digit in position i has as associated weight r^i
- The value of the number is the sum of the digits multiplied by the associated weight r^i :

$$D = \sum_{i=-n}^{p-1} d_i \times r^i$$

Number Systems Used in Computers

Name of Radix	Radix	Set of Digits	Example
Decimal	r=10	{0,1,2,3,4,5,6,7,8,9}	$(255)_{10}$
Binary	r=2	{0,1}	$(11111111)_2$
Octal	r= 8	{0,1,2,3,4,5,6,7}	$(377)_8$
Hexadecimal	r=16	{0,1,2,3,4,5,6,7,8,9,A, B, C, D, E, F}	$(FF)_{16}$

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

Decimal Numbers

A decimal number such as 7392 represents a quantity equal to 7 thousands plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc., are powers of 10 implied by the position of the coefficients. To be more exact, 7392 should be written as

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

The decimal number system is said to be of *base*, or *radix*, 10 because it uses 10 digits and the coefficients are multiplied by powers of 10.

How to Convert

From

Any Number Based System

To

Decimal

Binary numbers

- Digits: a binary digit representing a 0 or a 1.
- Binary numbers are base 2 as opposed to base 10 typically used.
- Instead of decimal places such as 1s, 10s, 100s, 1000s, etc., binary uses powers of two to have 1s, 2s, 4s, 8s, 16s, 32s, 64s, etc.

Binary to Decimal Conversion

- $(\mathbf{101})_2 = (1\times2^2)+(0\times2^1)+(1\times2^0)=4+0+1=(5)_{10}$
- $(\mathbf{10111})_2=(1\times2^4)+(0\times2^3)+(1\times2^2)+(1\times2^1)+(1\times2^0)=(23)_{10}$
- $(\mathbf{11010.11})_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = (26.75)_{10}$
- $(1010.011)_2 = 2^3 + 2^1 + 2^{-2} + 2^{-3} = (10.375)_{10}$

Octal Numbers

- Octal numbers are numbers of base 8.
- Octal numbers uses powers of 8.
- Digits: Octal digit representing {0, 1, 2, 3, 4, 5, 6, 7}.

Octal to Decimal Conversion

- a number expressed in base r can be converted to its decimal equivalent by multiplying each coefficient with the corresponding power of r and adding. The following is an example of octal to decimal conversion:

$$(630.4)_8 = 6 \times 8^2 + 3 \times 8 + 4 \times 8^{-1} = (408.5)_{10}$$

Hexadecimal Numbers

- Hex numbers are numbers of base 16.
- Hex numbers uses powers of 16.
- Digits: Hex digit representing :

  {0, 1, 2, 3, 4, 5, 6, 7,8,9,A,B,C,D,E,F}.

Hexadecimal to Decimal Conversion

Examples:

- $\mathbf{(B65F)_{16}} = B\times 16^3 + 6\times16^2 + 5\times16^1 + F\times16^0$

  $= 11\times 16^3 + 6\times16^2 + 5\times16^1 + 15\times16^0 = (56687)_{10}$
- $\mathbf{(2AF3)_{16}} = 2\times 16^3 + A\times16^2 + F\times16^1 + 3\times16^0$

  $= 2\times 16^3 + 10\times16^2 + 15\times16^1 + 3\times16^0 = (10995)_{10}$

The same rule for any base number

- **Convert the following number to Decimal:**

An example of a base 5 number is:

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

Note that coefficient values for base 5 can be only 0, 1, 2, 3, and 4.

It is customary to borrow the needed *r* digits for the coefficients from the decimal system when the base of the number is less than 10. The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10. For example, in the *hexadecimal* (base 16) number system, the first 10 digits are borrowed from the decimal system. The letters *A*, *B*, *C*, *D*, *E*, and *F* are used for digits 10, 11, 12, 13, 14, and 15, respectively.

