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Computer Logic Design

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Chapter 2:
Binary Numbers

Unsigned Binary Numbers

Complements

- Complements are used in digital computers for simplifying the subtraction operations and for logical manipulation.
- There are two types of complements for each base-r system.
  - The r's complement.
  - The (r-1)'s complement.
- When the value of the base is substituted, the two types receives the names:
  - For binary numbers: 2's and 1's complement.
  - For decimal numbers: 10's and 9's complement.

The
r's Complement

r's Complement
For Decimal Numbers:

- Given a positive number N in base r with an integer part of n digits, the r's complement of N is defined as (r^n-N) for N ≠ 0 and 0 for N=0.
- **Example: Find the r's complement for the following:**

  (in other word: find the 10's complement)
- $(52520)_{10}$ = 10^5-52520 =47480

  The number of digits in the number is n=5
- $(0.3267)_{10}$ = 1- 0.3267 = 0.6733

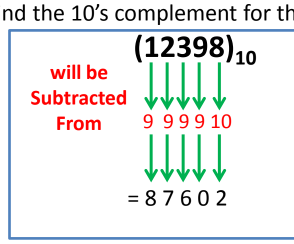
  No integer part so $10^n=10^0=1$
- $(25.639)_{10}$ = 10^2-25.639=74.361

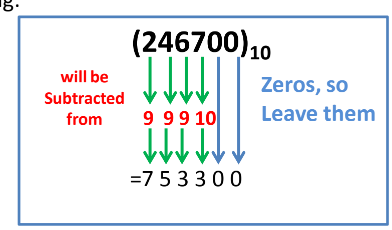
r's Complement
For Decimal Numbers

- Another way:

From the definition and the examples it is clear that the 10's complement of a decimal number can be formed by leaving all least significant zeros unchanged, subtracting the first nonzero least significant digit from 10, and then subtracting all other higher significant digits from 9.

Example: Find the 10's complement for the following:





r's Complement
For Binary Numbers

- Given a positive number N in base r with an integer part of n digits, the r's complement of N is defined as $(r^n\text{-}N)$.

- **Example: Find the r's complement for the following:**

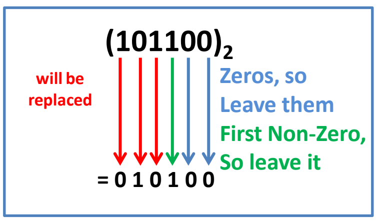
  (in other word: find the 2's complement)
- $(101100)_2 = (2^6)_{10} - (101100)_2 = (1000000 - 101100) = 010100$
- $(0.0110)_2 = (1-0.0110)_2 = 0.1010$

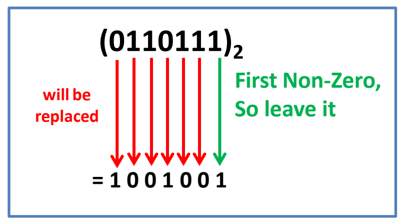
r's Complement
For Binary Numbers

- **Another way:**

2's complement can be formed by leaving all least significant zeros and the first nonzero digit unchanged, and then replacing 1's by 0's and 0's by 1's in all other higher significant digits.

- **Example:** Find the 2's complement for following:





The (r-1)'s Complement

(r-1)'s Complement
For Decimal Numbers

- Given a positive number N in base r with an integer part of n digits, the (r-1)'s complement of N is defined ($r^n - r^{-m} - N$).
- **Example:** Find the (r-1)'s complement for the following:

  (in other word: find the 9's complement)
- **$(52520)_{10}$** = (10^5-1-52520)=99999-52520=47479.

  No fraction party, $10^{-m}=10^0=1$
- **$(0.3267)_{10}$** = (1-10^{-4}-0.3267)=0.9999-0.3267=0.6732

  No integer part, so 10n=10^0=1.
- **$(25.639)_{10}$** =(10^2-10^{-3}-25.639)=99.999-25.639=74.360

(r-1)'s Complement
For Decimal Numbers

- Another way:

9's complement of a decimal number is formed simply by subtracting every digit from 9.

- **Example:** Find the 9's complement for following:

$(52520)_{10}$

will be Subtracted From

9 9 9 9 9

= 4 7 4 7 9

(r-1)'s Complement
For Binary Numbers

- Given a positive number N in base r with an integer part of n digits, the (r-1)'s complement of N is defined $(r^n - r^{-m} - N)$.

- **Example:** Find the (r-1)'s complement for the following:
  (in other word: find the 1's complement)

- $\mathbf{(101100)_2}$ **=** $(2^6-1) - (101100) = (111111-101100) = 010011$
- $\mathbf{(0.0110)_2}$ $= (1-2^{-4})_{10} - (0.0110)\ 2 = (0.1111-0.0110)_2 = 0.1001$

(r-1)'s Complement
For Binary Numbers

- Another way:

The 1's complement of a binary number is even simpler to form: the 1's are changed into 0's and the 0's into 1's.

- **Example:** Find the 1's complement for following:

$\mathbf{(101100)_2}$

Will be replaced

= 0 1 0 0 1 1

Obtaining (r's) complement from (r-1)'s complement

- It is sometimes convenient to use (r-1)'s complement when the (r's) complement is desired.
- From the definitions and a comparison of the results obtained in the examples, it follows that the (r's) complement can be obtained from the (r-1)'s complement after the addition of (r^{-m}) to the least significant digit.
- For example, the 2's complement of (10110100) is obtained from the 1's complement (01001011) by adding 1 to give (01001100).