How to Convert

From

Decimal

To

Any Number Based System

Convert From

Decimal

To

Binary

Decimal-to-Binary Conversion

- Separate the decimal number into whole and fraction portions.
- To convert the whole number portion to binary, use successive division by 2 if there is a reminder put 1, if no reminder put 0.. until the quotient is 0. The remainders form the answer, with the first remainder as the *least significant bit (LSB)* and the last as the *most significant bit (MSB)*.
- Example: Convert $(179)_{10}$ to binary:

179	
89	1
44	1
22	0
11	0
5	1
2	1
1	0
0	1

$= (10110011)_2$

Decimal-to-Binary Conversion

- Example: Convert $(41)_{10}$ to binary:

41	
20	1
10	0
5	0
2	1
1	0
0	1

$= (101001)_2$

Decimal-to-Binary Conversion

Fraction Conversion

multiplication by 2 is used instead of division, and integers are accumulated instead of remainders.

EXAMPLE: Convert $(0.6875)_{10}$ to binary.

	integer	*fraction*
0.6875 X 2 =	1	0.3750
0.3750 X 2 =	0	0.7500
0.7500 X 2 =	1	0.5000
0.5000 X 2 =	1	0.0000

We stop because it reached 0.

$= (0.1011)_2$

Decimal-to-Binary examples

Try the following:

- $(108)_{10} = (1101100)_2$
- $(90)_{10} = (1011010)_2$
- $(7)_{10} = (111)_2$
- $(11)_{10} = (1011)_2$

Convert From

Decimal

To

Octal

Decimal-to-Octal Conversion

- Separate the decimal number into whole and fraction portions.
- To convert the whole number portion to Octal, use successive division by 8 if there is a reminder put it, if no reminder put 0.. until the quotient is less than 8, if it is <8 put it to reminders taken.
- Examples: Convert $(153)_{10}$ to Octal:

153	**reminder**
19	1
2	3
	2

Less than 8, we stop

$= (231)_8$

Decimal-to-Octal Conversion

- Example: Convert $(891)_{10}$ to Octal:

891	Reminder
111	3
13	7
1	5
	1

$= (1573)_8$

$$(891)_{10} = (1573)_8$$

Convert From

Decimal

To

Hexadecimal

Decimal-to-Hex Conversion

- Separate the decimal number into whole and fraction portions.
- To convert the whole number portion to Hex, use successive division by 16 if there is a reminder put it, if no reminder put 0.. until the quotient is less than 16, if it is<16 put it to reminders taken.
- Examples: Convert $(256)_{10}$ to Hex:

256	**Reminder**
16	0
1	0
	1

= $(100)_{16}$

Decimal-to-Hex Conversion

- Examples: Convert $(2500)_{10}$ to Hex:

2500	**Reminder**
156	4
9	12 → C
	9

$= (9C4)_{16}$

$(2500)_{10} = (9C4)_{16}$

Convert From

Binary

To

Octal or Hexadecimal

Binary to Octal

1 digit in Octal corresponds to 3 digits in binary, Because $2^3=8$.

The conversion accomplished by:
partitioning the binary number into groups of three digits, each starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group.

Example: convert $(10110001101011.111100000110)_2$ to Octal.

$$(10\ 110\ 001\ 101\ 011\ .\ 111\ 100\ 000\ 110)_2 = (26153.7406)_8$$

10	110	001	101	011	.	111	100	000	110
2	6	1	5	3		7	4	0	6

Binary to Hexadecimal

1 digit in Hex corresponds to 4 digits in binary, Because $2^4=16$.

The conversion accomplished by:
partitioning the binary number into groups of Four digits, each starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group.

Example: **Convert $(10110001101011.11110010)_2$ to Hexadecimal:**

$$(10\ 1100\ 0110\ 1011\ .\ 1111\ 0010)_2 = (2C6B.F2)_{16}$$

10	1100	0110	1011	.	1111	0010
2	C	6	B	.	F	2

Convert From

Octal or Hexadecimal

To

Binary

Octal to Binary

Conversion from octal to binary is done by :
Each octal digit is converted to its three- digit binary equivalent.

Example: Convert $(673.124)_8$ to Binary.

$$(673.124)_8 = (\underbrace{110}_{6}\ \underbrace{111}_{7}\ \underbrace{011}_{3} \cdot \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{100}_{4})_2$$

Hex to Binary

Conversion from octal to binary is done by :
Each octal digit is converted to its Four - digit binary equivalent.

Example: Convert $(306.D)_{16}$ to Binary.

$$(306.\,D)_{16} = (\underbrace{0011}_{3}\,\underbrace{0000}_{0}\,\underbrace{0110}_{6} \cdot \underbrace{1101}_{D})_2$$