Subtraction with r's complement

Subtraction with r's complement

- The direct method of subtraction taught in elementary schools uses the borrow concept.
- In this method, we borrow a 1 from a higher significant position when the minuend digit is smaller than the corresponding subtrahend digit.
- This seems to be easiest when people perform subtraction with paper and pencil. When subtraction is implemented by means of digital components, this method is found to be less efficient than the method that uses complements and addition as stated below:
- The subtraction of two positive numbers (M-N), both of base r, may be done as follows:

Subtraction with 10's complement

1) Add the minuend M to the r's complement of the subtrahend N.
2) Inspect the result obtained in step 1 for an end carry:
   - if there is an end carry discard it.
   - If No end carry then take the r's complement of the number obtained in step 1 and place a negative sign in front.

- Example: subtract (72532-3250) using r's complement :

$$\begin{array}{r} 72532 \\ 03250\ - \\ \hline \end{array} \quad \xrightarrow{\text{r's complement}} \quad \begin{array}{r} 72532 \\ 96750\ + \\ \hline 1\ /\ 69282 \end{array}$$

End carry,
Discard it.

Answer = 69282

Subtraction with 10's complement

- Example: subtract (3250-72532) using r's complement :

```
03250  ------------------->  03250
72532 -  r's complement      27468  +
      ------------------->  --------
No End carry, ---->  0  /  30718
                             |
                             | Find r's complement
                             v
                           69282
                             | Add negative sign
                             v
                          - 69282
```

Answer = - 69282

Subtraction with 2's complement

Use 2's complement to perform subtraction with the given binary numbers.

- 1010100 - 1000100

```
1010100  ----------------->  1010100
1000100 -  2's complement    0111100  +
         ----------------->  -------
End carry->  1  /  0010000
```

So, Answer= 0010000

- 1000100- 1010100

```
1000100  ----------------->  1000100
1010100 -  2's complement    0101100  +
         ----------------->  -------
No carry  0  /  1110000
```

So, Answer= 2's complement of (1110000)= -0010000

Subtraction with (r-1)'s complement

Subtraction with (r-1)'s complement

- The procedure for subtraction with the (r-1)'s complement is exactly the same as oe variation, called, "end-around carry" as shown below. The subtraction of M-N, both positive numbers in base r, may be calculated in the following manner.

1) Add the minuend M to the (r-1)'s complement of the subtrahend N.
2) Inspect the result obtained in step 1 for an end carry
   - if there is end carry occurs, add 1 to the least significant digit (end-around carry)
   - if No end carry, take the (r-1)'s complement of the number obtained in step 1 and place a negative sign in front.

Subtraction with (9)'s complement

- Example: subtract (72532-3250) using 9's complement :

```
  72532   ------------------->    72532
  03250 -   9's complement   ->   96749  +
                                 -------
End carry  ->  1               / 69281
                    Add it  ->       1  +
                                 -------
                                  69282
```

Answer= 69282

Subtraction with (9)'s complement

- Example: subtract (3250-72532) using 9's complement :

```
  03250   ------------------->    03250
  72532 -   9's complement   ->   27467  +
                                 -------
No End carry  ->  0            / 30717
                                    | Find 9's complement
                                  69282
                                    | Add negative sign
                                - 69282
```

Answer= -69282

Subtraction with (1)'s complement

- Example: subtract (1010100 - 1000100) using 1's complement :

```
1010100  ------------------->   1010100
1000100 -   1's complement  ->  0111011 +
                              -----------
            End carry ->  1 /   0001111
                  Add it ---->        1 +
                              -----------
                                0010000
```

Answer= 10000

Subtraction with (1)'s complement

- Example: subtract (3250-72532) using 1's complement :

```
1000100  ------------------->   1000100 +
1010100 -   1's complement  ->  0101011
                              -----------
         No End carry ->  0 /   1101111
                                   |  Find 1's complement
                                   v
                                0010000
                                   |  Add negative sign
                                   v
                              - 0010000
```

Answer= - 0010000

Signed Binary Numbers

Signed Binary Numbers

- To represent negative integers, we need a notation for negative values.
- It is customary to represent the sign with a bit placed in the leftmost position of the number since binary digits.
- The convention is to make the sign bit 0 for positive and 1 for negative.
- Three methods are the sign/magnitude representation , the 1's
- complement and the 2's complement method of representation.
- Example: to represent the signed number (-9)

Signed-magnitude representation:	10001001
Signed-1's-complement representation:	11110110
Signed-2's-complement representation:	11110111

Signed Binary Numbers

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Signed Binary Numbers addition

- Arithmetic addition
  - The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. **If the signs are the same**, we add the two magnitudes and give the sum the common sign. **If the signs are different**, we subtract the smaller magnitude from the larger and give the difference the sign if the larger magnitude.
  - The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits.
  - A carry out of the sign-bit position is discarded.
- Example:

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+ 19	00010011	+ 7	00000111
+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	− 19	11101101

Signed Binary Numbers
Subtraction

- Arithmetic Subtraction
  - In 2's-complement form:

1. Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including sign bit).
2. A carry out of sign-bit position is discarded.

$$(\pm A)-(+B)=(\pm A)+(-B)$$
$$(\pm A)-(-B)=(\pm A)+(+B)$$

- Example:

(– 6) – (– 13) → (11111010 – 11110011)

→ (11111010 + 00001101)

→ 00000111 (+ 7)